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lens for five minutes, that the thermometer might assume the temperature of its situation. The result was as follows :

The lens covered	0'	No. 3.	} Here, in one minute, the thermometer received
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	I	176	

112 degrees of heat, which came with the coloured part of the solar spectrum, and were refracted to a focus; so that, if the coloured rays themselves are not of a heat-making nature, they are at least accompanied with rays that have a power of heating bodies, and are subject to certain laws of refraction, which cannot differ much from those affecting light.

[To be continued.]

VII. *An Examination of ST. PIERRE's Hypothesis respecting the Cause of the Tides, which, in opposition to the received Theory, attributes them to supposed periodical Effusions of the Polar Ices.* By SAMUEL WOODS, Esq. Read before the Askejian Society November 5, 1799.

THE tides are two periodical motions actuating the ocean (called the flux and reflux, or ebb and flow), which succeed each other alternately at an interval of about six hours; the period of a flux and reflux being, upon an average, 12 hours 24 minutes, the double of which, 24 hours 48 minutes, corresponds to that of a lunar day, or the time elapsing between the moon's passing a meridian and coming to it again. These alternate elevations and depressions of the ocean so exactly correspond with the course of the sun and moon, as to time and quantity, that the influence of those luminaries has in all ages been considered as the cause of their production; but it was reserved for modern times to ascertain the principle of their laws, and to calculate, with precision, the effects produced by the different situations of the sun and moon, and the proportions of their power. This principle is no other than gravitation. It is evident that, if the earth were entirely fluid and quiescent, its particles, by their mutual gravity, would form the whole mass into a perfect sphere: now, if any power be supposed to act on all the particles of this sphere with

with equal force, and in parallel directions, the whole mass would be moved together without experiencing any alteration in its figure. But this is not the case with respect to the moon's action on our globe: the power of gravity diminishes as the square of the distance increases, and therefore the waters (at Z, Plate V. fig. 1.) on the side of the earth (A,B,C,D) next the moon (M), are more attracted by the moon than the central parts of the earth (O), and the central parts more attracted than the waters on the opposite side (at n); and therefore the distance between the earth's centre and the waters on its surface under and opposite to the moon will be increased. For, suppose three bodies in the same line (H,O,D), if they are all equally attracted by any power (as M), they will all move towards it with equal rapidity, their mutual distances continuing the same; but if the attraction of this power (M) is unequal, the body most forcibly attracted will move fastest, and their reciprocal distances will be proportionally increased: thus, the power of gravitation acting unequally on the three bodies (H,O,D), the distance of the first (H) from the second (O), and of the second from the third (D), will be increased in proportion to the difference of the gravitating power at the distance of the three bodies (H,O,D) respectively: now, suppose a number of bodies (A,B,C,D) placed round the centre (O) so as to form a fluid ring, unequally attracted by some power (M), the parts nearest and furthest (H and D) from this power will have their distance from the centre (O) increased, while the sides of this ring (B and F), being nearly equidistant from the power (M), the centre (O) will not recede, but rather approach the centre (O), and form an ellipsis (nLIZN). To apply this reasoning to the case under consideration, while the earth, by its gravity, tends toward the moon, the water directly below her will swell and rise gradually; the water on the opposite side will recede from the centre (or, more properly, the centre will advance), and rise, or appear to rise, while the water at the sides is depressed, and falls below the former level: hence, as the earth revolves on its axis from the moon to the moon again in 24 hours 50 minutes, there will be two tides of ebb and two of flood in that period. In consequence

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sequence of the earth's motion on her axis, the most elevated part of the water is carried beyond the moon in the direction of the rotation, and continues to rise after it has passed directly under the moon, not attaining its greatest elevation till it has got about half a quadrant further. It continues also to descend, after it has passed at 90° distance from the point below the moon, to a like distance of about half a quadrant; and therefore in open seas, where the water flows freely, the time of high water does not exactly coincide with the time of the moon's coming to the meridian, but is some time after. Besides, the tides do not always answer to the same distance of the moon from the meridian, since they are variously affected by the sun's action, which brings them on sooner when the moon is in her first and third quarters, and keeps them back later when she is in her second and fourth: because, in the former case, the tide raised by the sun alone would be earlier than the tide raised by the moon: in the latter case, later.

We have hitherto considered the moon as the principal agent in producing tides, but it is obvious that the inequality of the sun's action must produce a similar effect; so that, in reality, there are two tides every natural day occasioned by the sun, as well as two tides every lunar day occasioned by the moon, and subject to the same laws: on account, however, of the sun's immense distance, his action is considerably inferior to that of the moon. By comparing the spring and neap tides at the mouth of the Avon, below Bristol, Sir Isaac Newton calculates the proportion of the moon's force to the sun's as 9 to 2 nearly. Dr. Horsley, in his edition of the *Principia*, estimates it as 5,0469 to 1; and, considering the elevation of the waters by this force as an effect similar to the elevation of the equatorial above the polar parts of the earth, it will be found that the moon is capable of producing an elevation of about ten feet, the sun of about two feet; which corresponds pretty nearly to experience.

In order to understand the cause of spring and neap tides, we must consider, that the moon, revolving round the earth in an elliptic orbit, approaches nearer and recedes further from it, than her mean distance, in every revolution or lunar month. When nearest, her attraction is strongest, and *vice versa*:

versâ: when both luminaries are in the equator, and the moon in perigeo, the tides rise highest, particularly at opposition and conjunction: at the change, when the attractive forces of the sun and moon are combined, the tide is raised to a greater height: at the full, when the moon raises the tide under and opposite to her, the sun, acting in the same line, raises the tides under and opposite to him, whence their conjunct effect is the same as at the change, and in both cases occasions what we call spring tides: but at the quarters, the sun's action diminishes the effect of the moon's action, so that they rise a little under and opposite the sun, and fall as much under and opposite the moon, these two luminaries acting obliquely on each other, and producing what is called neap tides*.

The spring tides, however, do not happen precisely at the full and change of the moon, nor the neap tides at the quarters, but about two days later. In this, as in many other cases, the effects are not greatest, or least, when the immediate influence of the cause is greatest or least: as, for instance, the greatest heat of summer is not at the time of the solstice, but some weeks after; and if the actions of the sun and moon should be suddenly suspended, the tides would continue for some time in their usual course. The variations of the moon's distance from the earth produce a sensible difference in the tides. When the moon approaches the earth, her action on every part increases, and the differences of her action increase in a higher proportion as the moon's distances decrease. According to Sir Isaac Newton, the tides increase as the cubes of the distances decrease; so that the moon, at half her distance, would produce tides eight times as great. The sun being nearer the earth in winter than in summer, the spring tides are highest, and the neap tides lowest, about the time of the equinoxes, a little after the autumnal and before the vernal; and, on the contrary, the spring tides lowest and the neap tides highest at the solstices, when the

* In Fig. 2. Plate V. H Z O N represents the earth; A B C D the moon's orbit. At the full and change, the sun and moon act in the same line S P: at the quarters, the sun's influence in the line S O H counteracts that of the moon acting in the direction M Z N, and produces neap tides.

sun is most distant from the equator. When the moon is in the equator, the tides are equally high in both parts of the lunar day; but as the moon declines towards either pole, the tides are alternately higher and lower at places having north or south latitude: while the sun is in the northern signs, the greater of the two diurnal tides in our climates is that arising from the moon above the horizon: when the sun is in the southern signs, the greatest is that arising from the moon below the horizon. Thus the evening tides in summer are observed to exceed the morning tides, and in winter the morning tides exceed the evening tides: the difference at Bristol is found to be 15 inches, at Plymouth 12. It would be still greater, but that a fluid always retains an impressed motion for some time, and consequently the preceding tides always affect those that follow.

If the earth were covered all over with the sea to a great depth, the tides would be regularly subservient to these laws; but various causes combine to produce a great diversity of effect, according to the peculiar situation and circumstances of places, shoals, fords, and straits: thus, a slow and imperceptible motion of a large body of water, suppose two miles deep, will be sufficient to elevate its surface ten or twelve feet in a tide's time; whereas, if the same quantity of water is forced through a narrow channel forty or fifty fathoms deep, it produces a very rapid stream, and of course the tide is found to set strongest in those places where the sea grows narrowest, the same quantity of water being constrained to pass through a smaller passage, as in the straits between Portland and Cape la Hogue in Normandy; and it would be still more so between Dover and Calais, if the tide coming round the island did not check it.

The shoalness of the sea and the intercurrent continents are the reasons why the tides in the open ocean rise but to very inconsiderable heights, when compared to what they do in wide-mouthed rivers opening in the direction of the stream of the tide; and that high water is some hours after the moon's appulse to the meridian, as it is observed upon all the western coast of Europe and Africa from Ireland to the Cape of Good Hope; in all which a south-west moon makes
high

high water; and the same is said to be the case on the western coast of America: so that tides happen to different places at all distances of the moon from the meridian, and consequently at all hours of the day.

To allow the tides their full motion, the space in which they are produced ought to extend from east to west 90° at least; such being the distance between the places most raised and depressed by the moon's influence. Hence it appears that such tides can only be produced in large oceans, and why those of the Pacific exceed those of the Atlantic ocean: hence also it is obvious why the tides in the torrid zone between Africa and America, where the ocean is narrower, are exceeded by those of the temperate zones on either side: and hence we may comprehend why the tides are so small in islands at a great distance from the shores, since the water cannot rise on one shore without descending on the other: so that at the intermediate islands it must continue at a mean height between its elevations on those shores.

The tide produced on the western coast of Europe corresponds to this theory. Thus, it is high water on the western coasts of Ireland, Spain, and Portugal, about the third hour after the moon has passed the meridian; from thence it flows into the adjacent channels, as it finds the easiest passage. One current, for example, runs up by the south of England, and another by the north of Scotland; taking considerable time to move all this way, and occasioning high water sooner in the places at which it first arrives, and begins to fall at these places while the current is proceeding to others further distant in its course. On its return it is unable to raise a tide, because the water runs faster off than it returns, till, by the propagation of a new tide from the ocean, the current is stopped, and begins to rise again. The tide propagated by the moon in the German ocean, when she is three hours past the meridian, takes twelve hours to come from thence to London bridge; so that, when it is high water there, a new tide has already attained its height in the ocean, and in some intermediate place it must be low water at the same time. When the tide runs over shoals, and flows upon flat shores, the water

is elevated to a greater height than in open and deep oceans that have steep banks, because the force of its motion is not broken upon level shores till the water has attained a greater height. If a place communicates with two oceans, or by two different openings with the same ocean, one of which affords an easier and readier passage than the other, two tides may arrive at this place in different times, which, interfering together, may produce a great variety of phænomena.

At several places it is high water three hours before the moon comes to the meridian; but that tide which the moon drives, as it were, before her, is only the tide opposite to that produced by her when nine hours past the opposite meridian.

It would be tedious to enumerate all the particular solutions easily deducible from these doctrines: as, why lakes and seas, such as the Caspian and the Mediterranean, the Euxine and the Baltic, have little or no sensible tides; since, having no communication, or being connected by very narrow inlets with the great ocean, they cannot receive or discharge water sufficient to alter their surface sensibly. In general, when the time of high water at any place is mentioned, it is to be understood on the days of new and full moon: the times of high water in any place fall at nearly the same hours after a period of about fifteen days, or between one spring tide and another.

This theory, however, is not without objections and difficulties; which has encouraged a Frenchman of some eminence, St. Pierre, to frame a new and singular hypothesis, ascribing all the phænomena of the tides to the periodical effusions of the polar ices. I shall first mention the most material facts and considerations which appear to militate against the common theory, as stated by St. Pierre; and I shall then endeavour to explain the theory he has substituted (which it has cost me some pains to collect, abstract, and arrange), as nearly as possible in a literal translation of his own language.

IT is said that, if the moon acted by her attraction, her influence must extend to the Mediterranean, the Baltic, the Caspian,

pian, and the vast lakes of North America, in some degree at least; but all these have no sensible tides*. This tranquillity renders her attraction liable to suspicion; and we shall, perhaps, find that the greatest part of the tides in the ocean have nothing more than an apparent relation either to her influence or her course.

The phases of the moon do not correspond all over the globe with the movements of the seas. On our coasts the flux and reflux follow the moon rather than her real motion: in various places they are subject to different laws, which obliged Newton to admit (chap. 25.) "that in the periodical return of the tides there must be some other mixed cause, hitherto undiscovered."

The currents and tides in the vicinity of the polar circle come from the pole, as appears from the testimony of Fred. Martens, who asserts, that the currents amidst the ices set in towards the south; but adds, that he can state nothing with certainty respecting the flux and reflux of the tides.—*Voyage towards the North Pole*, 1671.

Henry Ellis observed that the tides in Hudson's bay came from the north, and were accelerated as the latitude increased. It is impossible these tides should come from the line or the Atlantic. He ascribes them to a pretended communication with the South sea. At Waigat's straits these north tides run at the rate of eight or ten leagues an hour. He compares them to the sluice of a mill.—*Voyage to Hudson's Bay*, 1746.

Linscotten, in 1594, made nearly the same remarks, and observes that in Waigat's straits the water was only brackish. He says the tides come from the east with great velocity, bringing with them large islands of ice.

W. Barents (1595) confirms this account.

All these effects can be produced by nothing else than the effusion of ices surrounding the pole. These ices, which melt and flow with such rapidity in the northern parts of America and Europe about the months of July and August,

* The Caspian sea is about 860 miles long, and, in one place, 260 miles broad: there are strong currents, but no tides.

There is no regular flux and reflux in the Baltic.

In some particular spots of the Mediterranean there is a small tide.

greatly

greatly contribute to our high equinoctial tides; and when these effusions cease in October, our tides begin to diminish.

If the tides depended on the action of the sun and moon on the equator, they ought to be much more considerable towards the focus of their movements than any where else. But this is contrary to fact (Dampier says). From Cape Blanc, from the third to 30° south lat. the flux and reflux of the sea does not exceed two feet. The tides in the East Indies rise not above a foot; near the poles they rise 20 or 25 feet.

In the road of the island Maffafuero ($33^{\circ} 46'$ south lat. $80^{\circ} 22'$ west long.) the sea runs twelve hours north, and then flows back twelve hours south: its tides, therefore, run towards the line.—*Byron, April 1765.*

At English Creek, on the coast of New Britain (5° south lat. 152° west long.) the tide has a flux and reflux once in 24 hours.—*Carteret, Aug. 1767.*

At the Bay of Isles, in New Zealand (35° south lat.), the tides set in from the south.—*Cook, Dec. 1769.*

At Endeavour River, in New Holland, neither the flood nor ebb tides were considerable, excepting once in 24 hours.—*June, 1770.*

At Christmas Harbour, in Kerguelen's Land, the flood came from the south-east, running two knots an hour.—*Cook, Dec. 1776.* It appears to have been regular and diurnal, *i. e.* a tide of twelve hours. The tide rises and falls about four feet.

At Otaheite the tides seldom rise more than twelve or fourteen inches; and it is high water nearly at noon, as well at the quarters as at the full and change of the moon.—*Cook, Dec. 1777.* It is evident, from a table of these tides for 26 days, that there was but one tide a day; and this, during the whole time, was at its mean height between 11 and 1. These tides, therefore, can have no relation to the phases of the moon.

Let us now take a cursory view of the effects produced by the tides in the northern part of the South sea. At the entrance of Nootka it is high water, on the days of new and full moon, at twenty minutes past twelve: the perpendicular rise and fall eight feet nine inches; which is to be understood of the

the day tides, and those which happen two or three days after the full and change. The night tides rise nearly two feet higher.—*Cook, April 1778*. These semidiurnal tides differ from ours in taking place at the same hour, and exhibiting no sensible rise till the second or third day after the full moon: all which is perfectly inexplicable on the lunar hypothesis.

These northern tides of the South sea, remarked in April, become, in higher latitudes, stronger in May, and still stronger in June; which cannot be referred to the moon's course then passing into the southern hemisphere, but must be ascribed to the sun's course passing into the northern hemisphere, and proceeding, as its heat increases, to fuse the ices of the north pole: besides, the direction of these northern tides towards the line constitutes a complete confirmation that they derive their origin from the pole.

At the entrance of Cook's River there was a strong tide setting out of the inlet at the rate of three or four knots an hour: higher up in the inlet, at a place four leagues broad, the tide ran with prodigious violence at the rate of five knots an hour. Here the marks of a river displayed themselves, the water proving considerably fresher.—*Cook, May 1778*.

What Cook calls a river, is nothing but a real northern sluice, through which the polar effusions are discharged into the ocean. Middleton (*Voyage to Hudson's Bay, 1741 and 1742*) found, between lat. 65° and 66° , a considerable inlet running west, which he calls Wager's River; and, after repeated trials of the tides for three weeks, found the flood constantly coming from the east. This is another of the northern sluices.

In Karakakooa Bay, Sandwich Islands, the tides are very regular, ebbing and flowing six hours each alternately.—*Clerke, March 1779*.

At the town of St. Peter and Paul, in Kamfchatka, the tides are very regular every twelve hours.—*Clerke, Oct. 1779*.

Mr. Wales (*Introduction to Cook's last Voyage*) acknowledges that the tides observed in the middle of the great Pacific ocean fall short full two-thirds of what might have been expected from calculation.

The course of the tides towards the equator in the South sea;

sea; their retardations and accelerations on these shores; their directions, sometimes eastward, sometimes westward, according to the monsoons; finally, their elevation, which increases in proportion as we approach the poles, and diminishes in proportion to the distances from it, even between the tropics, demonstrate that their focus is not under the line. The cause of their motions depends not on the attraction or pressure of the sun and moon on that part of the ocean, for their forces would undoubtedly act there with the greatest energy, and in periods as regular as the course of the two luminaries.

Why, then, are the tides between the tropics so feeble and so much retarded under the direct influence of the moon?

Why does the moon, by her attraction, give us two tides every 24 hours in the Atlantic ocean, and produce only one in many parts of the South sea, which is incomparably broader?

Why do the tides take place there constantly at the same hours, and rise to a regular height almost all the year round?

Why do some rise at the quarters just the same as at the full and change?

Why are they always stronger as you approach the poles, and frequently set in toward the line, contrary to the principle of lunar impulsion?

These problems, which it is impossible to explain by the lunar theory, admit an easy solution on the hypothesis of the alternate fusion of the polar ices.—

Such are the most material objections adduced to invalidate the lunar theory. How far they are conclusive, shall be left to future investigation.

But St. Pierre is not content with demolishing the old structure; he has judged proper to erect a new one; and a fair exposition of this system will enable us to determine, by comparison, to which we shall give our suffrage.

It is well known that Sir Isaac Newton and Cassini differed in their opinion respecting the figure of the earth: the former conceiving it to be an oblate spheroid, flattened at the poles; the latter contending it must be oblong, or elongated at the poles.

poles. To ascertain this point, some of the most celebrated mathematicians of Europe were appointed to determine, by actual measurement, the length of a degree both at the equator and at the pole. They found that the polar degrees exceeded the equatorial, and concluded they must consequently be parts of a larger circle, and, of course, that the earth was flattened at the poles. This was universally considered as decisive of the question, till the genius of our Frenchman detected a gross and palpable error in the calculation, which had escaped their accurate knowledge and penetration: but, as the elongation of the poles constitutes a leading feature in the new theory, I shall give it a more detailed examination.

This polar elongation, as he conceives, is supported by four direct and positive proofs:—the first geometrical, upon which he lays the greatest stress, and upon which he has staked his reputation; the 2d, atmospherical; the 3d, nautical; the 4th, astronomical: of all which in order.

The 1st, or geometrical proof, is what he calls a demonstration founded on the measurement of the earth, and admitting the polar degrees to exceed the equatorial: here follows the demonstration: If you place a degree of the meridian at the polar circle on a degree of the same meridian at the equator, the first degree, which measures 57,422 fathoms, will exceed the 2d, which is 56,748 fathoms, by 674; consequently, if you apply the arc of the meridian contained within the polar circle, being 47° ; to an arc of 47° of the same meridian at the equator, it would produce a considerable protuberance, its degrees being greater.

To render this more apparent, let us always suppose that the profile of the earth, at the poles, is an arc of a circle containing 47° ; is it not evident, if you trace a curve on the inside of this arc, as the academicians do when they flatten the earth at the poles, that it must be smaller than the arc within which it is described, being contained in it? And the more this curve is flattened the smaller it becomes. Of consequence, the 47° of this entire curve will be individually smaller than the 47° of the containing arc. But as the degrees of the polar curve exceed those of the arc of a circle, it must follow that the whole curve is of greater extent

than the arc of a circle : now to be of greater extent it must be more protuberant : the polar curve, of consequence, forms a lengthened ellipsis. Q. E. D.*

It must be acknowledged that this demonstration is very perspicuous and convincing. How the most celebrated academicians and mathematicians, for nearly half a century, could have overlooked a proposition so plain and simple, can only be ascribed, in the opinion of St. Pierre, to their obstinate and inveterate prejudices. He pursues his victory in a strain of vain and indecent exultation, which would dishonour a more respectable cause ; but, perhaps, a little attention will induce us to doubt at least whether the charge of gross ignorance may not, with justice, be retorted on their accuser.

It would have been indeed extraordinary, if men of science had been absurd enough to imagine that a larger arc might be included in a less ; but they might suppose, with propriety and justice, that the smaller arc of a larger circle can be included in the larger arc of a smaller circle, which, in the present instance, appears to be the case. In measuring a degree on the meridian, a certain spot is fixed upon, where the elevation of the polar star is taken by a quadrant ; from this spot they proceed in a direct line north, till the quadrant indicates an additional elevation of one degree. In proportion as this degree constitutes a part of a larger or smaller

* Let x , Fig. 3. Plate V. be the unknown arc of the meridian, comprehended above the arctic circle ABC ; and let DEF be the arc of the same meridian, comprehended between the tropics ; these two arcs are each 47° . According to our astronomers, a degree at the polar circle is greater by 674 fathoms than a degree of the same meridian near the equator ; the arc x therefore exceeds, in extent, the arc DEF by 674×47 , or 31,678 fathoms = $12\frac{3}{4}$ leagues. The question to be determined is, whether this unknown polar arc x is contained within the circle in the curve ABC , or coincides with it, as ABC , or falls without its circumference, as $A \times C$. The arc x cannot be contained within the circle, as ABC , for it would then be evidently smaller than the arc ABC ; and the more this curve ABC is flattened, the less will be its extent, as it will approach nearer and nearer to the straight line AC ; neither can it coincide with the arc ABC , for it exceeds it $12\frac{3}{4}$ leagues. It must belong, therefore, to a curve falling without the circle, as $A \times C$. The globe of the earth, therefore, is lengthened at the poles, since degrees of the meridian are greater there than at the equator.

circle,

circle, a greater or less portion of ground will be passed over before the desired elevation is observed; and the measurement of this ground unequivocally decides whether this degree is part of a larger or smaller circle. In this case the measurement is admitted, but the conclusion denied. St. Pierre seems to have supposed, that the academicians divided the polar arc into 47 parts, and then measured one of these parts: a thing impracticable and ridiculous. The fact is, that the polar arc, which, if the earth were a perfect sphere, would contain 47° , does not actually contain so many, but perhaps about 46° of a larger circle; and if the polar degrees are parts of a larger circle, as they certainly are, it is demonstrably evident that the real arc must be contained within the spherical arc, and, consequently, that the earth is flattened at the poles*.

[To be continued.]

* Let the circle $ABCD$, Fig. 4. represent the earth as a sphere, and let P represent the polar star, having no sensible parallax. Draw the diameter BD , prolonging it to P ; draw the transverse diameter CA , the tangent AP , and the line FP , parallel to AP ; bisect the quadrant AD equally at F ; draw the tangent KL perpendicular to the radius GF , and with the radius BF describe the circle $EOMN$, and let the segment HEF represent the earth flattened at the pole; draw the tangent SR to the circle $EOMN$, perpendicular to the radius BF . An observer at A will perceive the polar star P in the horizon; an observer at D or E will perceive it in the zenith, or at an elevation of 90° . If the earth be a sphere the tangent KL will be the horizon to an observer at F , and the angle of elevation PFK is, by construction, 45° ; but if the earth is not a sphere, but flattened towards the pole, as in the segment HEF , the tangent SR will be the horizon to an observer at F , and of course the angle PFS will be the angle of elevation. Now, the angle EGF is by construction 45° , consequently, the $\angle FGB = 135^{\circ}$, and the angles GBF and GFB each $22^{\circ} 30'$. Draw the dotted line EF . Now the triangle BEF is an isosceles triangle, and the angle GBF being found $= 22^{\circ} 30'$, it follows that the angles BEF and BFE are each $78^{\circ} 45'$; and the $\angle SFB$ being a right angle, the $\angle SFE$ is $90^{\circ} - 78^{\circ} 45' = 11^{\circ} 15'$. At E draw the tangent TV , and, for the same reason, the $\angle VEF = 11^{\circ} 15'$; and consequently the $\angle ERF = 157^{\circ} 30'$, and the $\angle VRF = 22^{\circ} 30'$. Now the $\angle FZE$ is a right angle, therefore the angle $PFS = 180^{\circ} - 90^{\circ} - 22^{\circ} 30'$, or $67^{\circ} 30'$. The difference, therefore, between the elevation at E and at F will be $90 - 67,30 = 22\ 30'$, but the difference between the elevation at F and D will be 45° ; whence it is evident that a larger measurement of ground will be included in a degree in proportion as HEF is the segment of a larger or smaller circle.