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“The Thermal Efficiency of Steam-Engines.”

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STEAM-engine economy has, during the last fourteen years, formed the subject of several Papers read before the Institution, and a considerable portion of them, and of the discussions which followed, have had reference to the standard with which the experimental results were compared. It is remarkable that, notwithstanding this repeated discussion, there cannot yet be said to be agreement amongst engineers as to the proper standard to be employed; and, in view of the inconvenience of such a state of things, the Author feels that little apology is needed for again bringing the question forward, in the hope of preparing the way for the adoption of some definite standard.

As to whether, in the first place, the absolute thermal efficiency of the actual engine should be used, or a comparison be made with an ideal engine, the Author considers that both efficiencies are required to give a clear view of the thermal performance of an engine, and that even erroneous conclusions may result if either be not taken into account.

The absolute thermal efficiency is defined as the ratio of the portion of the heat supplied which is converted into work, to the total heat supplied, and this efficiency must, for physical reasons, be very small. It is required for the following purposes:—

(1) To obtain the work-return of any particular engine without reference to others.

(2) To compare the conversion of heat into work obtained by various kinds of heat-engines, whether using saturated steam, superheated steam, gas, oil, or air, &c.

(3) To compare the conversion of heat into work obtained by

the same kind of heat-engine or even the same engine when working with different temperature ranges.

The standard efficiency can be defined as the ratio between the heat converted into work by the actual engine and that which an ideal standard engine, working under the same conditions, could convert into work, and varies between 50 per cent. and 80 per cent.; but there is no physical reason why it should not reach 100 per cent., although there are practical reasons to prevent a much greater efficiency than 80 per cent. being obtained. The standard efficiency indicates how nearly an actual engine has approached to what is physically possible under the conditions of working.

It will be observed that the absolute thermal efficiency and the standard thermal efficiency give information of different kinds, and moreover it frequently happens that engines that have a high absolute thermal efficiency have a low standard thermal efficiency. Thus a condensing steam-engine working with an admission pressure of 180 lbs. per square inch absolute, may have an absolute efficiency of about 17 per cent., whereas a non-condensing engine, working with the same admission pressure, cannot be expected to have a greater absolute efficiency than about 12 per cent. But the standard thermal efficiencies would be about 60 per cent. and 80 per cent. respectively; in other words, the condensing-engine, which of course gives the best absolute return, is nevertheless not doing so well (in proportion to its opportunities) as the non-condensing-engine.¹

¹ The following two extracts illustrate this point:—

“He did not think that the Author was justified in taking the efficiency in the way he had done. He thought he could show from the figures in the Paper that it led to a very decided fallacy. In Table I, ‘simple series,’ four expansions, 27·8 lbs. of water, the ‘efficiency’ was put down at 67·1 per cent. Working the same engine with less than two expansions, with 42·76 lbs. of water, the ‘efficiency’ was 81·08 per cent.; so that the more the water was wasted, the better was the ‘efficiency,’ so called, which was obviously an error.” —Mr. Cowper’s remarks on Mr. Willans’ Paper, “Economy Trials of a Non-condensing Steam-Engine,” Minutes of Proceedings Inst. C.E., vol. xciii. pp. 233-4.

“Mr. W. W. Beaumont and Mr. C. E. Cowper pointed out, during the discussion, that in many cases the comparison of water used with water theoretically required was misleading, the efficiency as given being higher in certain cases in which the feed-water was greater, and Mr. Mair and Mr. Cowper thought the efficiency should be based on a comparison of the heat supplied with the heat given up in useful work. He agreed with Mr. Cowper that this was the only way of taking the ‘absolute efficiency,’ but he might observe that this method of comparing the efficiency was quite as misleading as

STANDARD THERMAL EFFICIENCY.

Many views are held as to what should constitute the ideal engine required as a standard of comparison. It has been advanced,¹ that it is immaterial what ideal engine is adopted, so long as comparisons are universally made with it.² Undoubtedly, the adoption of any agreed standard would ensure the good result, that all figures of thermal efficiency would be directly comparable. But it is not less important that the standard should be so chosen that it may be readily seen how far the particular engine considered falls short of what is physically possible within the limits of temperature by which the engine is bound. To strictly fulfil this requirement the standard of comparison ought evidently to be the perfect steam-engine, that is, it ought to be the particular

a comparison of the feed-water; because it took no account of the limitations which the working conditions themselves imposed, whereby the temperatures at which heat was received and was rejected were unalterably fixed."—Mr. Willans' reply to Correspondence on Paper, "Economy Trials of a Non-condensing Steam-Engine," Minutes of Proceedings Inst. C.E., vol. xevi. p. 231.

¹ Minutes of Proceedings Inst. C.E., vol. xciii. p. 229; also *ibid.*, vol. xevi. p. 239.

² The following extract from Mr. Willans' reply to the Correspondence on his Paper, "Economy Trials of a Non-Condensing Engine," shows that it is of practical importance to select a proper standard of comparison:—

"In the technical journals, a few months back, particulars of an engine-trial were given, and it was stated that a non-condensing engine, working with 150 lbs. boiler-pressure, gave 1 HP. hour per 14·12 lbs. of water evaporated. Now the heat supplied to the above weight of water to convert it into steam taking the feed temperature as that of the exhaust steam, was 14,330 units. The heat converted into work per HP. hour = $42\cdot75 \times 60 = 2,565$ units, or an absolute efficiency of $\frac{2,565}{14,330}$, or 0·179. This was an excellent result, but still

one which Prof. Perry would consider that of a spendthrift engine; at any rate, it would seem to leave much to be desired. Comparing it with the work due from the Carnot cycle, however, the efficiency would suddenly jump up to 96·2 per cent. of theoretical possibilities, taking the Carnot cycle between the same limits of temperature as unity—a result which no doubt satisfied the experimenter.

"If, however, the result was compared with the work due from an ideally perfect steam-engine, namely, with the work represented by the area of the best diagram possible, as given by the formula of Clausius, it would be found that the actual engine gave 103·4 per cent. of the work theoretically possible, or 3·4 per cent. more power than could possibly be obtained from that weight of steam expanded between the boiler and atmospheric pressures. This result would probably, had he realized it, have induced the experimenter to make another trial, after overhauling his indicators and water-weighing apparatus."

ideal steam-engine which converts into work the maximum percentage possible of the heat-units supplied to it, having regard to the varying temperature at which the heat is supplied to the feed-water, and to the admission- and exhaust-temperatures. It will, however, be seen later that for practical reasons, which are universal and permanent, a slight modification is desirable in connection with the point to which the expansion is carried.

The cycle of the perfect steam-engine has been described by Clausius, Rankine, Tait and other authorities,¹ and is well known to be, (1) feed raised from exhaust temperature to admission temperature; (2) evaporation at constant admission temperature producing dry saturated steam; (3) adiabatic expansion carried down to back pressure; and (4) rejection of heat at constant temperature corresponding to back pressure.

The perfect steam-engine as thus defined was the standard advocated and adopted by the late Mr. Willans in both his Papers on steam-engine economy;² it may be said he made this standard peculiarly his own, and when it was criticised by several speakers in the discussion³ on the earlier Paper, Mr. Willans effectively⁴ maintained his position in his reply.

VARIOUS IDEAL ENGINES PROPOSED AS STANDARDS.

Reference will now be made to some of the standards of comparison that have been used by engineers.

Carnot Cycle.—A heat-engine working on the Carnot cycle is probably the most usually accepted standard of comparison. An engine working on this cycle receives all its heat at the higher temperature and rejects it all at the lower temperature. The perfect steam-engine above defined, however, receives a part of its heat at temperatures varying between the lower and the higher limit, namely, that portion of the heat required to raise the feed-water from the lower to the higher temperature, while the remainder of the heat (namely, the latent heat of evaporation) is put in at the higher temperature. The absolute thermal efficiency of the perfect steam-engine is therefore necessarily less than that of the Carnot cycle engine, as will be seen by com-

¹ Minutes of Proceedings Inst. C.E., vol. xciii. p. 132.

² *Ibid*, vol. xciii. p. 134, and vol. cxiv. p. 2.

³ *Ibid*, vol. xciii. pp. 227-229, 249 and 256.

⁴ *Ibid*, vol. xcvi. pp. 242-245.

paring the following expressions, or as exemplified by the $\theta \phi$ chart, *Fig. 2* :—

Absolute efficiency of Carnot cycle¹ = $\frac{\theta_a - \theta_e}{\theta_a}$;

„ „ of perfect steam-engine of Clausius—

$$= \frac{(\theta_a - \theta_e) \left(1 + \frac{L_a}{\theta_a}\right) - \theta_e \log_e \frac{\theta_a}{\theta_e}}{L_a + \theta_a - \theta_e},$$

where θ_a and θ_e are respectively the absolute admission- and exhaust-temperatures, and L_a is the latent heat at temperature θ_a .

Inasmuch then, as the absolute thermal efficiency of the perfect steam-engine is the maximum physically possible, the Carnot cycle is an ideal impossible of realization ; and hence, notwithstanding its general acceptance, it cannot be said to be a suitable standard of comparison when dealing with the actual steam-engines as at present constructed or contemplated. It is, in fact, a purely scientific abstraction, incapable of practical application without a dynamical feed-heater. The following Table shows the absolute efficiencies of the Clausius perfect steam-engine and of the Carnot cycle engine between various limits of temperature :—

—	Non-Condensing.		Condensing.		
	270° F.	370° F.	270° F.	320° F.	370° F.
Admission-, or higher, temperature, t_a	270° F.	370° F.	270° F.	320° F.	370° F.
Exhaust-, or lower, temperature, t_e .	212° F.	212° F.	110° F.	110° F.	110° F.
Clausius perfect steam-engine . .	0·075	0·176	0·207	0·248	0·284
Carnot heat-engine	0·079	0·190	0·219	0·269	0·313

It will be seen from this Table that the difference between the Carnot cycle and the perfect steam-engine is considerable, especially with condensing-engines working with high-pressure steam. The difference is even more marked with engines using superheated steam, as will be shown later. The greatest difference, however, occurs in the case of gas-engines. For instance, A C D E, *Fig. 1*, is the $\theta \phi$ diagram of an ideal gas-engine, compressing the charge adiabatically, receiving all the heat produced by the explosion at constant volume, expanding adiabatically to atmospheric pressure, and exhausting at that pressure. The absolute thermal efficiency is $\frac{\text{area A C D E}}{\text{area A}_1 \text{ A C C}_1} =$ about 50 per cent., whereas the absolute thermal

¹ Minutes of Proceedings Inst. C.E., vol. xcix. p. 224.

efficiency of the Carnot cycle, working between the same limits, is about 90 per cent. It is well to realise this, not to raise false hopes as to the possibilities of gas-engines.

Jacketed Engine.—If the expansion-line, instead of being adiabatic, follows the saturation-line, the cycle is that of the ideal jacketed engine, in which the steam is kept saturated and dry during expansion by added heat. The absolute efficiency¹ of this cycle is less than that of the Clausius steam-engine, as will be shown later. Therefore it is unsuitable as a standard, because the standard thermal efficiency of an actual engine would be exaggerated if compared with it. Prof. Osborne Reynolds adopted

Fig. 1.

the saturation-line in his standard of comparison,² and further limited the expansion so that the release pressure in the standard was the same as in the actual engine.³ This limitation in the expansion ought to prevent this cycle from being adopted as a standard for thermal efficiency, because one of the defects in actual engines which the standard is appointed to try, namely, incomplete expansion, is thereby altogether condoned; and no matter how defective in this respect the actual engine might be, the ideal engine is practically equally so. The objection is not, however, so much that the expansion is incomplete, as that it varies

¹ Minutes of Proceedings Inst. C.E., vol. xcix. pp. 226, 227.

² See under "Type Standard."

³ Minutes of Proceedings, Inst. C.E., vol. xcix. p. 181.

with the release pressure in the actual engine. It is even the fact that the worse the actual engine is, in regard to limiting the expansion unduly, the more efficient it will appear, relatively to other engines, because in this respect the standard changes just as the actual engine does. Mr. Henry Davey, in his Paper¹ on "Steam-Engine Economy," practically adopted the same standard as Prof. Reynolds.

The comparison of the expansion-line of an actual engine with the saturation-line, although, as already shown, incorrect if the object is to find the thermal efficiency of the engine, is nevertheless of value. If the saturation-line is drawn on the indicator diagram (or on a combined diagram in the case of an engine in which the steam is in more than one cylinder), not through the actual point of cut-off, but through a later point, representing as saturated steam the whole of the feed admitted per stroke, then at any other point in the expansion the dryness fraction of the steam can be determined by a comparison of the actual expansion-line with the saturation-line.

An ideal engine having an expansion-line following the hyperbolic law ($p v = \text{a constant}$) is also used for the purposes² of comparison. The absolute thermal efficiency of such an engine is, however, less than that of the Clausius steam-engine, working within the same temperature limits, and it is therefore unsuitable as a standard of comparison for thermal efficiency. The graphic comparison of the expansion-line of an actual engine with the curve $p v = \text{constant}$, drawn through the point of cut-off is, however, of use in other respects, because the expansion-line of a large majority of actual engines, owing to initial condensation and re-evaporation, approximates to this curve, so that any material discrepancy will indicate something wrong with such engines. The fact that an expansion-line following the law $p v = \text{constant}$ is the indication of the defect of initial condensation,³ is proved later, and for this reason alone it makes a most unsatisfactory standard.

Other standards of comparison have been suggested. For instance, Dr. A. B. Kennedy, F.R.S., mentioned three in his remarks⁴ on Mr. Willans' Paper on "Economy Trials of a Non-Condensing Steam-Engine," but those referred to will suffice to show that there is a considerable disparity of views on the subject.

¹ Minutes of Proceedings Inst. C.E., vol. cxxii. p. 1.

² *Ibid.*, vol. xciii. p. 249.

³ *Ibid.*, vol. xcvi. pp. 250, 251.

⁴ *Ibid.*, vol. xciii. pp. 227, 228.

The Author feels strongly, however, that the perfect steam-engine defined early in the Paper, but with the possible modification already referred to and discussed later, is the proper standard to use when it is desired to ascertain the degree of perfection reached by a particular engine, or when reporting the results of trials.

There is also another standard of comparison which is of use to the engine-designer, because it can be made to take account of the amount of clearance, compression, back-pressure, release, jacketing, etc., which are practically necessary in any particular type of engine under consideration. This standard might well be called the "Type Standard," and the difference in heat-units utilized by it and the standard steam-engine gives the absolutely unavoidable loss normal to the particular type of engine. Further, the difference in heat-units utilized by the actual engine and by the type standard shows the more or less avoidable loss, and may suggest improvement in the design of the engine. The comparison with this standard might be called the "Type Thermal Efficiency." The standard adopted by Professor Reynolds in his Paper "Triple-Expansion Engines and Engine Trials,"¹ is an example of a "Type Standard," and it was, no doubt, really in this connection that he used it, although this was only made clear in his reply, and some of his critics—notably Mr. Willans—were led to think that he intended it as a general standard applicable not only to the engine at Owens College, but to others.

COMPARISON OF THE VARIOUS STANDARDS BY THE $\theta\phi$ CHART.

The various ideal engines that have been considered can be compared² graphically by means of their $\theta\phi$ diagrams. The $\theta\phi$ chart, as usually drawn, refers to 1 lb. of H_2O .³ The ideal engine should therefore consist of a cylinder fitted with a piston, and containing at all points of the stroke 1 lb. of H_2O behind the piston. If it is conceded that heat may be introduced into or abstracted from this cylinder in any desired manner, the transformation of the 1 lb. of H_2O can be so arranged that any indicator diagram (or $p v$ diagram) whatsoever can be reproduced.⁴

Thus, in the case of the perfect steam-engine of Clausius, the

¹ Minutes of Proceedings Inst. C.E., vol. xcix. p. 152.

² *Ibid.*, vol. xcvi. p. 240.

³ The chemical symbol H_2O is used, as the cylinder contains a mixture of steam and water varying in proportion at each point of the stroke.

⁴ Minutes of Proceedings Institution of Mechanical Engineers, 1894, pp. 82-90; also *Engineering*, 3rd January, 1896 (translation of Prof. Boulvin's Paper).

cylinder contains at the starting-point 1 lb. of water at the lower temperature θ_1 , and the state of the water is represented by the point A on the $\theta \phi$ chart, *Fig. 2*, and by the point a on the $p v$ diagram, *Fig. 3*. Heat being now introduced into the cylinder, the temperature of the water is raised to the higher limit of temperature θ_2 . During this stage the state point of the water moves

Fig. 2.

Fig. 3.

along the "water-line" of the $\theta\phi$ chart to the point B, corresponding to the point b on the $p v$ diagram, and the number of heat-units added is represented graphically by the areas $K + M$. The heat-supply being continued, the water is gradually evaporated at the higher temperature θ_a , corresponding to pressure p_a . Immediately steam is formed, the piston moves forward under constant pressure, and continues to do so until all the water is evaporated; there is then 1 lb. of saturated steam behind the piston, which has reached

the point of cut-off represented by C on the $\theta\phi$ chart, and by c on the $p v$ diagram. The heat added during this stage, namely, the latent heat, is shown by the area L + N, *Fig. 2*. The heat-supply now ceases, and the piston moves forward under a continually diminishing pressure; in other words, the steam expands adiabatically until the temperature has reached the lower limit θ_c . During this stage the state point travels along the straight line CD on the $\theta\phi$ chart, and along the expansion line cd on the $p v$ diagram. No heat is either added or subtracted during this stage, and when the point D is reached the lb. of H_2O consists of $\frac{A D}{A E}$ lb. of steam and $\frac{D E}{A E}$ lb. of water. The last stage is to condense the steam, namely, $\frac{A D}{A E}$ lb., at constant pressure and temperature by abstraction of heat until the piston has returned to its initial position, and the 1 lb. of H_2O is wholly converted into 1 lb. of water at θ_c . During this stage the state point moves from D to A on the $\theta\phi$ chart, and from d to a on the $p v$ diagram. The heat abstracted is represented by the area M + N. The cycle is now complete, and evidently K + L heat-units have been transformed into work as represented by the $p v$ diagram. The heat represented by (K + L) is the maximum quantity that can be converted into work by a steam-engine working between the limits θ_a and θ_c . In *Fig. 2*, θ_a has been taken as 110° F. and θ_c as 370° F., and it is proposed to retain the same values for the several ideal engines for the sake of obtaining an arithmetical comparison.

The absolute thermal efficiency of the Clausius steam-engine is therefore $\frac{K + L}{K + M + L + N}$, which, with the temperatures chosen, becomes $\frac{318}{1119} = 0.284$.

The Carnot cycle can be traced on the $\theta\phi$ chart as follows:—Before the piston begins to move, the cylinder contains 1 lb. of water at θ_a temperature, represented by the point B on the chart, *Fig. 4*, and by the point b on the $p v$ diagram, *Fig. 5*. Heat is then applied, and the water is evaporated at temperature θ_a , corresponding to pressure p_a . When L + N heat-units have been added, all the water has been transformed into steam, and the point C is reached, corresponding to the point of cut-off, c. The steam is now expanded adiabatically to D, after which heat is abstracted at constant temperature θ_c , corresponding to pressure p_c , until the point E is reached, E being vertically below B.

The amount of heat abstracted is represented by the area N. The piston has now attained the position e , and the cylinder contains a mixture of steam and water at a temperature θ_e , the weight of steam being $\frac{A E}{A F}$ lb., and the weight of water

Fig. 4.

Fig. 5.

being $\frac{EF}{AF}$ lb., together equal to 1 lb. of H_2O . The next step is to compress the steam adiabatically as represented by EB and eb . No heat is introduced into or abstracted from the cylinder, but work, represented by the area bba_ea , is done, and by means of this work the 1 lb. of H_2O in the cylinder at temperature θ_a is converted into 1 lb. of water at temperature θ_a , and the cycle is completed. The heat supplied during the cycle is

$N + L$, and the heat abstracted is N , so that the portion L is converted into work; hence the absolute efficiency is $\frac{L}{N + L}$, which is evidently greater than the efficiency of the Clausius steam-engine. Therefore, as already pointed out, the Carnot cycle is an unsuitable standard, because it is impossible of realization with the steam-engine as at present constructed, requiring a mechanical feed-heater¹ to produce the portion EB of the cycle. This part of the cycle should not be confounded with the steam compression in an actual steam-engine, because in the latter case only the

Fig. 6.

steam retained in the cylinder is raised to the higher temperature θ_a by the compression, and the remainder of the 1 lb. of H_2O originally contained in the cylinder is exhausted from the cylinder at the lower temperature θ_c , and it, or a corresponding weight of water, has to be raised from θ_c to θ_a in the boiler.

Therefore, if an ideal engine expels the proportion $\frac{EF}{AF}$ of its H_2O during the return stroke in the same way that an actual engine does, and gives a pv diagram $bcd e$, Fig. 5, corresponding to the $\theta \phi$ diagram $BCDE$, Fig. 4, heat has to be supplied in the boiler to raise the temperature of the portion

¹ Minutes of Proceedings Inst. C.E., vol. xcvi. p. 236.

$\frac{EF}{AF}$ of the feed-water, and this heat is represented by the areas $K_1 + M_1$. ($A_1 E$ is equal to $A E \times \frac{EF}{AF}$, and the horizontal intercepts of the two curves are in the same proportion.) The absolute efficiency of such an ideal steam-engine is $\frac{L}{K_1 + M_1 + L + N}$, which is evidently not only less than that of the Carnot-cycle engine, but less than that of the perfect steam-engine.

The cycle of the ideal jacketed-engine is shown in *Fig. 6*. It is unnecessary to follow the cycle completely, as it only differs in certain particulars from those previously discussed. During the whole of the expansion from the point of cut-off C , heat has to be introduced into the cylinder to maintain the steam in a state of saturation, and the amount of heat so introduced is represented on the $\theta \phi$ chart by the areas O and P , of which the portion O is converted into work and the remainder rejected, so that the absolute thermal efficiency of the heat supplied by the jacket is only $\frac{O}{O + P}$. The absolute thermal efficiency of the whole cycle is $\frac{K + L + O}{K + M + L + O + N + P}$, which is evidently less than that of the Clausius steam-engine working between the same temperature limits.

In the case of Prof. Osborne-Reynolds' standard, and also in that adopted by Mr. Davey, the expansion continues along the line CR (or cr) until the release-pressure is reached at the point R on the $\theta \phi$ chart, *Fig. 7*, and at r on the $p v$ diagram, *Fig. 8*. The heat added during the expansion is shown on the $\theta \phi$ chart by the areas O_1 and P_1 , and it will be observed that of this amount only the small portion O_1 is converted into work. At R (r) the release is effected at constant volume which is represented on the $\theta \phi$ chart by the portion ER of the constant volume-line drawn through R to E . From E to A the exhaust takes place at constant pressure and temperature. The absolute efficiency of this ideal steam-engine is

$\frac{K + L + O_1}{K + L + O_1 + M + N + P_1}$; clearly less than that of the ideal jacketed-engine, and *à fortiori* less than that of the Clausius steam-engine, so that the efficiency of an actual engine would be much exaggerated if this cycle were adopted as a standard.

The cycle of the ideal engine, the expansion in which follows the law $p v = \text{constant}$, is shown on the $\theta \phi$ chart in *Fig. 9*. It will be observed that it is necessary, as in the case of the

ideal jacketed-engine, to introduce heat into the cylinder during the whole of the expansion and to a greater extent. The heat so added is represented by the areas S and T , of which S is converted into the work. The absolute thermal efficiency of this ideal engine is $\frac{K + L + S}{K + M + S + L + N + T}$, which is evidently less than that of the Clausius cycle, and it will be noticed that the steam has to be superheated during the whole of the expansion.

Fig. 7.

Fig. 8.

In actual engines, in which the expansion-line follows the law $p v = \text{constant}$, there is never any superheat at any point of the expansion; the steam at cut-off must therefore be wet, but it becomes drier as the expansion proceeds, and is, in fact, often nearly dry at release. If quite dry at release, the expansion line

cylinder up to the point of cut-off will always be less than the boiler-pressure. This defect in pressure is in part due to throttling in the valve-passages,¹ and in part to the energy required to put the steam in motion in the steam-pipes. The former part of this defect is due to the engine, and any resulting loss is rightly charged to the engine, but the latter is due to the steam-pipes, and ought not to be charged against the engine. Account should, however, be taken of the energy in the steam due to its velocity; but if the steam-pipes are of sufficient size this energy is so small a percentage that it can be neglected. Thus if the velocity of the steam is 50 feet per second the energy due to velocity is 0·05 B. T. U. per lb., or only about 0·005 per cent. of the total energy in the steam. It thus appears that the higher limit of temperature ought to be taken as the temperature of the steam in the steam-pipe close to the admission into the engine, that is, at the engine stop-valve.

Expansion.—The expansion in the Clausius steam-engine is continued to the back pressure; in an actual engine, however, there is loss in respect of effective horse-power if the expansion is carried beyond the pressure (referred to the low-pressure piston) required to drive the engine itself. A standard, in which the expansion is carried down to the back pressure would therefore tend towards the use of engines too large for their work. It would be a mistake to establish a standard that would tend to such a result. On the other hand, if the expansion is limited so that the release pressure is equal to the back pressure plus the mean pressure corresponding to the friction of the engine, a premium is, so to speak, awarded to a stiff engine instead of a penalty, which is clearly not right. Moreover, the standard, instead of being dependent only on the range of temperature, is made to vary with the stiffness of the actual engine. Both these objections can be in great measure obviated by agreeing that the expansion in the standard of comparison shall be carried down to the back-pressure plus a pressure to be decided upon, a suggestion originally made by Mr. Macfarlane Gray.² This additional pressure should be somewhat less than the mean pressure required to drive the most mechanically efficient engine it is reasonably possible to make, and the Author suggests that this pressure should be taken at 0·15 atmosphere, which corresponds to a mechanical efficiency of 93·5 per cent. when the mean pressure referred to the low-pressure piston is 34 lbs. per

¹ Radiation and conduction losses in the steam-pipes, &c., cause condensation of a portion of the steam, and the water thus produced is supposed to be drained off and allowed for in reckoning up the feed.

² Minutes of Proceedings Inst. C.E., vol. cxiv. pp. 73, 102 and 103.

square inch.¹ The reduction in the work done by the standard of comparison by thus limiting its expansion is shown in *Figs. 10 and 11*, both for a condensing, and for a non-condensing engine. It will be observed that the reduction is far less for the latter than for the former,² and this leads to another advantage of limiting the expansion of the steam in the standard engine as suggested, for if the Clausius perfect steam-engine is taken as the standard, that is, if the expansion is continued to the back pressure, a condensing engine, if properly proportioned to its work, must of necessity have a considerably less standard thermal efficiency than a non-condensing engine, whereas if the expansion is limited a more real comparison is established between the two varieties of engine.

Fig. 10.

Fig. 11.

Back Pressure.—The view may be taken that for a non-condensing engine the back pressure, and the corresponding lower limit of temperature, should be taken at atmospheric pressure, and that for a condensing engine a limit should be fixed, based on some such grounds, that the gain due to improved water-consumption, owing to improvement in vacuum, is equal to the increase in cost of driving the air-pumps, etc. This degree of vacuum must vary in every case; possibly, however, not to any very great extent.

Another view may however be taken, namely, that as regards a condensing engine, a defect in vacuum outside the engine is the fault of the condensing arrangements, and that it would be as

¹ Mr. Michael Longridge also takes this view of the matter. See his remarks on Mr. Davey's Paper on "Steam-Engine Economy," Minutes of Proceedings Inst. C.E., vol. cxxii. p. 32.

² Minutes of Proceedings Inst. C.E., vol. cxiv. pp. 10 and 56.

reasonable to take the lower limit at, say, 110° F. for a non-condensing engine, which is hampered by the atmospheric pressure, as it would be to take the same limit for a condensing engine which is hampered by a back pressure of 3 lbs. (140° F.) due to a defective air-pump or condenser, or, as in many cases, due to warm injection water. At any rate, it is only a question of degree.

The object of the standard under discussion is to compare the actual engine with an ideal steam-engine working under the same conditions of temperature, and to make this comparison an accurate one the lower limit of temperature should be that in the exhaust just outside the engine, a temperature which would have to be observed. In this manner defects in the exhaust pipes, or in the condensing arrangements, are not reckoned against the engine, and thus a true ratio is obtained of the transformation of heat into work effected by the actual engine proper, as compared with what is possible by the standard working under the same conditions. This way of taking the lower limit of temperature also includes the not unusual case of an engine working against a considerable back pressure, and also the less usual case of working at a high elevation in an atmosphere of less pressure.

Compression.—There appears to be an unanimity of opinion that the standard of comparison should have no clearance, and therefore no compression. Such an engine is of course practically unattainable, but it is possible with actual engines to have the clearance so small that the gain by expunging the clearance altogether is minute. The compression line of the standard should therefore follow the water line A B on the $\theta \phi$ chart.

PROPOSED STANDARD OF COMPARISON.

The preceding considerations tend to indicate the following definition of the most satisfactory ideal steam-engine for a standard of comparison.

The heat is received into the feed-water at varying temperatures, gradually increasing from the temperature in the exhaust just outside the engine to the temperature of the steam at the engine stop-valve. The evaporation is considered to take place at a constant temperature equal to that at the engine stop-valve, and dry saturated steam is used. The expansion is adiabatic and is carried to a pressure equal to 0.15 atmosphere more than the pressure in the exhaust pipes immediately outside the engine. The lower limit of temperature at which the heat is rejected is constant, and is that corresponding to the pressure in the exhaust

pipe of the actual engine at the outlet from the engine. There is no compression and no clearance.

The Author suggests this ideal steam-engine as the standard of comparison, and its cycle is graphically shown on the $\theta \phi$ chart in *Fig. 12*, by the lines A B, B C, C D, D E, and E A. It is the Clausius perfect steam-engine, with a certain portion of the toe of the $p v$ diagram cut off.

The absolute thermal efficiency of this standard steam-engine is $\frac{\text{area A B C D E}}{\text{area A}_1 \text{ A B C C}_1}$, and in symbols it is expressed by—

$$\frac{(\theta_a - \theta_r) \left(1 + \frac{L_a}{\theta_a}\right) - \theta_r \log_e \frac{\theta_a}{\theta_r} + \frac{144 (p_r - p_e) V_r}{778}}{L_a + \theta_a - \theta_r},$$

where θ_a , θ_r , and θ_e are respectively the absolute temperatures of

admission, release and exhaust, L_a is the latent heat at temperature θ_a , p_r and p_e are respectively the absolute pressures at temperatures θ_r and θ_e ($p_r - p_e = 0.15$ atmosphere), and V_r is the volume of steam at temperature θ_r .

As will be evident from an examination of the above formula, the calculations required to obtain the absolute thermal efficiency of the proposed standard are of considerable length and complication, for it will be noticed that θ_r is so related to θ_e that the difference of the pressures corresponding to these temperatures is 0.15 atmosphere, and that V_r is the volume of 1 lb. of H_2O after adiabatic expansion from θ_a to θ_r . Possibly the expression might

be thrown into a more workable form, but the Author has not made any attempt to do this, because a solution can readily be obtained by means of the $\theta \phi$ chart, as explained in Appendix I. Even by this method, however, the calculations are considerably longer than those required to obtain the absolute thermal efficiency of the Carnot cycle, and this is possibly one of the reasons why the Carnot cycle has been so generally adopted as a standard. To obviate this objection to the standard engine proposed, the Author has prepared a set of curves, which are given in *Fig. 13*, by means of which the absolute thermal efficiency of the standard can be read off without any calculation.

Fig. 13.

CURVES OF ABSOLUTE THERMAL EFFICIENCY OF PROPOSED STANDARD ENGINE WITH DIFFERENT ADMISSION- AND EXHAUST-TEMPERATURES, AND WITH SATURATED AND SUPERHEATED STEAM.

The following numerical example shows how to obtain the absolute thermal efficiency and the standard thermal efficiency of an actual engine working under the following conditions :—

Admission-temperature, $\theta_a = 370^\circ$; exhaust-temperature, $\theta_e = 110^\circ$; feed-water per I.H.P., 13.4 lbs.

The amount of heat required to produce saturated steam at 370° F. at constant pressure from a temperature of 110° , is 1,117 B.T.U., but 42.4×60 B.T.U. correspond to 1 HP. per hour ;

hence absolute thermal efficiency of actual engine = $\frac{42.4 \times 60}{1,117 \times 13.4}$
= 17 per cent.

From *Fig. 13* it appears that the absolute thermal efficiency of the standard engine is 26·8 per cent. Hence the standard thermal efficiency of the actual engine is $\frac{17}{26\cdot8} = 63\cdot5$ per cent.

Although probably unnecessary, it may be well to state that the standard engine is assumed to be free from mere internal defects of design; for instance, there is no clearance in the cylinders or ports, no incomplete expansion by reason of cylinder dimensions restricted for practical convenience; no back pressure due to friction in exhaust ports, or passages within the engine itself, and no losses from either condensation, radiation, conduction or leakage.

The relation which the various ideal engines bear to each other is clearly brought out in the following Table, in which an actual engine working between the temperature limits of 370° and 110° F., and of which the absolute thermal efficiency is 17 per cent., is compared with the other standards.

Description.	Absolute Thermal Efficiency.		Standard Thermal Efficiency of actual Engine compared with the various ideal Engines.
	On $\theta \phi$ Chart.	Of Numerical Example.	
Actual engine	..	17·0	
Clausius perfect steam-engine . {	$\frac{K + L}{K + M + L + N}$	28·3	60·6
Carnot cycle heat-engine . . {	$\frac{L}{M + N}$	31·3	54·4
Ideal jacketed-engine . . . {	$\frac{K + L + O}{K + M + O + L + N + P}$	25·9	65·7
Reynolds ideal engine . . . {	$\frac{K + L + O_1}{K + M + O_1 + L + N + P_1}$	22·5	75·6
Proposed standard ideal engine	..	26·8	63·5

As a summary it appears that there are three different thermal efficiencies, which are useful when considering the thermal performance of an engine :—

I. The absolute thermal efficiency—

$$\frac{\text{Heat-units converted into work by the actual engine}}{\text{Heat-units supplied to the engine}}$$

II. The standard thermal efficiency—

$$\frac{\text{Heat-units converted into work by the actual engine}}{\text{Heat-units which the corresponding standard steam-engine can convert into work}}$$

or—

$$\frac{\text{The absolute thermal efficiency of the actual engine}}{\text{The absolute thermal efficiency of the corresponding standard engine}}$$

III. The type thermal efficiency—

$$\frac{\text{Heat-units converted into work by the actual engine}}{\text{Heat-units which the corresponding ideal steam-engine of the same type can convert into work}}$$

or—

$$\frac{\text{The absolute thermal efficiency of the actual engine}}{\text{The absolute thermal efficiency of the corresponding type standard}}$$

The uses of the absolute thermal efficiency and of the standard thermal efficiency have already been mentioned, and in the Author's opinion both these efficiencies should be stated in reporting trials of engines. The type thermal efficiency is only useful to the designer of the particular type in question, and has therefore not further been discussed.

STANDARD OF COMPARISON FOR SUPERHEATED STEAM-ENGINES.

The absolute thermal efficiency is as applicable to a superheated steam-engine as it is to a saturated steam-engine. As regards the standard thermal efficiency, the perfect superheated steam-engine will be the same in principle as the perfect saturated steam-engine of Clausius, and it only remains to find what is its absolute thermal efficiency.

Returning to first principles, let the ideal cylinder contain 1 lb. of water at temperature θ_s and pressure p_s . On the application of heat, the temperature of the water will be raised until the pressure p_a is reached, precisely as in the case of the saturated steam-engine, its state being then defined by B on the $\theta \phi$ chart, Fig. 14. On further application of heat to the cylinder, the water is evaporated at constant pressure p_a until the point C, representing saturation is reached, then the state of the steam follows the constant pressure-line CS (pressure p_a) until the temperature of superheat, θ_s , is reached at the state point S. The cylinder then contains 1 lb. of superheated steam at pressure p_a and temperature θ_s which has been formed at the constant pressure p_a according to the graph A B C S, and the heat required

against the engine. That is to say, the temperature θ , should be the steam temperature just outside the engine stop-valve. An addition ought to be made for the energy contained in the steam due to its velocity in the steam-pipe, but as already seen, the proportionate amount of this energy is so small that it may be neglected, provided the steam-pipe is of sufficient dimensions, so that the velocity of the steam does not exceed, say, 50 feet to 60 feet per second.

To continue the cycle: at S adiabatic expansion commences, represented by the straight line S D, on the $\theta \phi$ chart (*Fig. 14*), and by sd , on the corresponding $p v$ diagram, and continues to the point D, that is, until a pressure of 0.15 atmosphere above the exhaust pressure has been reached, adopting the same release pressure as in the case of the saturated steam-engine.¹ It will be noticed that at the point O the expansion crosses the saturation line, and from this point onwards, therefore, the steam is wet. The remainder of the cycle is the same as in the case of the standard saturated steam-engine. It will be seen that the standard for superheated steam converts into work the heat-units represented by the area A B C S D E, so that its absolute thermal efficiency is = $\frac{A B C S D E}{A_1 A B C S S_1}$.

The following numerical example is worked out as an illustration, the temperature of steam at entrance to the engine being 420° F., the pressure, 150 lbs. absolute, and the temperature of exhaust, 110° F.

The $\theta \phi$ diagram of the standard of comparison for this numerical example is given in *Fig. 14*, and it is found that the area A B C S D E is equal to 299 B. T. U., and that the area $A_1 A B C S S_1$ is 1,143 B. T. U., so that the absolute thermal efficiency is $\frac{299}{1,143} = 26.2$ per cent.

This result can, however, be more readily obtained by means of the curves in *Fig. 14*, as the following will show.

The temperature of saturated steam corresponding to a pressure of 150 lbs. absolute is 358° F.; and from the curves (*Fig. 13*) the absolute thermal efficiency of the saturated steam-engine with an admission temperature of 358° and exhaust temperature of 110° is found to be 0.260. But the addition read off the curves marked "superheated steam" is 0.002; so that the absolute thermal

¹ In the perfect superheated steam-engine the expansion continues to the back pressure.

efficiency of this superheated steam-engine is 0.262, or 26.2 per cent. as before.

As already shown, the heat required to produce superheated steam at 420° F., and 150 lbs. pressure from 110° F. is 1,143 B.T.U. If this energy were used to make saturated steam the pressure would be 448 lbs. and the temperature 456° F., and the portion of this energy that the corresponding standard of comparison for saturated steam could convert into work is 363 B.T.U., shown by the dotted diagram in *Fig. 14*. It has just been shown, however, that the standard of comparison for superheated steam converts less than this into work; namely, 299 B.T.U., from which it appears that the superheated steam-engine does not make as good use of the heat as the saturated steam-engine, so far as the standards of comparison are concerned.

Another numerical example will illustrate this point further. One of the trials of the Schmidt engine by Professor Schröter¹ gave the following results: admission temperature $t_a = 650^\circ \text{F.}$; pressure = 185 lbs. absolute per square inch; exhaust temperature $t_e = 100^\circ \text{F.}$; feed-water per I.H.P. per hour 10.2 lbs. From these figures it works out that the absolute thermal efficiency of the actual engine was $\frac{2,543}{12,820}$, or 19.9 per cent. But the absolute thermal efficiency of the standard superheated steam-engine working under the same conditions is 29.1 per cent., so that the standard thermal efficiency of the Schmidt engine in this experiment is $\frac{19.9}{29.1}$, or 68.5 per cent.

The absolute thermal efficiency of the Carnot cycle heat-engine, working under the same range of temperature, will be found to be 49.5 per cent., much greater, it will be noticed, than that of the standard superheated steam-engine. *Fig. 14* shows the reason, namely, how small a portion of the heat is supplied in the neighbourhood of the higher temperature.

COMMERCIAL CRITERION OF ENGINE ECONOMY.

The two efficiencies which have been considered in the Paper, namely, the absolute thermal efficiency and the standard thermal efficiency, apart from being the basis on which to compare various engine-trials, are principally useful to the designer of steam-engines, especially the standard thermal efficiency, as it points

¹ See *Engineering*, vol. lix. p. 391.

out the extent to which an actual engine fails to reach perfection. The usual commercial criterion as to an engine's thermal economy is, however, the number of pounds of feed-water per hour per I.H.P., and although this measure of the economy is not accurate, because the number of heat-units per lb. of steam depends upon the pressure, yet the error is inconsiderable in the case of saturated steam for the usual ranges of pressure used; thus, 1,182 B.T.U. are required to produce steam at 100 lbs. absolute pressure, and 1,198 B.T.U. are required to produce steam at 200 lbs. absolute pressure, in both cases from 32° F., so that the error in reckoning in lbs. of feed per I.H.P. per hour is only 1·4 per cent.

With superheated steam, however, the error may be considerable. Supposing, for instance, that steam at 185 lbs. absolute pressure is superheated to 650° F.: to do this, 132 B.T.U. per lb. have to be added, so that altogether 1,328 B.T.U. per lb. are required, and the error in reckoning in lbs. of feed-water per I.H.P. is no less than 12·4 per cent. when compared with saturated steam at 100 lbs. pressure. Thus if it is found by measurement that an engine using superheated steam at 650° F. requires 10·2 lbs. of feed-water per I.H.P., this, so far as heat expenditure is concerned, which of course means coal-consumption, is equivalent to 11·46 lbs. of feed-water when compared with an engine using saturated steam at 100 lbs. pressure.

	Lbs. of Feed- Water per I.H.P. per Hour.	Apparent Com- parison.	Heat-Units per I.H.P. per Hour from 32° F.	True Com- parison.
Saturated steam-engine :—				
100 lbs. per square inch pressure . .	16·0	1·57	18,910	1·39
200 lbs. „ „ „ „	12·0	1·18	14,380	1·06
Superheated steam-engine :—				
185 lbs. per square inch pressure, super- heated to 650° F.	10·2	1·00	13,550	1·00

All condensing.

The true criterion of an engine's economy is of course heat-units required per horse-power-hour, or per kilowatt-hour, as was pointed out by Mr. Mair-Rumley in his Paper "On the Independent Testing of Steam-engines and the Measurement of Heat used," where he advocates¹ stating the number of heat-units required per I.H.P.

¹ Minutes of Proceedings Inst. C.E., vol. lxx. p. 315, and Table I, line 65 (Pamphlet Edition only).

per hour reckoned from 32° F. Thus, in the case of the super-heated steam-engine above referred to, 13,550 B.T.U. are required per I.H.P. per hour,¹ and the comparison of three engines by the two methods would be as shown in the Table on the preceding page.

The Paper is accompanied by thirteen drawings from which the *Figs.* in the text have been prepared.

¹ On the assumption that specific heat of steam is 0.48—a figure which is open to some doubt.

APPENDIXES.

APPENDIX I.

USE OF THE $\theta \phi$ CHART FOR FINDING THE ABSOLUTE THERMAL EFFICIENCY OF THE PROPOSED STANDARD ENGINE.

To find the absolute thermal efficiency of the proposed standard engine readily by means of the $\theta \phi$ chart, it is necessary to draw constant-pressure and constant-volume lines on the chart as shown in *Fig. 16*. To illustrate the method, it is proposed to find the absolute thermal efficiency of the standard

Fig. 16.

One British thermal unit is represented by the area of a square of side equal to 0.115 inch, or 0.0133 square inch.

 $\theta \phi$ CHART FOR 1 LB. OF H_2O .

for saturated steam when $\theta_a = 370^\circ + 461$, and $\theta_s = 110^\circ + 461$. Referring to *Fig. 12*, a vertical is drawn through the intersection, C, of the 370° temperature line with the saturation line; this vertical represents adiabatic expansion. From the intersection, D₁, of this vertical with the 110° temperature line, a pressure of 0.15 atmosphere (2.2 lbs. per square inch) is plotted back by means of the constant-pressure lines, and so the point D is determined, giving the

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temperature at release $\theta_r = 608^\circ \text{ F.}$, and the volume at release $V_r = 82.5$ cubic feet; thus all the data required for the formula given at page 200 have been found.

A graphic method can, however, be followed by drawing in the constant-volume line, DE , and determining the area, $ABCDE$ in any convenient manner.

It is, however, easier in practice to proceed thus. The area $A_1 A B C C_1$ is equal to the total heat of steam at C , less the water-heat at A ,

$$\begin{aligned} &= 1,195 - 110 + 32. \\ &= 1,117 \text{ B.T.U.} \end{aligned}$$

The area $A_1 A D_1 C_1$ is equal to $\theta_e \times A D_1$, and $A D_1$ can be measured off the entropy scale, and is found to be $= 1.402$. Therefore

$$\begin{aligned} \text{Area } A_1 A D_1 C_1 &= (110 + 461) \times 1.402 = 801 \text{ B.T.U.} \\ \text{Hence area } A B C D_1 &= 1,117 - 801 \\ &= 316 \text{ B.T.U.} \end{aligned}$$

The area $ED D_1$ is now measured by a planimeter, or otherwise, and is found to equal 17 B.T.U., so that

$$\begin{aligned} A B C D E &= 316 - 17 \\ &= 299 \text{ B.T.U.} \end{aligned}$$

and finally, the absolute thermal-efficiency of the standard $t_a = 370^\circ$, and $t^e = 110^\circ$.

$$= \frac{299}{1,117} = 26.8 \text{ per cent.}$$

Evidently the same method is applicable to find the absolute thermal efficiency of the standard for the superheated steam-engine.

APPENDIX II.

The efficiency of the fluid in heat-engines in general has been investigated by Rankine.¹ The Author would like to show that Rankine's result can readily

$\phi, \phi + d\phi$ respectively."

Rankine treats $abcd$ as the diagram of an elementary engine, which in the previous paragraph (265) he defines as an engine in which the reception of heat by the fluid takes place wholly at one absolute temperature (θ_a) and its

¹ "Manual of the Steam-Engine and other Prime Movers," paragraph 266.

rejection at another absolute temperature (θ_c); in other words, an elementary engine is an engine following the Carnot cycle.

Fig. 18 shows the $\theta\phi$ diagram corresponding to the $p v$ diagram $A a b B c d$ in *Fig. 17*. Considering the elementary engine whose $\theta\phi$ diagram is $a b c d$ (which corresponds to the $p v$ diagram $a b c d$, *Fig. 16*), it is clear that the heat supplied is $a b n m$, and the heat utilized is $a b c d$, so that the thermal efficiency of this small portion of the whole engine is $\frac{a b c d}{a b n m}$, which is equal to

$\left(\frac{(\theta_a - \theta_c) \delta \phi}{\theta_a \delta \phi} \right) = \frac{\theta_a - \theta_c}{\theta_a}$, which is a proof, if a proof were required, that the element of the engine is an "elementary" engine; that is, its cycle is the Carnot cycle. It will be noticed how easily this result is obtained from the $\theta\phi$ chart.

Further, since the heat utilized by the element of the engine is $(\theta_a - \theta_c) d\phi$ the heat utilized by the whole engine is,

$$\int_{\phi_A}^{\phi_B} (\theta_a - \theta_c) d\phi,$$

where ϕ_A and ϕ_B are the entropies at A and B. The total heat supplied to the engine is clearly

$$\int_{\phi_A}^{\phi_B} \theta_a d\phi,$$

so that the absolute thermal efficiency of the engine is

$$\frac{\int_{\phi_A}^{\phi_B} (\theta_a - \theta_c) d\phi}{\int_{\phi_A}^{\phi_B} \theta_a d\phi}$$

the result given by Rankine, and from which the absolute thermal efficiency of all the ideal engines referred to in the Paper can be obtained.

The cycle of the element of the engine can be followed on the $\theta\phi$ chart as follows:—Let the piston be considered at the moment it is in the position represented by a , *Fig. 17*. The space in the cylinder behind the piston now contains 1 lb. of H_2O in the condition represented by the state point a on the $\theta\phi$ chart, that is, the proportion of steam to water, the pressure, temperature and volume can be read off the $\theta\phi$ chart when mapped out with constant volume and constant pressure lines, etc., as in *Fig. 16*. An element of heat, namely, $\theta_a d\phi$, is now introduced, and under its influence the piston moves to b , *Fig. 17* and the state-point to b , *Fig. 18*. The heat-supply now ceases, and the piston moves forward owing to the expansion of the steam; the expansion line is adiabatic, represented by bc both on the $p v$ diagram and on the $\theta\phi$ chart. When the piston reaches c an element of heat, $\theta_c d\phi$, is abstracted, and the piston moves back to d , *Fig. 17*. The piston is now forced back, doing work on the 1 lb. of H_2O and compressing it adiabatically along da in *Figs. 17* and *18*. When a is reached the cycle of the element of the engine is complete, and the net work done is represented by $abcd$.