XVI.—*Effects of Strain on Electric Conductivity.* By AUGUST WITKOWSKI.
(Communicated by SIR WILLIAM THOMSON.)

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In the following communication an account is given of some experiments made in the Physical Laboratory of the Glasgow University, whose object was the measurement of the change of conductivity produced by strain in a metal conductor.

Two constants are required in order to determine this alteration in the most general case of a homogeneous isotropic conductor, strained in any way, as may be easily inferred from the analogy with elastic strains and stresses, constants expressing the change produced by uniform cubical compression and by simple distortion. The determination of the latter for brass is the object of the present paper.

The method proposed by SIR WILLIAM THOMSON for this purpose is shortly described in the Proceedings of the Roy. Soc. 1879. SIR WILLIAM THOMSON has added arrangements admitting of accurate measurements, consisting of apparatus represented in the annexed diagram, which may be described as follows :—

FIG. 1.

R is a brass tube (length = 17·8, external diam. = 1·48, internal diam. = 1·29 cm.) fastened firmly by means of a collar and screws to the wooden support P,

and bearing on its free end a lever D, whose ends are the points of application of twisting forces, produced by weights and transmitted by a system of cords and pulleys, sufficiently indicated in the diagram. The amount of twist is measured by means of a telescope, and a scale reflected in the mirror Z. An electric current, produced by a single DANIELL'S cell of several square meters surface, and therefore very small internal resistance, is conducted along the tube. Within the tube there is suspended at the end of a brass rod MN, resting on the fork-shaped supports F, one of SIR WILLIAM THOMSON'S well known galvanometer mirrors, with magnets on the back. The magnets are perpendicular to the axis of the tube, and the stream lines being parallel to it, the current has no tendency to deflect the magnet. When the tube is twisted a directing force is found to be experienced by the magnet, the direction being that of a force exerted by helical stream lines, following the lines of greatest compression of the twisted material.*

The remaining parts of the apparatus, to be presently described, are intended to measure the intensity of this force. C is a stout copper strip, bent so as to form a circular arc of nearly 360 degrees. Its ends are insulated from one another by a piece of vulcanite; and slide over two brass rods which transmit the current. The circle C is supported by suitable means, so as to remain always in a plane perpendicular to the axis of the tube, the axis passing through the centre of the tube. The circle thus forms part of the circuit and exerts a force on the magnet. Another nearly circular coil S on the opposite side of the magnet, arranged so as to have its diameter easily altered, carries the current in such a direction as to compensate the force exerted on M by the coil C, when placed at a convenient distance from the magnet. This distance must be altered, of course, when the tube is twisted, and from the amount of shifting the intensity of the magnetic force exerted by the helical stream lines may be calculated.

Now, the explanation given by Sir WILLIAM THOMSON of the phenomenon above described is this—When the tube is twisted, the material forming it is compressed in one direction and pulled out in another, the lines of greatest compression and pull being inclined at angles of 45° to the axis of the tube, in a plane tangential to a coaxial cylinder, passing through the point in question. Denote by $\frac{c}{\sqrt{2}}$ the components (equal of course) of the current c , per unit area, along these directions in the unstrained state. Then we shall have

$$\frac{c}{\sqrt{2}} (1 + e) \text{ and } \frac{c}{\sqrt{2}} (1 - e)$$

* In making the experiments described below, it was found necessary to reduce the earth's magnetic force nearly to nothing by means of steel magnets properly arranged for the purpose.

as components along these lines, when the twist is applied, which being now different in amount give a component $=ce$ in a direction perpendicular to the axis and bisecting the angle between them, e denoting a coefficient depending on the strain. Thus the effect is that of a set of stream lines parallel to the axis, and a true solenoid superimposed upon them (see Trans. Roy. Soc., 1879, Part I. p. 74).

Let us now define the constant connecting the conductivity with the strain. Consider a cube of the conducting material, whose conductivity, uniform in all directions, is k , and let it be distorted to the amount indicated by the dotted lines (fig. 2), then we shall have

$$k(1+a\sigma) \text{ and } k(1-a\sigma)$$

as expressions for the conductivity in the strained material in the directions $a'd$ and cb' , σ denoting a constant and a the amount of distortion, represented by the angle aca' . In the case of a tube, twisted at the rate ϕ per length L , the distortion in a cylindrical layer, radius ρ , is $\frac{\rho\phi}{L}$, and therefore the quantity denoted above by e is equal to $\sigma \frac{\rho\phi}{L}$.

Now, the magnetic force exerted by a solenoid of length L and radius ρ , carrying a current $=c\sigma \frac{\rho\phi}{L} d\rho$ per unit length, on a point in the middle of its axis is

$$\frac{2\pi c\sigma\phi\rho d\rho}{\sqrt{\rho^2 + \left(\frac{L}{2}\right)^2}}.$$

Denoting by $d\rho$ the thickness of the layer, we find for the action of the tube

$$F = 2\pi c\sigma\phi \left\{ \sqrt{R_2^2 + \left(\frac{L}{2}\right)^2} - \sqrt{R_1^2 + \left(\frac{L}{2}\right)^2} \right\}$$

R_1 and R_2 being its inner and outer radius ; or introducing the strength of the current $i = c\pi(R_2^2 - R_1^2)$,

$$F = \frac{2 \left\{ \sqrt{R_2^2 + \left(\frac{L}{2}\right)^2} - \sqrt{R_1^2 + \left(\frac{L}{2}\right)^2} \right\}}{R_2^2 - R_1^2} \sigma\phi i.$$

Denote by D_1 the distance from the magnet at which the circle C must be placed in order to compensate the magnetic force Mi , exerted by the circle S (and possibly also by other unmovable parts of the apparatus) before twisting ; by D_2 the corresponding distance, when the weights are put on ; and by

FIG. 2.

r the radius of the circle C ($=5.2$ cm.), then we have the following equations of equilibrium:—

$$\frac{2\pi r^2 i}{(r^2 + D_1^2)^{\frac{3}{2}}} = Mi$$

and

$$\frac{2\pi r^2 i}{(r^2 + D_2^2)^{\frac{3}{2}}} = Mi + F$$

hence

$$\sigma = \frac{K}{\phi} \left\{ \frac{1}{(r^2 + D_2^2)^{\frac{3}{2}}} - \frac{1}{(r^2 + D_1^2)^{\frac{3}{2}}} \right\}$$

where

$$K = \frac{\pi r^2 (R_2^2 - R_1^2)}{\sqrt{R_2^2 + \left(\frac{L}{2}\right)^2} - \sqrt{R_1^2 + \left(\frac{L}{2}\right)^2}}$$

a constant depending on the dimensions of the apparatus. For the apparatus described above we have—

$$K = 1515.9655.$$

The mode of experimenting is now obvious. The magnet, attached to the light mirror, is maintained in a plane perpendicular to the axis of the tube, and this position ascertained by observing the spot of light reflected on a scale from a lamp placed in the prolongation of the axis.

A few further remarks appear necessary. Although the proceeding described is a true “zero method,” the exact positions of the compensating circle C cannot be easily determined by simply shifting it along the scale, and even the deflections of the spot of light caused by twisting the tube are so small as to be scarcely visible. The circuit contained for that reason a reversing-key K, and after the approximate position of the circle had been found, more exact readings were taken by comparing the vibrations of the magnet, produced by reversing the current, while the circle C occupied two positions on both sides of the presumed zero position.

Another remark that should be made is this. The effects of permanent strain are well known to interfere in all experiments concerned with elastic bodies. They do so in the present case more than in others, inasmuch as the constant σ for permanent strains seems to exceed considerably the value found for elastic strains. Thus the distances D_1 and D_2 were found variable to a considerable extent, and the positions of the compensating coil, corresponding to equilibrium, were successively shifted, the shifting coinciding in direction with the quasi-elastic shifting, when the tube was twisted again and again in the same way. Several trials were necessary to bring the apparatus into such a condition that both the tube and the compensating coil should return to their initial positions when the weights were removed, with the precision that might

be expected. It is to this elastic part of the phenomenon that the value of σ given below is related.

The following is an instance of a set of experiments :—

Weights: on each side 8 lbs.; distance of mirror from scale, 253 cms.

Weights off,		Weights on.	
D ₁ cms.	Reading on Scale.	D ₂ cms.	Reading on Scale.
52.2	50.0	46.3	62.4
50.9	50.3	46.0	62.5
50.4	50.3	45.9	62.5
50.4	50.3	45.8	62.4
50.3	50.3	45.9	62.5
50.4	50.3	45.9	62.5
50.4	50.3	45.9	62.5

For the distances D₁ and D₂ the numbers 50.4 and 45.9 being adopted, we calculate $\phi = 0.02409$ and $\sigma = 0.155$.

The weights were varied in different experiments from 4 to 28 lbs., and the proportionality of change of conductivity and strain, assumed in the above formula, was between these limits sufficiently proved.

The main average value obtained for the constant σ and the used specimen of brass is

$$\sigma = 0.158.$$

Now, the way followed usually in determining coefficients of elasticity might be used also in the present case, and the other constant alluded to above determined by observing the change of resistance of a stretched wire.

I find for that resistance the approximate formula,

$$r = R(1 + s\lambda)$$

R being the resistance of the unstrained wire, r that of the strained, λ the longitudinal strain, and s expressed as follows :—

$$s = 1 + 2\mu + \frac{4}{3} \sigma(1 + \mu) + \rho(1 - 2\mu).$$

Here μ denotes the “Poisson’s ratio,” σ the constant forming the subject of the experiments described above, and ρ is to be defined as follows :—Take a

homogeneous isotropic conductor, strain it (without altering its shape) so as to diminish its volume in the ratio $\frac{1}{1-\theta}$, then its conductivity, being k in the unstrained state, becomes $=k(1+\rho\theta)$.

An approximate evaluation of this constant is now possible, experiments on increase of resistance for stretched brass wires being made by Mr TOMLINSON (Proc. Roy. Soc., 1877). According to his results a brass cube-centimeter opposing a resistance $=719.4 \times 10^{-8}$ (mean) to the passage of a current between its faces, being longitudinally strained to an extent $\lambda=10120 \times 10^{-13}$ exhibits an increase of resistance (in the direction of strain) equal to 1704×10^{-17} .

Mr TOMLINSON adopts for brass the value,

$$s=2.203.$$

Now, taking for brass $\mu=0.34$ (rather uncertain), and substituting for σ the number given above, we calculate

$$\rho=0.75.$$