In fact, since we have

$$m = \Sigma \frac{(m-\lambda_2)(m-\lambda_3)\dots(m-\lambda_n)}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)\dots(\lambda_1-\lambda_n)}\lambda_1,$$

we see that the matrix is expressible linearly in terms of its latent roots as follows,

$$m = e_1\lambda_1 + e_2\lambda_2 + \dots + e_n\lambda_n,$$

and the above expression for the differential follows immediately from this.

In the paper* referred to in the postscript to my former note, I was dealing with binary matrices, and took the equation

$$\frac{2m-\lambda_1-\lambda_2}{\lambda_1-\lambda_2} = \text{const.}$$

as the required condition. This, of course, is a consequence of the two conditions that should hold, but it is not sufficient in itself. It will, however, be seen that, in the application made, the two conditions are separately satisfied, so that the work is quite valid.

Note on some Properties of Bessel's Functions. By E. W. Hobson.

Received and communicated January 14th, 1897.

An integral theorem involving Bessel's functions is here given, which contains some special cases of interest. It is well known that, for integral values of m, there are an odd number of positive roots of the equation

$$J_{m+1}(x) = 0$$

lying between consecutive positive roots of the equation

$$J_m(x) = 0,$$

and this is easily seen to be true for fractional values of m; it does

* "On the Application of the Theory of Matrices to the Discussion of Linear Differential Equations with Constant Coefficients," *Proc. Camb. Phil. Soc.*, Vol. VIII., pp. 201-210.

not, however, appear to have been strictly proved that this odd number must in all cases be unity;* the truth of this theorem is here deduced from the integral theorem. The roots of the equation

$$x^{-m} J_m(x) = 0$$

for unrestricted real values of m have been discussed† by Hurwitz, who has shown that, if $m > -1$, the roots are all real; he has also shown that, if m is between -1 and -2, all the roots are real except one pair of purely imaginary roots, and also that, if m lies between the two negative integers $-2k$, $-2k-2$, the equation has k conjugate pairs of complex roots as well as an infinite number of real roots, and also one pair of purely imaginary roots in case m lies between $-2k-1$ and $-2k-2$.

1. Let u_m be any solution of Bessel's equation of order m, and v_n any solution of the equation of order n; it can then be shown that $\frac{u_m v_n}{x}$ is a perfect differential of a simple function involving u_m, v_n and their differential coefficients.

We have
$$\frac{d^2 u_m}{dx^2} + \frac{1}{x}\frac{du_m}{dx} + \left(1 - \frac{m^2}{x^2}\right) u_m = 0,$$

$$\frac{d^2 v_n}{dx^2} + \frac{1}{x}\frac{dv_n}{dx} + \left(1 - \frac{n^2}{x^2}\right) v_n = 0;$$

hence
$$x\left(u_m \frac{d^2 v_n}{dx^2} - v_n \frac{d^2 u_m}{dx^2}\right) + \left(u_m \frac{dv_n}{dx} - v_n \frac{du_m}{dx}\right) = (n^2 - m^2)\frac{u_m v_n}{x},$$

or
$$(n^2 - m^2)\frac{u_m v_n}{x} = \frac{d}{dx}\left\{x\left(u_m \frac{dv_n}{dx} - v_n \frac{du_m}{dx}\right)\right\};$$

hence
$$(n^2 - m^2)\int_\alpha^\beta \frac{u_m v_n}{x}\,dx$$

$$= \left[x\left(u_m \frac{dv_n}{dx} - v_n \frac{du_m}{dx}\right)\right]_{x=\beta} - \left[x\left(u_m \frac{dv_n}{dx} - v_n \frac{du_m}{dx}\right)\right]_{x=\alpha}, \quad (1)$$

where α, β are any quantities such that the integral is convergent at each limit.

Let
$$u_m = J_m(x), \quad v_n = J_n(x);$$

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* See Gray and Mathews, *Treatise on Bessel Functions*, p. 50.

† See *Mathematische Annalen*, Vol. XXXIII. (1889).

then, if $m+n$ is positive, we have

$$(n^2-m^2)\int_0^a \frac{J_m(x)J_n(x)}{x}\,dx = a\left[J_m(a)\frac{dJ_n(a)}{da} - J_n(a)\frac{dJ_m(a)}{da}\right],$$

or, using the relation

$$\frac{dJ_m}{da} = \frac{m}{a}J_m - J_{m+1},$$

this becomes

$$(n^2-m^2)\int_0^a \frac{J_m(x)J_n(x)}{x}\,dx$$
$$= (n-m)J_m(a)J_n(a) + a\left\{J_n(a)J_{m+1}(a) - J_m(a)J_{n+1}(a)\right\}. \quad (2)$$

This theorem seems an obvious one, but I have not been able to find it in any writings on the subject.*

Whatever m and n are, we have

$$(n^2-m^2)\int_a^\beta \frac{J_m(x)J_n(x)}{x}\,dx = (n-m)\left\{J_m(\beta)J_n(\beta) - J_m(a)J_n(a)\right\}$$
$$+\beta\left\{J_n(\beta)J_{m+1}(\beta) - J_m(\beta)J_{n+1}(\beta)\right\}$$
$$-a\left\{J_n(a)J_{m+1}(a) - J_m(a)J_{n+1}(a)\right\}.$$

Let a, β be roots of the equation

$$x^{-n}J_n(a) = 0;$$

this then becomes

$$(n^2-m^2)\int_a^\beta \frac{J_m(x)J_n(x)}{x}\,dx = aJ_m(a)J_{n+1}(a) - \beta J_m(\beta)J_{n+1}(\beta).$$

In particular, let $n = m+1$, then use the equation

$$J_{m+2} = \frac{2m+2}{x}J_{m+1} - J_m,$$

and we have, since

$$J_{m+1}(a) = 0, \quad J_{m+1}(\beta) = 0,$$

$$(2m+1)\int_a^\beta \frac{J_m(x)J_{m+1}(x)}{x}\,dx = \beta J_m^2(\beta) - aJ_n^2(a), \quad (3)$$

where a, β are roots of the equation

$$x^{-m-1}J_{m+1}(x) = 0.$$

3. We shall now apply the theorem (3) to show that there can only be one positive zero of the function $x^{-m-1}J_{m+1}(x)$ between two

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* A referee has pointed out that the theorem (1) has been given by Sonine, *Mathematische Annalen*, Vol. XVI.

consecutive positive zeros of the function $x^{-m}J_m(x)$. The relation

$$x^{-m-1}J_{m+1}(x) = -2x\frac{d}{d(x^2)}\{x^{-m}J_m(x)\}$$

shows that the number of zeros must be odd, and that they are at the maxima and minima of $x^{-m}J_m(x)$. If possible, let there be three or more zeros $\alpha, \beta, \gamma, \ldots$ in ascending order and taken positive, all lying between consecutive zeros of $x^{-m}J_m(x)$. We have then

$$(2m+1)\int_\alpha^\beta \frac{J_m(x)J_{m+1}(x)}{x}\,dx = \beta J_m^2(\beta) - \alpha J_m^2(\alpha),$$

$$(2m+1)\int_\beta^\gamma \frac{J_m(x)J_{m+1}(x)}{x}\,dx = \gamma J_m^2(\gamma) - \beta J_m^2(\beta).$$

Now $J_m(x)$ is of invariable sign within the interval α to γ, and $J_{m+1}(x)$ is of opposite signs in the two intervals α to β and β to γ. Hence, if $2m+1$ is positive, each side of that one of these equations is negative for which $J_m(x)$, $J_{m+1}(x)$ have opposite signs; if $2m+1$ is negative, each side of that equation is negative for which $J_m(x)$, $J_{m+1}(x)$ have the same signs.

Also
$$\frac{d}{dx}\{xJ_m^2(x)\} = J_m(x)\left\{J_m(x) + 2x\frac{dJ_m}{dx}\right\}$$
$$= J_m(x)\{(2m+1)J_m(x) - 2xJ_{m+1}(x)\}.$$

Hence, if $2m+1$ is positive, and $J_m(x)$, $J_{m+1}(x)$ have opposite signs, $xJ_m^2(x)$ increases as x increases; this is contrary to what has been shown above, that the one of the expressions

$$\beta J_m^2(\beta) - \alpha J_m^2(\alpha), \quad \gamma J_m^2(\gamma) - \beta J_m^2(\beta)$$

is negative for which in the integral $J_m(x)$, $J_{m+1}(x)$ have opposite signs. Similarly, if $2m+1$ is negative, $xJ_m^2(x)$ decreases as x increases, if $J_m(x)$, $J_{m+1}(x)$ have the same sign; this is also contrary to what has been shown above.

It has thus been shown that there cannot be more than one positive zero of $x^{-m-1}J_{m+1}(x)$ between two consecutive positive zeros of $x^{-m}J_m(x)$.

4. If, in (1), we let $u_m = Y_m(x)$, $u_n = Y_n(x)$ where Y denotes the function of the second kind, we have

$$(n^2-m^2)\int_a^\infty \frac{Y_m(x)\,Y_n(x)}{x}\,dx = -(n-m)\,Y_m(a)\,Y_n(a)$$
$$+a\{Y_m(a)\,Y_{n+1}(a) - Y_n(a)\,Y_{m+1}(a)\}, \quad (4)$$

where $a > 0$, the integral being convergent at the upper limit.

We have also, as a special case of (1),

$$(n^2-m^2)\int_a^\infty \frac{Y_n(x)\,J_m(x)}{x}\,dx$$

$$= -(n-m)\,Y_n(a)\,J_m(a) + a\left\{J_m(a)\,Y_{n+1}(a) - Y_n(a)\,J_{m+1}(a)\right\}. \quad (5)$$

For the function $K_m(x)$, which is a Bessel's function of argument ix, and is so chosen that $K_m(\infty) = 0$, we have

$$(n^2-m^2)\int_a^\infty \frac{K_m(x)\,K_n(x)}{x}\,dx$$

expressed in a similar form.

We have, from (2),

$$(n^2-m^2)\int_0^\infty \frac{J_m(x)\,J_n(x)}{x} = \underset{a=\infty}{L}\, a\left\{J_n(a)\,J_{m+1}(a) - J_m(a)\,J_{n+1}(a)\right\}.$$

Substituting on the right-hand side the asymptotic values of the functions, the expression becomes

$$\frac{2}{\pi}\left\{\cos\left(\frac{2n+1}{4}\pi - a\right)\cos\left(\frac{2m+3}{4}\pi - a\right)\right.$$

$$\left. - \cos\left(\frac{2m+1}{4}\pi - a\right)\cos\left(\frac{2n+3}{4}\pi - a\right)\right\},$$

or

$$\frac{1}{\pi}\left\{\cos\frac{n-m-1}{2}\pi - \cos\frac{n-m+1}{2}\pi\right\},$$

which is

$$\frac{2}{\pi}\sin\frac{n-m}{2}\pi;$$

thus

$$\int_0^\infty \frac{J_m(x)\,J_n(x)}{x}\,dx = \frac{2}{n^2-m^2}\,\frac{1}{\pi}\sin\frac{n-m}{2}\pi. \quad (6)$$

If $n-m$ is an even integer,

$$\int_0^\infty \frac{J_m(x)\,J_n(x)}{x}\,dx = 0; \quad (7)$$

and, if $n = m$, and m is positive,

$$\int_0^\infty \frac{J_m^2(x)}{x}\,dx = \frac{1}{2m}. \quad (8)$$

The theorems (7) and (8) have been obtained otherwise by Heine.*

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* See *Kugelfunctionen*, Vol. I., p. 255.

5. In (2), let $n = m+\delta m$, and suppose a to be a root of the equation $x^{-m} J_m(x) = 0$; we have then

$$2m\,\delta m \int_0^a \frac{J_m^2(x)}{x}\,dx = a\left\{-J_{m+1}(a)\,\frac{dJ_m(a)}{da}\,\delta a\right\},$$

where $a+\delta a$ is such that $J_{m+\delta m}(a+\delta a) = 0$, and the squares of δm, δa are omitted.

Hence, since $$\frac{dJ_m(a)}{da} = -J_{m+1}(a),$$

when $J_m(a) = 0$, we have

$$\frac{da}{dm} = \frac{2m}{a\left\{J_{m+1}(a)\right\}^2}\int_0^a \frac{\left\{J_m(x)\right\}^2}{x}\,dx, \tag{9}$$

a theorem which has been given without proof by Schläfli.*

The theorem (9) shows that a root a, considered as a function of the order m, increases as the order increases.

P.S.—Since writing the above I have received from the author, a copy of a paper just published in *The American Journal of Mathematics*† which contains a proof of the theorem proved above, that the positive roots of the equations $J_m(x) = 0$ and $J_{m+1}(x) = 0$ occur alternately. As the method given above is an entirely different one, I have not thought it desirable to suppress the note, especially as it contains other results of some interest.

Thursday, February 11th, 1897.

Prof. ELLIOTT, F.R.S., President, in the Chair.

Ten members present.

Mr. Macaulay read a paper "On a Theorem in non-Euclidean Geometry." An animated discussion followed, in which Messrs. Kempe, Cunningham, Love, and the President, joined with the author.

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* *Mathematische Annalen*, Vol. x.

† "On the Roots of Bessel and P Functions," by E. B. Van Vleck, Vol. xix.

An impromptu communication was made by Mr. Kempe in connexion with Prof. Sylvester's communication at the January meeting. The President and Major MacMahon spoke on the subject.

The President (Major MacMahon, Vice-President, in the Chair) gave a short account of Mr. Segar's theorem that the product of the differences of n unequal numbers is divisible by the product of the differences of 0, 1, 2, ... $(n-1)$, and showed also that the product of the differences of n unequal square numbers is divisible by the product of the differences of $0^2, 1^2, 2^2, \ldots (n-1)^2$.*

Lt.-Col. Cunningham brought forward some high primes.

A paper by Mr. H. M. Taylor "On the Degeneration of a Cubic Curve" was communicated by reading its title.

The following presents were made to the Library :—

"Catalogue of the Michigan Mining School, 1894-6," 8vo; Houghton, Michigan, 1896.

"Proceedings of the Royal Society," Vol. LX., No. 365.

"Proceedings of the Royal Irish Academy," Vol. IV., No. 1; Dublin, 1896.

Emmens, S.—"The Argentaurum Papers, No. 1, Some Remarks concerning Gravitation."

"Journal of the Institute of Actuaries," Vol. XXXIII., Pt. II., No. 181; London, January, 1897.

"Beiblätter zu den Annalen der Physik und Chemie," Bd. XX., St. 12, 1896, and Bd. XXI., St. 1; Leipzig, 1897.

"Proceedings of the Physical Society of London," Vol. XV., Pt. 1, No. 76; January, 1897.

"Revue Semestrielle des Publications Mathématiques," Tome V., Partie 1e; April-October, 1896.

"Monatshefte für Mathematik und Physik," Jahrgang VIII., 1897, Pt. I.; Wien.

"Archives Néerlandaises des Sciences Exactes et Naturelles," Tome XXX., Liv. 4; Harlem, 1896.

"Rendiconto dell' Accademia delle Scienze Fisiche e Matematiche," Serie 3, Vol. II., Fasc. 12, December, 1896; Napoli.

"Bulletin de la Société Mathématique de France," Tome XXIV., No. 8 et dernier; Paris.

"Bulletin of the American Mathematical Society," 2nd Series, Vol. III., No. 4; January, 1897; New York.

"Bulletin des Sciences Mathématiques," Tome XXI., January, 1897; Paris.

"Atti della Reale Accademia dei Lincei—Rendiconti," Sem. 2, Vol. V., Fasc. 12, 1896, Sem. 1, Vol. VI., Fasc. 1, 2, 1897; Roma.

"Educational Times," February, 1897.

"La Naturaleza," Tome VIII., Núm. 1; Madrid.

Kantor, S.—"Theorie der periodischen cubischen Transformationen im Raume R_3,"

* *Cf.*, *Messenger of Mathematics*, CCCXIII., pp. 12–15.

4to pamphlet (from "American Journal," Vol. xix., No. 1); "Ueber Collineationsgruppen an Kummer'schen Flächen," 4to pamphlet.

"Indian Engineering," Vol. xx., No. 26, Vol. xxi., Nos. 1, 2, 3; December 26, 1896–January 10, 1897.

The following is the note communicated by Lt.-Col. Cunningham:—

The 43 high primes (p) on the list below are all beyond the present factor-tables, *i.e.*, are all > 9 million. They are all of form

$$p = 10\varpi + 1,$$

and were found as divisors of numbers of form

$$N = (x^5 + y^5) \div (x + y),$$

where $$y = \pm 1;$$

their *prime* character was recognised from this property by aid of a MS. Table, prepared by the author, giving the base x of *all* numbers of form N with prime factors p of form $(10\varpi + 1) \not> 5060$. The Table below shows the values of x, y giving the number N from which they were derived.

The fact of being factors of such numbers (N) facilitates the discovery* of the *residuacity* of the base 2 to such primes: this is shown in the Table by the letters q, n.

q denotes that 2 is a *quintic residue* of p; *i.e.*, $2^{\frac{1}{5}(p-1)} \equiv 1 \pmod{p}$,

n denotes that 2 is a *quintic non-residue* of p; *i.e.*, $2^{\frac{1}{5}(p-1)} \not\equiv 1 \pmod{p}$.

Three of these primes were previously† known, as factors of numbers of form

$$N = 2^{\xi} - 1,$$

viz., $p =$ 10,567,201; 13,334,701; 18,837,001;

$\xi =$ 75; 300; 90.

The author's present research confirms the fact that they are *primes*: the remaining forty are believed to be *new primes*. In three cases, it has been found possible to determine the *minimum* exponent (ξ), for which $2^{\xi} \equiv 1 \pmod{p}$, and therewith the *maximum* residue-index

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* *See* Mr. Bickmore's paper "On the Numerical Factors of $(a^n - 1)$," in the *Messenger of Mathematics*, Vol. xxvi., Art. 21, *et seq.*

† *See* Ed. Lucas's paper *Sur la Série Récurrente de Fermat*, pp. 8–10, Rome, 1879. The mode of determining the *prime* character is not stated.

$(p-1) \div \xi$, which gives the residuacity of the base 2, viz.,

$$p = 9{,}792{,}191;\quad 13{,}133{,}291;\quad 14{,}106{,}971;$$
$$\xi = 5\,.\,979{,}219;\quad 2\,.\,131{,}329;\quad 2\,.\,1{,}410{,}697;$$
$$(p-1)\div\xi = \quad 2;\qquad 5;\qquad 5.$$

HIGH PRIMES.

p	x	y		p	x	y	
9,170,881	139	$\bar{1}$	*n*	16,007,041	63	$\bar{1}$	*n*
9,367,291	101	$\bar{1}$	*n*	16,039,531	115	$\bar{1}$	*n*
9,384,251	3125	$\bar{1}$	*n*	16,776,481	411	$\bar{1}$	*n*
9,498,581	380	$\bar{1}$	*n*	17,501,251	188	1	*n*
9,792,191	189	$\bar{1}$	*n*	17,562,031	153	1	*n*
10,332,211	103	$\bar{1}$	*n*	17,994,481	309	1	*n*
10,487,921	262	$\bar{1}$	*n*	18,304,611	4205	$\bar{1}$	*n*
10,567,201	32^3	$\bar{1}$	*n*	18,837,001	512	1	*n*
11,006,851	146	1	*q*	19,019,801	99	$\bar{1}$	*q*
11,106,421	241	$\bar{1}$	*n*	19,622,651	181	$\bar{1}$	*n*
11,249,741	287	1	*q*	20,411,341	310	1	*n*
11,295,311	231	1	*q*	20,640,071	123	1	*q*
11,498,111	311	1	*n*	21,700,501	68	$\bar{1}$	*n*
11,501,761	163	1	*n*	22,199,431	199	$\bar{1}$	*n*
12,483,671	108	$\bar{1}$	*q*	22,284,781	295	$\bar{1}$	*q*
12,827,531	341	1	?	22,996,651	243	1	*n*
13,133,291	244	$\bar{1}$	*q*	24,253,721	233	1	*n*
13,334,701	1024^3	1	*n*	25,058,741	71	1	*q*
14,106,971	321	$\bar{1}$	*q*	25,410,401	5120	$\bar{1}$	*q*
14,715,341	230	$\bar{1}$	*n*	25,447,421	269	1	*n*
14,961,091	147	1	*n*	25,621,901	194	1	*n*
15,018,571	62	$\bar{1}$	*n*				

Thursday, March 11th, 1897.

Prof. ELLIOTT, F.R.S., President, in the Chair.

Eleven members present.

The following gentlemen were elected members of the Society: Paul Jerome Kirkby, M.A. Hertford College, Oxford, Lecturer in

Mathematics at St. David's College, Lampeter; Frederick William Lawrence, B.A., Scholar of Trinity College, Cambridge; and Alfred Young, B.A., Scholar of Clare College, Cambridge, of Ridley Hall, Cambridge.

The President referred to a letter received from the President of the Royal Society with reference to the Victoria Research Fund, which it is proposed to institute in commemoration of Her Majesty's long reign; and commended the fund to the generous consideration of members. He then spoke briefly on the loss the mathematical world has sustained by the recent death of Prof. Weierstrass.*

Mr. M. Jenkins, Vice-President, having taken the Chair, the President communicated a paper by Mr. J. E. Campbell, "On a Law of Combination of Operators bearing on the Theory of Continuous Transformation Groups."

The President, having resumed the Chair, read some "Notes on Symmetric Functions," by Mr. W. H. Metzler.

The senior Secretary briefly communicated a "Note on a System of Circles associated with a Triangle," by Prof. J. E. A. Steggall.

Lt.-Col. Cunningham announced (at this and the subsequent meeting) the eight † following high *primes*, and gave a sketch of the methods he had employed in determining them:—

$$47,763,361 = \frac{(3^{30}+1)(3^2+1)}{(3^{10}+1)(3^6+1)}; \quad 85,280,581, \text{ the large factor of } (5^{35}-1);$$

$$234,750,601 = (5^{14}+1) \div (5^2+1);$$

$$\frac{5^{13}-1}{5-1} = 305,175,781; \quad \frac{1}{409}\,\frac{5^{17}-1}{5-1} = 466,344,409;$$

$$550,554,229 = \text{the large factor of } (3^{63}+1);$$

$$632,133,361 = \frac{1}{241}\,\frac{5^{20}+1}{5^4+1}; \quad 2,413,941,289 = \frac{3^{33}-1}{3^{11}-1}\cdot\frac{3^1-1}{3^3-1};$$

also the *completion* of the resolution of $(3^{105}+1) \div (3^{35}+1)$ previously given by Mr. C. E. Bickmore ‡ as

$$81,365,467,681 \,.\, (211 \,.\, 1051 \,.\, 3,454,081),$$

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* See *Nature* (March 11th, 1897, p. 443).

† Three (the 2nd, 3rd, and 8th) were announced at this meeting, and the rest at the meeting of 8th April; but it has been thought most convenient to collect them all together here.

‡ In *Messenger of Mathematics*, Vol. xxv., paper "On the Numerical Factors of (a^n-1)," foot of Art. 7.

into its *prime* factors, now found to be

$$(24151 . 3,369,031)(211 . 1051 . 3,454,081).$$

This number $(3^{105}+1)$ is interesting as being one of the largest ever completely resolved into *prime* factors.

The following presents were made to the Library :—

"Jahrbuch über die Fortschritte der Mathematik," Bd. xxv., Heft 3, Jahrgang 1893 and 1894 ; Berlin, 1897.

"Proceedings of the Royal Society," Vol. LX., No. 366.

"Beiblätter zu den Annalen der Physik und Chemie," Bd. XXI., St. 2 ; Leipzig, 1897.

"Proceedings of the Cambridge Philosophical Society," Vol. IX., Pt. 4 (Michaelmas Term, 1896) ; 1897.

Smyth, A. H.—"Memoir of Henry Phillips, Jr.," pamphlet, 8vo ; 1896 (American Philosophical Society, November, 1896).

"Proceedings of the Physical Society," Vol. xv., Pt. 2 (February, 1897), Pt. 3 (March, 1897) ; London.

"Memoirs and Proceedings of the Manchester Literary and Philosophical Society," Vol. XLI., Pt. 11, 1897.

"Berichte über die Verhandlungen der Königl. Sächs. Gesellschaft der Wissenschaften zu Leipzig" (Mathematisch-Physische Klasse, 1896), IV. ; 1897.

"Bulletin des Sciences Mathématiques," Tome XXI., Fev., 1897 ; Paris.

Kantor, S.—"Über n Momente von R_i Complexen im R_r," pamphlet, 8vo ; München, 1897.

"Nyt Tidsskrift for Matematik," A. Aargang 7, Nr. 8, 1896 ; B. Aargang 7, Nr. 4, Aargang 8, Nr. 1, 1897 ; Copenhagen.

"Bulletin of the American Mathematical Society," 2nd Series, Vol. III., No. 5, February, 1897 ; New York.

"Nachrichten von der Königl. Gesellschaft der Wissenschaften zu Göttingen," 1896, Heft 4.

"Rendiconti del Circolo Matematico di Palermo," Tomo XI., Fasc. 1 and 2 ; 1897.

"Rendiconto dell' Accademia delle Scienze Fisiche e Matematiche," Vol. III., Fasc. I. ; Napoli, 1897.

"Atti della Reale Accademia dei Lincei—Rendiconti," Sem. 1, Vol. VI., Fasc. 3 ; Roma, 1897.

Wessel, C.—"Essai sur la Représentation analytique de la Direction," 4to ; Copenhague, 1897.

"Educational Times," March, 1897.

"Annals of Mathematics," Vol. XI., No. 2.

"Journal für die reine und angewandte Mathematik," Bd. CXVII., Heft 3 ; Berlin, 1897.

"Indian Engineering," Vol. XXI., Nos. 4-7, Jan. 23-Feb. 13, 1897.