XLIII.—*On the Knowledge of Distance given by Binocular Vision.* By Sir David Brewster, K.H., D.C.L., F.R.S., and V.P.R.S. Edinburgh.

(Read 15th April 1844.)

In analysing Mr Wheatstone's beautiful discovery, that in binocular vision we see all objects of three dimensions by means of two dissimilar pictures on the retina, I trust I have satisfied the Society that the dissimilarity of these two pictures is in no respect the cause of our vivid perception of such objects, but, on the contrary, an unavoidable accompaniment of binocular vision, which renders it less perfect than vision with one eye. On the other hand, it is quite true that, in Mr Wheatstone's experiment of producing the perception of objects of three dimensions by the apparent coalescence of two dissimilar representations of such objects *in plano,* the dissimilarity of the pictures is necessary in the exhibition of that beautiful phenomenon.

In performing, with the eye alone, the various experiments detailed in a former paper, I was very much struck with the fact, that the apparent solid figure, produced by the union of its dissimilar pictures, never took its right position in absolute space: that is, in place of appearing suspended between the eye and the plane upon which the dissimilar figures were drawn, the base of the solid seemed to rest on that plane, whether its apex was nearer the eye or more remote than its component plane figures.

With the view of finding the cause of this, I placed the component figures on a plate of glass suspended in the air, so as to have no vision of the surface on which they rested, and after uniting these figures by binocular vision, and concealing the two outstanding single figures, I obtained results which, though not entirely satisfactory, proved that there existed some disturbing cause which prevented the united image from placing itself in the *binocular centre,* or the intersection of the optical axes. This disturbing cause was simply the influence of other objects in the same field of view, whose distance was known to the observer.

In order to avoid all such influences, and to study the subject under a more general aspect, it occurred to me that these objects would be gained by using a numerous series of plane figures, such as those of flowers or geometrical patterns upon carpets or paper-hangings. These figures being always at equal distances from each other, and almost perfectly equal and similar, the coalescence of any pair of them, by directing the optic axes to a point between the paper-hangings and the eye, is accompanied with the coalescence of every other pair. When the observer, therefore, places himself in front of that side of a papered room in which there are neither doors nor windows, and conceals from his eye the floor, the roof, and the right and left hand sides of the room, the whole of his retina will be covered with the images of the united plane figures, and there will be no interposing objects to prevent him from judging of the distance of the picture that may be presented to him.

Let the observer, therefore, now place himself *three* feet in front of the papered wall, and unite two of the figures, suppose two flowers, at the distance of *twelve* inches. The whole wall will now be presented to his view, consisting of flowers as before, but each flower will be composed of two flowers superimposed at the binocular centre, or the point of convergence of the optical axes. If we call D the distance of the eyes from the wall or *three* feet, C the distance between the eyes or two-and-half inches, and d the distance between the similar parts of the two flowers, we shall have x the distance of the binocular centre from the wall, $x = \frac{\mathrm{D}\,d}{\mathrm{C}+d} = 30$ inches nearly, and $\mathrm{D} - x = 6$ inches, the distance of the binocular centre from the middle point between the two eyes.

Hence the whole papered wall, with all its flowers, in place of being seen, as in ordinary vision, at the distance of *three* feet, *is now suspended in the air, at the distance of six inches from the observer.* In maintaining this view of the wall, the eye will, at first, experience a disagreeable sensation; but after a few experiments the sensation will disappear, and the observer will contemplate the new picture with the same satisfaction and absence of all strain as if he were looking directly at the wall itself: for there is a natural tendency in the eyes to unite two similar pictures, and to keep them united, provided they are not too distant.

When this picture is at first seized by the observer, he does not, for a while, decide upon its distance from himself. It sometimes appears to advance from the wall to its true position in the binocular centre, and, when it has taken its place, it has a very extraordinary character :—the surface seems slightly convex towards the eye; it has a sort of silvery transparent aspect, and looks more beautiful than the real paper; it moves, with the slightest motion of the head, either laterally or to or from the wall. If the observer, who is now *three* feet from the wall, retires from it, the suspended wall of flowers will follow him, moving farther and farther from the real wall, and also, but very slightly, farther and farther from the observer: that is, the distance of the observer from the real wall increases faster than the distance of the suspended wall from it, according to the law expressed by the preceding formula. The binocular centre, therefore, recedes from the eye as the observer retires, and the strain consequently diminishes.

in order to observe these phenomena in the most perfect manner, the paper

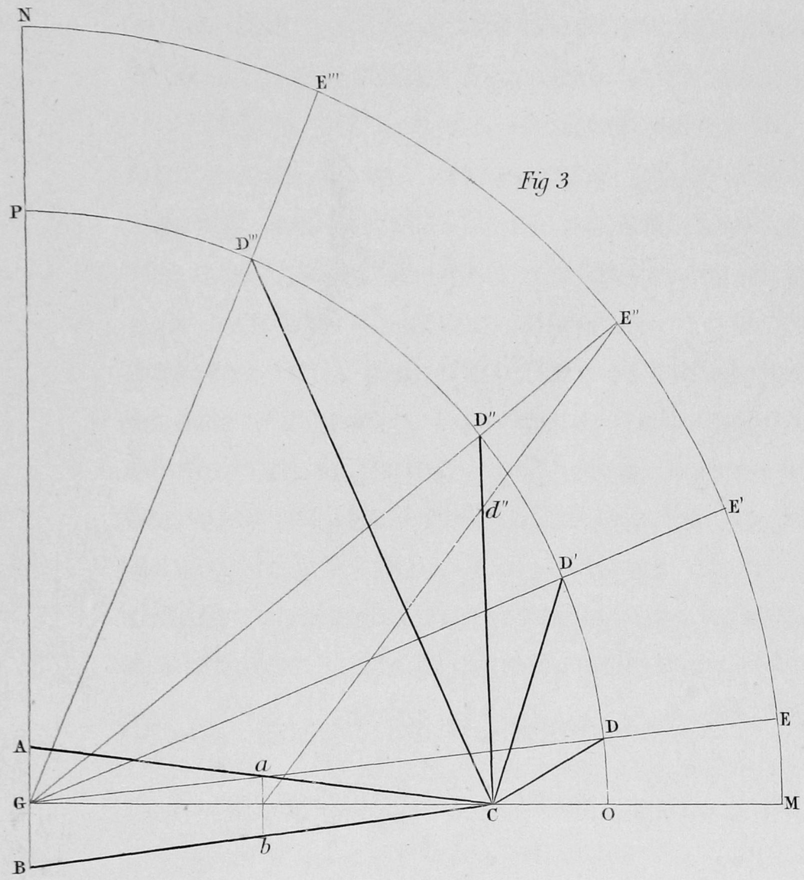


Fig 3

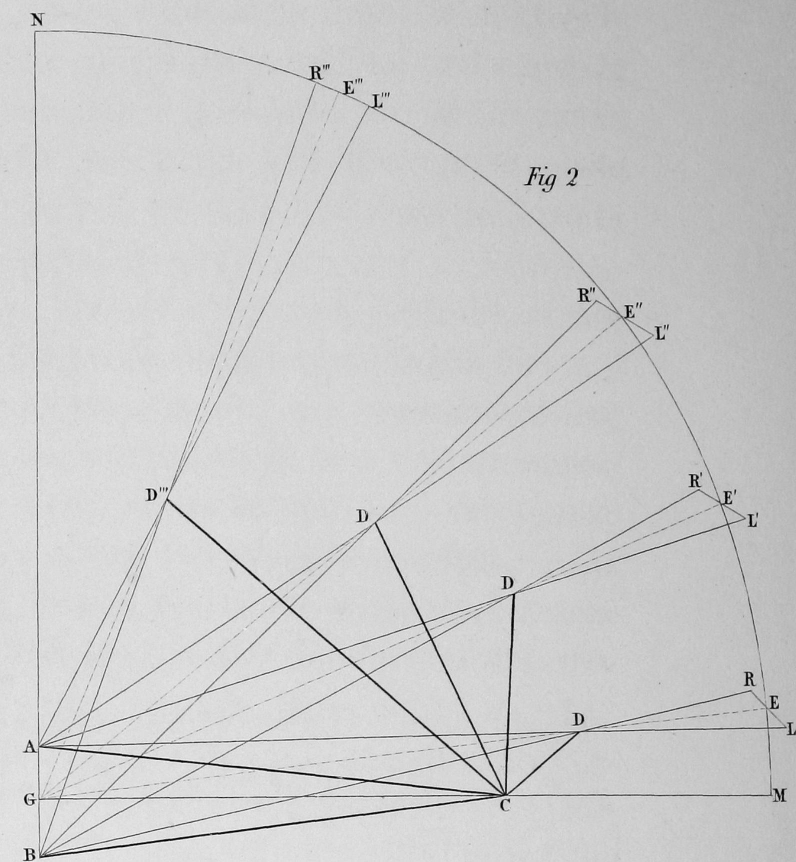


Fig 2

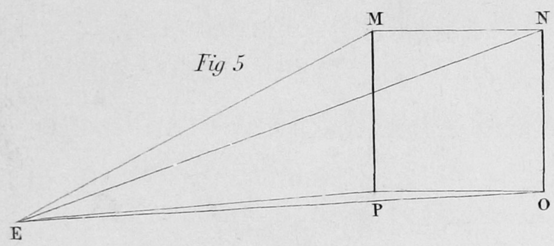


Fig 5

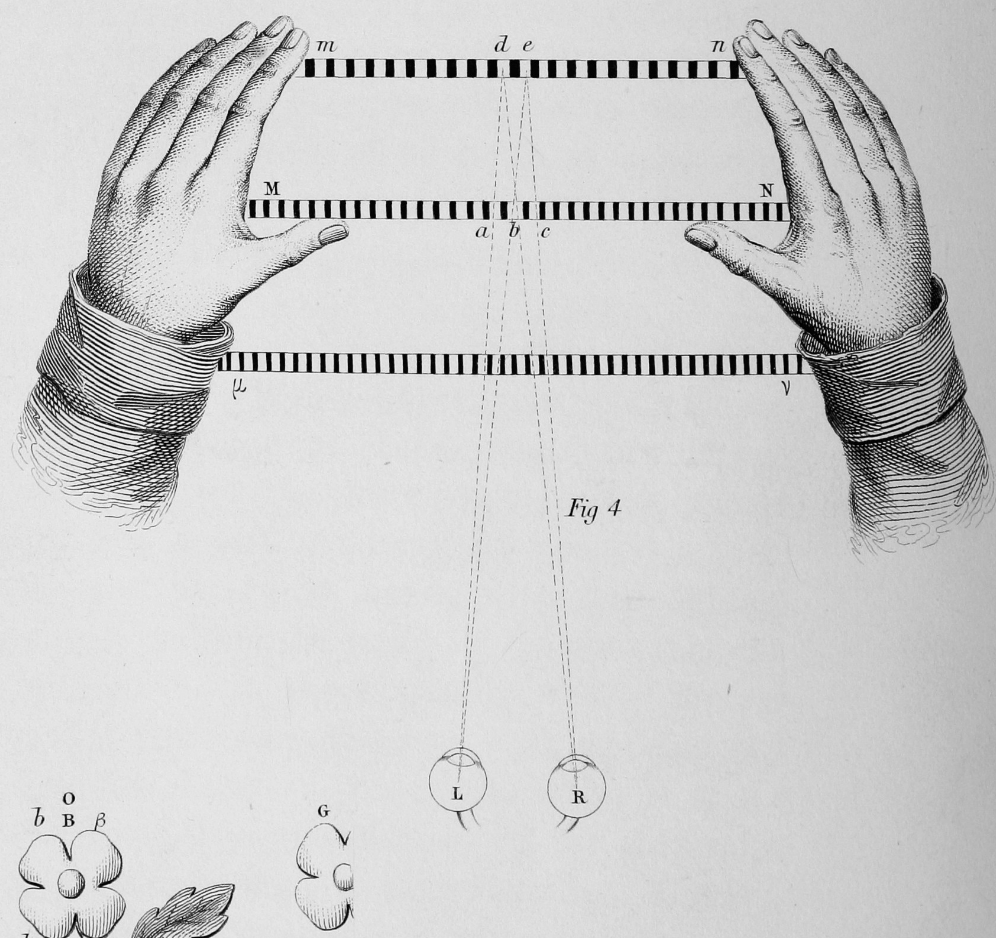
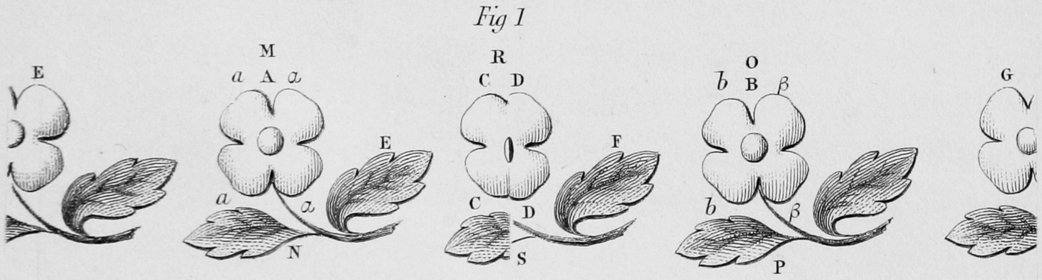


Fig 4

Fig 1



Engd by W. & A. K. Johnston

should be pasted upon a large screen, previously unseen by the observer, unconnected with the roof or the floor, and placed in a large apartment. The deception will then be complete; and when the picture stands suspended before the observer, and within a few inches of himself, he may stretch out his hand and place it on the other side of the picture, and even hold a candle on the other side of it, so as to satisfy himself that in both cases the picture is between his hand and himself.

When we survey this picture with attention, several very curious phenomena present themselves. Some of the flowers, when narrowly examined, appear somewhat like real flowers. In some the stalk gradually retires from the general plane of the picture; in others, it rises above it: one leaf will come farther out than another, or the flower will appear thicker and more solid, deviating considerably from the plane representation of it seen by each eye separately. All this arises from slight and accidental irregularities in the two figures which are united, thus producing an approximation to three dimensions in the picture. If the distance, for example, of the ends of two stalks in two coalescing flowers is greater than the distance of corresponding points in other parts of the stalk, the end of the stalk will rise from the general surface of the figure, and *vice versa*. In like manner, if the distance between two corresponding leaves is greater than the distance between other two corresponding leaves, then the two first, when united, will appear nearer the eye than the other two, and hence the appearance of a solid flower is partially given to the combination. These effects are better seen in old and imperfectly made paper-hangings than in those which are more carefully executed.

In continuing our survey of the suspended image, another curious phenomenon presents itself: a part of one of the pieces of paper, and sometimes a whole stripe from the roof to the floor, will retire behind the general plane of the image, or rise above it; thus displaying, on a large scale, an imperfection in the workmanship which it would have required a very narrow inspection to discover. This defect arises from the paper-hanger having cut off too much of the white margin of one or more of the adjoining pieces, so that when the two halves of a flower are united, part of the middle of the flower is left out; and hence when this defective flower is united with the one on the right hand of it, and the one on the left hand united with the defective one, the united or corresponding portion, being at a less distance, will appear farther from the eye than those parts of the suspended image composed of complete flowers. In like manner, if the two portions of the flowers are not brought together, but separated by a small space, the opposite effect will be produced. This will be understood from Fig. 1 (Plate 17), where M N, O P represent portions of two separate pieces of paper, each twenty-one inches wide. In this specimen, there are only two flowers in each piece, namely one white flower, A or B, and two halves. If the two halves

C, D, are united as in the figure, it is obvious that the flower is incomplete, a part of the central circle of the corolla having been cut off from each half. If we now, by straining the eye, unite C D with B, and also with A, then, at the same time, E will be united with the second or left hand image of A, and G with the second or right hand image of B. But since a piece has been cut out of C D, the half $\alpha\,\alpha$ of A is nearer the half D D than the other half $a\,a$ is to the other half C C; and, in like manner, the half $b\,b$ of B is nearer the half C C than the other half $\beta\,\beta$ is to the other half D D. Hence, when the strained eyes unite $\alpha\,\alpha$ to D D, the binocular centre is more remote than when $a\,a$ is united to C, and the same is true of the other halves; consequently, the halves D D and $b\,b$ must appear, as it were, sunk in the wall, or as farther removed from the observer; and if the defective cutting exists along the line R S from the floor to the ceiling, the whole stripe of paper between R S and O P, from the floor to the ceiling, will appear sunk in the papered wall. But if the defect is confined to a portion only of the flowers, then a rectangular space of the breadth R O, and of a height equal to the defective portion, will appear sunk in the paper. If every junction has the same defect as that at R S, then the whole will appear to consist of equal stripes, every alternate one being raised and the other depressed.

In the preceding example, there are only *two* flowers in a breadth, and their distance is $10\frac{1}{2}$ inches, which is also the breadth of the sunk stripes. But if the flowers are three or four in number, and their distance $\frac{21}{3}$, $\frac{21}{4}$ inches, the sunk stripes will vary according as we unite two flowers whose distances are in the one case 7 or 14 inches, and $5\frac{1}{4}$ or $10\frac{1}{2}$ or $16\frac{3}{4}$ or 21 in the other. Calling B the breadth of the paper, n the number of flowers or figures in that breadth, and W the width of the sunk stripe, then we have $W=\frac{B}{n}$ or $\frac{2\,B}{n}$ or $\frac{3\,B}{n}$ according as we unite the two nearest, or the first and second flower, the first and third, or the first and fourth. When $W=B$, the sunk stripes will cover the whole paper, and all the flowers will lie in the same plane.

These results afford an accurate method of examining and discovering defects in the workmanship of paper-hangers, carpet-makers, painters, and other artists whose profession it is to combine a series of similar patterns in order to form an uniform and ornamental surface. The smallest defect in the similarity and equality in the figures or lines which compose a pattern, and any difference in the distance of the single figures, is instantly detected; and, what is remarkable, a small inequality of distance in a line perpendicular to the axis of vision, or in one dimension of space, is exhibited in a magnified form as a distance coincident with the axis of vision, and in an opposite dimension of space!

At the commencement of this class of experiments, it is difficult to realize, and very easy to dissolve, the singular binocular picture which we have been describing; but after the eyes have been drilled for a while to this species of exercise, the pictures become very persistent. Although the air-suspended image might be expected to disappear after closing one eye, and still more after having closed and re-opened both, yet I have found it in its original position in this latter case, and even after rubbing my eyes and shaking my head; and I have sometimes experienced a difficulty in ascertaining, after these operations, whether it was the real or the air-suspended wall that was before me. On some occasions a singular effect was produced. When the flowers on the paper are distant six inches, we may either unite two six inches distant, or two twelve inches distant. In the latter case, when the eyes have been accustomed to survey the suspended picture, I have found that, after shutting and opening them, I neither saw the picture formed by the two flowers twelve inches distant, nor the papered wall itself, but a picture formed by uniting the flowers six inches distant! The binocular centre had shifted its place, and instead of advancing to the wall, as is generally the case, and giving us ordinary vision of it, it advanced exactly as much as to unite the nearest flowers, just as on a ratchet wheel the detent slips over one tooth at a time; or, to speak more correctly, the binocular centre advanced in order to relieve the eyes from their strain, and when the eyes were opened, it had just reached that point which corresponded with the union of the flowers six inches distant.

In the construction of complex geometrical diagrams consisting only of fine lines, and in which similar figures are repeated at equal distances, it is very difficult to attain minute accuracy. The points of the compasses sink to different depths in the paper, and the lines which join such points seldom pass through their centres. Hence arises a general inaccuracy which the eye cannot detect; but if we examine such diagrams by strained binocular vision, their imperfections will be instantly displayed. Some parts will rise higher than others above the general level, and the whole will appear like several cobwebs placed at the distance of a tenth or a twelfth of an inch behind each other.*

In all the experiments made by Mr WHEATSTONE by the stereoscope, and in those described in my former paper, the dissimilar figures are viewed in a direction perpendicular to the plane on which they are drawn. A series of very interesting results, however, are obtained by uniting the images of lines meeting at an angular point, when the eye is placed at different heights above the plane of the paper, and at different distances from the angular point.

Let A C, B C be two lines meeting at C, the plane passing through them being the plane of the paper, and let them be viewed by the eyes at E‴, E″, E′, E at different heights in a plane G M N perpendicular to the plane of the paper.

* This effect is finely seen in the diagram of the Homogeneous Curve, which forms Plate IX. of Mr HAY's work "On the Harmony of Form."

Let R be the right eye and L the left eye, and when at E‴ let them be strained so as to unite the points A, B. The united image of these points will be seen at the binocular centre D‴, and the united lines A C B C will have the position D‴ C. In like manner, when the eye descends to E‴, E′, E, the united image D‴ C will rise and diminish, taking the positions D″ C, D′ C, D C till it disappears on the line C M, when the eyes reach M. If the eye deviates from the vertical plane G M N the united image will also deviate from it, and is always in a plane passing through the eye and the line G M.

If at any altitude E M the eye advances towards A C B in the line E G, the binocular centre D will also advance towards A C B in the line E G, and the image D C will rise and become shorter as its extremity D moves along D G, and after passing the perpendicular to G E it will increase in length. If the eye, on the other hand, recedes from A C B in the line G E, the binocular centre D will also recede, and the image D C will descend to the plane C M and increase in length.

The preceding diagram is, for the purpose of illustration, drawn in a sort of perspective, and therefore does not give the true positions and lengths of the united images. This defect, however, is remedied in Fig. 3, where E, E′, E″, E‴ is the middle point between the two eyes, the plane G M N being, as before, perpendicular to the plane passing through A C B. Now, as the distance of the eye from G is supposed to be the same, and as A B is invariable as well as the distance between the eyes, the distance of the binocular centres O, D, D′, D″, D‴, P, from G will also be invariable, and lie in a circle O D P whose centre is G, and whose radius is G O, the point O being determined by the formula $G\,O = G\,D = \frac{G\,M \times A\,B}{A\,B + R\,L}$. Hence, in order to find the binocular centres D, D′, D″, D‴, &c., at any altitude E, E′ &c., we have only to join E G, E′ G, &c., and the points of intersection D, D′, &c., will be the binocular centres, and the lines D C, D′ C, &c., drawn to C, will be the real lengths and inclinations of the united images of the lines A C, B C.

When G O is greater than G C there is obviously some angle A, or E″ G M at which D″ C is perpendicular to G C. This takes place when $\cos. A = \frac{G\,C}{G\,O}$. When O coincides with C, the images C D, C D′, &c., will have the same positions and magnitudes as the chords of the altitudes A of the eyes above the plane G C. In this case, the raised or united images will just reach the perpendicular when the eye is in the plane G C M, for since G C = G O, cos. A = 1, and A = 0°.

When the eye at any position, E″ for example, sees the points A and B united at D″, it sees also the whole lines A C B C forming the image D″ C. The binocular centre must, therefore, run rapidly along the line D″ C: that is, the inclination of the optic axis must gradually diminish till the binocular centre reaches C, when all strain is removed. The vision of the image D′ C, however, is carried on so rapidly, that the binocular centre returns to D″ without the eye being sensible of the removal and resumption of the strain which is required in maintaining a view of the united image D″ C.

If we now suppose A B to diminish, the binocular centre will advance towards G, and the length and inclination of the united images D C, D′ C, &c., will diminish also, and *vice versa*. If the distance R L (Fig. 2) between the eyes diminishes, the binocular centre will retire towards E, and the length and inclination of the images will increase. Hence persons with eyes more or less distant will see the united images in different places and of different sizes, though the quantities A and A B be invariable.

While the eyes at E′ are running along the lines A C, B C, let us suppose them to rest upon the points *a*, *b* equidistant from C. Join *a b*, and from the point *g*, where *a b* intersects G C, draw the line *g* E″, and find the point *d″* from the formula $g\,d'' = \frac{g\,E'' \times a\,b}{a\,b + R\,L}$. Hence the two points *a*, *b* will be united at *d″*, and when the angle E″ G C is such that the line joining D and C is perpendicular to G C, the line joining *d″* C will also be perpendicular to G C, the loci of the points D″ *d″ d′ d* will be in that perpendicular, and the image D C, seen by successive movements of the binocular centre from D″ to C, will be a straight line.

In the preceding observations we have supposed that the binocular centre D″, &c., is between the eye and the lines A C B C; but the points A, C, and all the other points of these lines, may be united by fixing the binocular centre beyond A B. Let the eyes, for example, be at E″; then if we unite A B when the eyes converge to a point, Δ'' (not seen in the figure), beyond G, we shall have $G\,\Delta'' = \frac{G\,E \times A\,B}{R\,L - A\,B'}$ and if we join the point Δ'' thus found and C, the line Δ' C will be the united image of A C and B C, the binocular centre ranging from Δ'' to C, in order to see it as one line. In like manner, we may find the position and length of the image Δ''' C, Δ' C, and Δ C corresponding to the position of the eyes at E‴ E and E. Hence all the united images of A C, B C: viz. C Δ''', C Δ'', &c., will lie below the plane of A B C, and extend beyond a vertical line N B continued; and they will grow larger and larger, and approximate in direction to C G as the eyes descend from E‴ to M. When the eyes are near to M, and a little above the plane of A B C, the line, when not carefully observed, will have the appearance of coinciding with C G, but stretching a great way beyond G. This extreme case represents the celebrated experiment with the compasses described by Dr Smith, and referred to by Professor Wheatstone. He took a pair of compasses, which may be represented by A C B, A B being their points, A C B C their legs, and C their joint; and having placed his eyes about E above their plane, he made the following experiment:—" Having opened the points of a pair of compasses somewhat wider than the interval of your eyes, with your arm extended, hold the head or joint in the ball of your hand with the points outwards, and equidistant from your eyes, and somewhat higher than the joint. Then, *fixing your eyes upon any remote object* lying in the plane that bisects the interval of the points, you will first perceive two pair of compasses (each by being doubled with their inner legs crossing each other, not unlike the old shape of the letter W.) But by compressing the legs with your hand, the two inner points will come nearer to each other; and when they unite (having stopt the compression), the two inner legs will also entirely coincide and bisect the angle under the outward ones, and will appear more vivid, thicker and larger, than they do, so as to reach from your hand to the remotest object in view even in the horizon itself, if the points be exactly coincident."* Owing to his imperfect apprehension of the nature of this phenomenon, Dr SMITH has omitted to notice that the united legs of the compasses lie below the plane of A B C, and that they never can extend farther than the binocular centre at which their points A and B are united.

There is another variation of these experiments which possesses some interest, in consequence of its extreme case having been made the basis of a new theory of visible direction by the late Dr WELLS.† Let us suppose the eyes of the observer to advance from E to N, and to descend along the opposite quadrant on the left hand of N G, but not drawn in fig. 3 (plate 17), then the united image of A C, B C, will gradually descend towards C G, and become larger and larger. When the eyes are a very little above the plane of A B C, and so far to the left hand of A B, that C A points nearly to the left eye, and C B to the right eye, then we have the circumstances under which Dr WELLS made the following experiment:—" If we hold two thin rules in such a manner that their sharp edges (A C, B C in Fig. 3) shall be in the optic axes, one in each, or rather a little below them, *the two edges will be seen united in the common axis* (G C in Fig. 3); and this apparent edge will seem of the same length with that of either of the real edges, when seen alone by the eye in the axis of which it is placed." This experiment, it will be seen, is the same with that of Dr SMITH, with this difference only, that the points of the compasses are directed towards the eyes. Like Dr SMITH, he has omitted to notice that the united image rises above G H, and he commits the opposite error of Dr SMITH, in making the length of the united image too short.

If in this form of the experiment we fix the binocular centre beyond C, then the united images of A C, B C descend below G C, and vary in their length, and in their inclination to G C, according to the height of the eye above the plane of A B C, and its distance from A B.

It is a remarkable circumstance, that no examples have been recorded of false estimates of the distance of near objects, in consequence of the *accidental*

* SMITH's Optics, vol. ii. p. 388, § 977.

† Essay on Single Vision, &c., p. 44.

binocular union of similar images. This has, no doubt, arisen from the rare occurrence of these circumstances or conditions, under which alone such illusions can be produced. In a room where the paper hangings have a small pattern, or similar figures recurring at the distance of 1, $1\frac{1}{2}$, or 2 inches, a short-sighted person might very readily turn his eyes on the wall, when their axes converged to some point between him and the wall, which would unite one pair of the similar images; and, in this case, he would see the wall nearer him than the real wall, and moving with the motion of his head like something aerial. In like manner, a long-sighted person, with his optical axes converged to a point beyond the wall, might see an image of the wall more distant, and of an aerial character;—or a person who has taken too much wine, which often fixes the optical axes in opposition to the will, might, according to the nature of his sight, witness either of the illusions above mentioned.

In the preceding observations, we have confined ourselves to the binocular union of figures upon an opaque ground. This limitation almost necessarily precluded us from observing the results when the binocular centre is beyond the plane where these figures are situated, because it is not easy to adjust the eyes to a distant object, unless we look through the surfaces containing the figures. Now, this is by far the most interesting form of the experiment, and it has the advantage of putting scarcely any strain upon the eyes, not only because the binocular centre is more distant, but because we cannot, in this way, unite figures whose distance exceeds $2\frac{1}{2}$ inches, the interval between the eyes. Transparent patterns for these experiments may be cut out of stiff card paper, or thin plates of metal, or they may be made of paper pasted upon large panes of glass. Experiments may be made with trellis work, or with windows composed of small squares or lozenges; but the readiest pattern is the cane bottom of a chair, and I have performed my experiments by simply placing such a chair upon a high table, with its cane bottom in a vertical position. The distance of the centres of the eight-sided open figures in the direction of the width or depth of the chair, varies in different patterns from 0.54 to 0·76 of an inch. In order to simplify the calculations, we shall take the distance at 0·5, or half an inch. Then let

D = 12 inches be the distance of the pattern from the eyes.

d = 0·5 the distance of the centres of the similar figures.

$+\Delta$ = distance of suspended image from, and in front of, the pattern.

$-\Delta'$ = distance of suspended image from, and behind, the pattern.

C = 2·5 the distance between the eyes.

Then we shall have

$$+\Delta = \frac{D\,d}{C+d} \text{ and } -\Delta' = \frac{D\,d}{C-d}. \quad \text{Hence}$$

$D - \Delta$ = distance of suspended image from the eye, and in front of the pattern, and

$D + \Delta'$ = its distance from the eye, and behind the pattern.

From these formulæ we have computed the following table, adapted to similar figures, whose centres are distant $\frac{1}{2}$ an inch, 1, $1\frac{1}{2}$, 2, and $2\frac{1}{2}$ inches; but in reference to the positive values of Δ and D, we may consider them as feet, 0·5 being in that case = 6 inches.

D Inches.	$d = 0·5$		$d = 1·0$		$d = 1·5$		$d = 2·0$		$d = 2·4$		$d = 2·5$	
	$+\Delta$	$-\Delta$	$+\Delta$	$-\Delta$	$+\Delta$	$-\Delta$	$+\Delta$	$-\Delta$	$+\Delta$	$-\Delta$	$+\Delta$	$-\Delta$
6	1	1·5	1·72	4	2·25	9	2·66	24	2·94	144	3	Infin.
12	2	3	3·43		4·50	18	5·33	48	5·88	288	6	Infin.
24	4	6	6·86	16	9	36	10·66	96	11·76	576	12	Infin.
48	8	12	13·7	32	18	72	21·33	192	23·52	1152	24	Infin.

Taking the case where D is 12 inches, and uniting the two nearest openings where d is 0·5, let M N (Fig. 4, Pl. 17) be a section of the transparent pattern, L, R the left and right eyes, L a d, L b e lines drawn through the centres of two of the open figures a b, and R b d, R c e lines drawn through the centre of b and c, and meeting L a d, L b e at d and e, d being the binocular centre when we look at it through a and b, and e the binocular centre when we look at it through b and c. Now, the right eye R sees the opening b at d, and the left eye L sees the opening a at d, hence the image at d consists of the similar images of a and b united. In like manner e consists of b and c united, and so on with all the rest, so that the observer at L R no longer sees the real pattern M N, but a suspended image of it at m n, *three* inches behind M N. If the observer now approaches M N, the image m n will approach to him, and if he recedes, m n will recede, being $1\frac{1}{2}$ inches distant from M N when the observer is 6 inches from M N, and 12 inches from M N when he is 48 inches from M N, the image m n moving from M N with a velocity $\frac{1}{4}$th of that with which the observer recedes. These two velocities are in the ratio of D to $\frac{\mathrm{D}\,d}{\mathrm{C} - d}$.

Resuming the position in the figure where the observer is 12 inches distant from M N, let us consider the important results to which this experiment cannot fail to lead us. If the observer, with his eyes at L R, grasp the cane bottom or pattern at M N, as shewn in Fig. 4, pl. 17, his thumbs pressing upon M N, and his fingers trying to grasp m n, he will then *feel what he does not see*, and *see what he does not feel!* The real pattern is absolutely invisible at M N, and stands fixed at m n. The fingers may be passed through and through—now seen on this side of it—now in the middle of it, and now on the other side of it. If we next place the palms of each hand upon M N, feeling it all over, the result will be the same.

No knowledge derived from touch—no measurement of real distances—no actual demonstration from previous or subsequent vision, that there is a real solid body at M N, and nothing at all at *m n*, will remove or shake the infallible conviction of the sense of sight that the object is at *m n*, and that *d* L or *d* R is its real distance from the observer. If the binocular centre be now drawn back to M N, the image *seen* will disappear, and the real object be seen at M N. If it be brought still farther back to *f*, the object M N will again disappear, and will be seen at $\mu\ \nu$, as described in a former part of this paper.

In making these experiments, the observer cannot fail to be struck with the remarkable fact, that though the openings at M N, *m n*, and $\mu\ \nu$, have all the same angular magnitude, that is, subtend the same angle at the eye, viz., *d* L *e*, *d* R *e*, yet those at *m n* appear larger than those at M N, and those at $\mu\ \nu$ smaller. If we cause the image *m n* to recede, and $\mu\ \nu$ to approach, the figures in *m n* will invariably *increase* as they *recede*, and those in $\mu\ \nu$ will *diminish* as they *approach* the eye, and their *visual magnitudes*, as we shall call them, will depend on the respective distances at which the observer, whether right or wrong in his estimate, conceives them to be placed.

Now, this is an universal fact, which the preceding experiments demonstrate; and though the estimate of magnitude thus formed is an erroneous one, yet it is one which neither reason nor experience is able to correct.

When we look at two equal lines, whose difference of distance is distinctly appreciable by the eye, either directly, or by inference, but whose difference of angular magnitude is not appreciable, the most remote must necessarily appear the smallest. For the same reason, if the remoter of two lines is really smaller than the nearer, and, therefore, its angular magnitude also smaller from both these causes, yet, even in this case, if the eye does not perceive distinctly the difference, the smaller and more remote line will appear the larger.*

The law of visual magnitude, which regulates this class of phenomena, may be thus expressed.

If we call A the angular magnitude of the *nearest* of two lines or magnitudes

* MALEBRANCHE seems to have been the first who introduced the *apparent* distance of objects as an element in our estimate of *apparent* magnitude. *De la Recherche de la Verité*, tom. i. liv. i.; tom. iii. p. 354. See also Bouguer, *Mem. Acad. Par.* 1755, p. 99. These views, however, have been abandoned by several subsequent writers, and the *real distance* of objects has been substituted for their *apparent* distance. VARIGNON, *Mem. Acad. Par.* 1717, p. 88. M. LEHOT, for example, says, " L'expression de la grandeur visuelle d'un corps est egale à la grandeur reelle, multipliée par le logarithme de la *distance reelle* divisée par cette distance." *Nouvelle Théorie de la Vision*, 1er *Mem.* Suppl. p. 7, 8. Paris, 1823. This estimate of distance is incompatible with experiment and observation.

whose apparent distance is d, a the angular magnitude of the remoter line, whose apparent distance is D, and V, v the visual magnitudes of the two lines, then

$$V : v = A \times d : a \times D.$$

Now, let the two lines M O, N P, be the two sides of a quadrilateral figure seen obliquely by an eye at E, then, if the apparent distances of M O, N P, are such, that

$$A \times d > a \times D, \text{ then } V > v,$$

and the lines M N, O P, will converge to a vanishing point beyond N P. But if

$$A \times d = a \times D, \text{ then } V = v,$$

and the line M N, O P, will appear to be parallel. And if

$$A \times d < a \times D, \text{ then } V < v,$$

and the lines M N, O P, will converge to a vanishing point between M O and the observer.

These results may be considered as laying the foundation of a new art, to which we may give the name of Visual Perspective, in contradistinction to Geometrical Perspective. This art furnishes us with an immediate explanation of a great variety of optical illusions which have never yet been explained; and there is reason to believe that some of its principles were known to ancient architects, and even employed in modifying the nature and position of the lines and forms which enter into the construction of their finest edifices.

St Leonard's College, St Andrews,
April 10. 1844.

Appendix.

APPENDIX.

When I wrote the paragraph in page 647, I had no expectation of learning that any example of such an illusion had ever occurred. A friend, however, to whom I had occasion to shew the experiments, and who is short-sighted, mentioned to me that he had been on two occasions greatly perplexed by the vision of these suspended images. Having taken too much wine, and being in a papered room, he saw the wall suspended near him in the air; and on another occasion, when kneeling and resting his arms on a cane-bottomed chair, he had fixed his eyes on the carpet, which accidentally united the two images of the open-work, and threw the suspended image of the chair bottom to a distance, and beyond the plane on which his arms rested.

The following case, communicated to me by Professor Christison, is still more interesting. "Some years ago, when I resided in a house where several rooms are papered with rather formally recurring patterns, and one, in particular, with stars only, I used occasionally to be much plagued with the wall suddenly standing out upon me, and waving, as you describe, with the movements of the head. I was sensible that the cause was an error as to the point of union of the visual axes of the two eyes; but I remember it sometimes cost me a considerable effort to rectify the error; and I found that the best way was to increase still more the deviation in the first instance. As this accident occurred most frequently while I was recovering from a severe attack of fever, I thought my near-sighted eyes were threatened with some new mischief; and this opinion was justified in finding that, after removal to my present house—where, however, the papers have no very formal pattern—no such occurrence has ever taken place. The reason is now easily understood from your researches."