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On the several Mortality Tables
employed by Life Assurance Companies
in the Valuation of
Annuity Contracts.

BY

GERALD H. RYAN, F.I.A., F.S.S.

ASSISTANT ACTUARY TO THE MARINE AND GENERAL
MUTUAL LIFE ASSURANCE SOCIETY.

*On the several Mortality Tables employed by
Life Assurance Companies in the Valua-
tion of their Annuity Contracts. By*
GERALD H. RYAN, F.I.A., F.S.S.

IN presenting to the Members of the Actuarial Society of Edinburgh the results of my investigation into so practical a matter as that described in the title of this paper, I feel that whatever may be their opinion with regard to the manner in which I have proceeded, and the methods which I have employed in my researches, I am at least absolved from the necessity of apologising for the nature of my subject. A glance at the syllabus of the Society for the current Session is sufficient to establish the fact that scientific inquiries upon theoretical points, and the discussion of matters of practical importance, are alike welcomed as acceptable contributions to its *Transactions*. 'But, if I may be permitted to anticipate in a slight measure the course of this investigation, as set forth in the following pages, I would take the opportunity of expressing the hope that my paper may be regarded as possessing some slight interest from the theoretical as well as the practical point of view. It will be my aim to afford such statistics of the Annuity business transacted in this country as the valuable returns to the Board of Trade—an endless source of statistical information upon every point connected with the operations of our Life Assurance Companies—will supply, more particularly with respect to the valuation of such contracts; to construct from data thus obtained a common base, which will enable the relative reserves according to certain mortality tables to be measured—or, as it is technically termed, a "Model Office;" and, finally, to employ this instrument in endeavouring to show the financial results yielded by the application of each of the tables in question. The former portion of my scheme is therefore of a decidedly practical character, while the latter may be regarded as theoretical in scope.

It is a circumstance worthy of note that, whereas numerous investigations have been conducted into the subject of Life

Assurance (using the word in its restricted sense) in every phase, the cognate questions relating to the grant of Annuities have hitherto escaped attention. Nor are the reasons for this obvious. It is true that the experience of some Companies has, in late years, resulted unfavourably; but whether this circumstance is to be considered as satisfactory proof that the issue of Annuities at current rates is unprofitable, is a matter on which opinions may be expected to differ. But, quitting the field of speculation for that of statistical facts, the Annuity business of the country is, unquestionably, on the increase. In evidence of this, I have compiled the following table (No. I.) from the Board of Trade Returns during the past twelve years. In this table we find given the total amount of immediate Annuities in course of payment by the thirty-five Offices who transact the largest business of this kind, as at the date of their last valuation returns, together with the number of such Annuities and the reserves held against them. Similar statistics are also given, having reference to the previous valuation returns. The dates to which the figures in each group refer are not synchronous; but the return nevertheless shows, with tolerable trustworthiness, the increase that has taken place in an intervaluation period of, on the average, about five years. The table may be summarised as follows—the years 1875 and 1880 being selected as those to which the two divisions may be assumed respectively to have reference:—

35 Companies.

	Group No. 1 (1875).	Group No. 2 (1880).
No. of Annuities in force, .	9,141	11,707
Amount of such Annuities, .	£389,461 9 0	£499,774 4 0
Net Liability thereon, .	3,004,998 0 0	4,045,256 8 0

Dealing with percentages, it will be seen that the number of annuities in force has increased by 28·1 per cent., the amount of such annuities by 28·3 per cent., and the net liability thereon by 34·6 per cent.

The corresponding figures in regard to the Companies forming the basis of the Model Office are—

10 Companies forming the Model Office.

	Group No. 1 (1875).	Group No. 2 (1880).
No. of Annuities in force, .	5,578	6,446
Amount of such Annuities, .	£245,375 0 0	£294,129 6 0
Net Liability thereon, .	1,903,643 6 0	2,445,400 10 0

TABLE I.

PARTICULARS OF LAST TWO VALUATIONS OF OFFICES HAVING MORE THAN 50 IMMEDIATE ANNUITIES IN FORCE.

TEN LARGEST SELECTED TO FORM MODEL OFFICE.

Particulars of Annuity Business of Thirty-five Companies.

No. of Office.	GROUP No. I.				GROUP No. II.				BASIS OF VALUATION.
	Date of Valuation.	Number of Immediate Annuities.	Amount of Immediate Annuities.	Net Liability.	Date of Valuation.	Number of Immediate Annuities.	Amount of Immediate Annuities.	Net Liability.	
1	31.12.74	151	5,831.1	34,463.1	31.12.81	161	7,023.7	59,124.0	{ English, No. 1, } Males, 4 per cent.
2	31.12.78	79	1,810.3	13,497.2	31.12.81	82	2,073.5	14,939.9	{ Carlisle, 3½ " (1878 Val.)
3	31.12.76	66	2,451.0	17,933.2	31.12.81	84	3,275.5	24,963.2	{ Hæ, 3 " (1881 ")
4	31.1.77	162	3,382.0	35,444.0	31.12.82	153	3,955.0	32,258.0	{ Government Annuity, . 3½ " "
5	31.12.76	142	3,381.0	24,100.0	31.12.81	186	4,201.0	30,696.1	{ Government Annuity, 1860, 3 " "
6	5.1.79	44	2,456.9	24,088.3	5.1.82	74	2,687.6	25,222.6	{ Swedish, 3 " "
7	25.3.77	120	4,095.9	34,753.7	25.3.82	121	3,773.0	30,820.0	{ Davies's Equitable, 3 per cent. for Males, 2½ per cent. for Females, except for ages over 79, for which Government granting rate (5.35 years' purchase) was set.
*8	30.6.79	931	41,770.6	331,730.8	30.6.82	1,685	76,932.2	663,983.4	{ 17 Offices' Experience, . 4½ per cent.(1879 Val.)
*9	20.11.77	461	13,897.6	126,383.8	20.11.82	516	16,306.3	150,016.2	{ Do, 4 " (1882 ")
10	25.12.75	103	4,773.3	35,984.5	25.12.80	101	4,506.4	35,575.0	{ Special Table, 3 " "
11	31.12.75	81	2,945.5	28,893.0	31.12.80	84	3,251.5	30,236.0	{ Government Annuity, . 3 " "
12	31.12.75	269	7,384.0	{ 62,264.0 } { 3,113.0 }	31.12.80	269	7,144.0	{ 58,693.0 } { 2,935.0 }	{ Equitable, 3 " "
13	31.12.75	243	11,971.0	73,453.0	31.12.80	223	9,946.0	64,620.0	{ Government Annuity, 1860, 3½ " (1880 Val.)
*14	15.11.75	486	23,159.8	195,423.1	15.11.80	485	27,520.7	246,889.9	{ Addition for Expenses, . 4 " (1875 ")
15	31.12.75	89	2,087.3	16,791.0	31.12.80	128	3,165.0	22,625.0	{ Government Annuity, 1860, 4 " "
*16	31.12.75	699	33,064.2	265,890.6	31.12.80	775	39,667.5	325,387.8	{ Carlisle, 3 " (1880 Val.)
									{ Government Annuity, 1829, 3½ " "

* The Offices marked with an asterisk in the left-hand margin of the above statement contributed the data on which the Model Office is based.

TABLE I.—Continued.

No. of Office.	GROUP No. I.				GROUP No. II.				BASIS OF VALUATION.
	Date of Valuation.	Number of Immediate Annuities.	Amount of Immediate Annuities.	Net Liability.	Date of Valuation.	Number of Immediate Annuities.	Amount of Immediate Annuities.	Net Liability.	
17	31.12.73	150	6,811.0	50,822.1	31.12.80	174	6,774.3	47,419.0	{ HM, 3½ per cent. (1880 Val.)
*18	30. 6.76	553	20,078.5	120,230.9	30. 6.81	318	11,619.2	66,796.3	{ Carlisle, 3
19	29. 2.72	646	14,081.1	114,873.0	28. 2.81	719	17,122.7	133,830.0	{ Government Annuitants, 1860, . . . 4
*20	5. 4.76	428	18,866.5	131,060.1	5. 4.81	453	19,190.8	134,122.2	{ Male and Female, 3½
21	31.12.77	124	7,551.8	54,631.0	31.12.80	136	7,662.8	59,215.0	{ Carlisle, 3
*22	31.12.74	455	15,710.9	130,470.5	31.12.79	537	18,022.0	147,279.7	{ Special Table, nearly identical with Equitable.
23	30.11.74	122	5,928.9	42,934.2	30.11.79	159	7,788.8	57,231.5	{ Davies's Equitable, 3 per cent.
*24	31.12.74	95	7,657.7	58,837.4	31.12.79	154	14,931.2	146,250.9	{ Davies's Equitable, 4
25	1. 3.75	178	5,769.5	44,647.9	1. 3.80	171	5,259.1	43,817.2	{ Office Tables, giving larger re- serves than Carlisle Table, . . . 3
*26	31.12.73	1,198	56,089.0	420,914.0	31.12.78	1,232	54,309.0	438,262.0	{ Carlisle, 3
27	31.12.73	67	1,676.5	16,477.0	31.12.78	140	4,199.6	34,130.0	{ Davies's Equitable, 3½
28	20. 1.74	143	5,607.0	38,053.0	20. 1.79	134	5,263.0	39,372.0	{ Government Annuitants 1860, . . . 4
29	31. 3.71	272	15,080.2	122,702.1	31. 3.78	291	15,630.4	126,412.1	{ Carlisle, 4
30	13. 5.71	59	1,656.6	14,118.1	14. 5.78	64	3,428.6	31,255.0	{ Government Annuitants, 4
31	30. 6.77	94	7,690.5	57,778.0	31.12.82	55	3,528.0	24,833.0	{ Carlisle, 4
32	31.12.76	220	22,896.0	164,596.0	31.12.80	1,454	75,788.0	579,815.0	{ American, 4½
33	20.11.77	62	1,742.8	14,885.1	20.11.82	61	1,677.8	13,456.6	{ Davies's Equitable, 4
34	31.12.77	83	2,643.6	20,466.3	31.12.82	92	3,207.2	23,625.2	{ Carlisle (1877), 4
35	31.12.77	66	6,962.3	62,335.0	31.12.82	236	8,933.0	79,149.0	{ Hr (1882), 4
		9,141	389,461.9	3,004,998.0		11,707	499,774.4	4,045,256.8	{ Carlisle, 3
Total for Ten Offices forming basis of Model Office.									
10 Companies,		5,578	245,375.0	1,903,643.3		6,446	294,129.3	2,445,400.5	

* The Offices marked with an asterisk in the left-hand margin of the above statement contributed the data on which the Model Office was based.

The ratios of increase here are 15·6 per cent. for the number of annuities, 19·9 for the amount, and 28·4 for the net liability thereon. The increase shown in these figures is very marked, and proves beyond doubt the growing character of the business. In addition to this, however, we gather from the above statement that this department of our Life Assurance Companies' business is of important magnitude, and of such an extent, indeed, as to call for closer scientific study and analysis than has been accorded it.

An examination of the basis of valuation of these thirty-five Companies shows them to be divisible into the classes set out on the following page :—

STATEMENT SHOWING CLASSIFICATION OF THE BASIS OF
VALUATION OF THIRTY-FIVE COMPANIES.

VALUATION BASIS.	No. of Companies adopting Basis.	
	Group 1 (1875).	Group 2 (1880).
Table of Mortality.		
Carlisle,	11	5
Davies's Equitable,	6	6
Government Annuitants',	11	14
H ^M ,	1	2
H ^F ,	...	1
English Life No. 3,	1	1
17 Offices',	1	1
Swedish,	1	1
American,	1	1
Special Tables,	2	3
Total,	35	35
Rate of Interest.		
3 per cent.,	14	13
3½ „	6	7
4 „	13	14
4½ „	2	1
Total,	35	35

Note.—One Company employs 2½ per cent. interest for Annuities on Females.

In regard to this classification, it may be stated that, were we to examine the basis employed at successive valuations (coincident, in point of time, with the dates represented by Group 1 and Group 2), of Life Assurance contracts, we should observe in all probability a material strengthening in the tables chosen to measure the reserves. A large number of Companies would be found to have assumed a lower rate of interest at the last-named date; while in many Companies a change to a stronger table of mortality would be recorded. These conditions are not disclosed in the analysis I have made with respect to Annuities; for the short statement just given shows that there has been no decrease in the rate of interest assumed, nor any apparent increase of strength in the mortality tables employed. Additional light will, however, be thrown upon this point, when the relative effect of the several mortality tables, taken in conjunction with the usual rates of interest, is investigated. It will, therefore, be unnecessary at this juncture, to pay closer attention to the matter.

Having now the needful materials in serviceable form, the next step in my scheme was the construction of the Model Office. For this purpose the list of Companies was carefully examined, and as many of them were found to have contributed but a very small proportion of the total observations, I selected, to form the foundation of my assumed Company, the ten British Offices whose annual payments on account of annuities were in excess of £10,000. The last valuation returns of these Companies were then again resorted to, and, from this source, the annuities payable (on single lives) at each age from 50 upwards were brought together, and the sum taken for all the ten Companies. The resulting totals, showing the aggregate amount of annuities in course of payment at each age from 50, were then taken by me as the basis of my Model Office. The figures, however, exhibiting some irregularity, and smoothness being desirable for such a purpose as mine, they were subjected to graduation, the method adopted being that of Mr. John Finlaison (see *J. I. A.* xx. 172, 173). A saving of labour and the retention of the great simplicity of the self-verifying columnar method were secured by resorting to the ingenious device proposed by Mr. T. G. Ackland, (*J. I. A.* xxiii. 356; see also foot-note on the same page). The *modus operandi* may be briefly explained as follows:—Finlaison's formula may be applied by summing the rough series, in groups of five, three times, the resulting figures being

divided by 125. But as, in the summation, account is only taken of those ages which form the centre of a *full* group of five values, there remain certain values to be supplied at the two extremes of the table; and for this purpose* interpolation is employed. By Mr. Ackland's method the series obtained by successive summation, instead of being contracted, are extended, as the sum is set against each age, not only when five values are included in the processes of summing, but when there is even a single value. The additional terms thus obtained are folded back, and added to the values opposite the commencing and ending ages. By this means the sum of the graduated values is made equal to the sum of the ungraduated series, and a convenient check upon the arithmetical work obtained. No need to call in the aid of interpolation exists: the whole operation is conducted in a homogeneous way, and is free from difficulty and complication.

The graduated "amounts of annuities in course of payment" being thus derived, it became necessary, in order to render the basis applicable to those mortality tables in which separate experiences have been taken of male and female lives, to divide the annuities at each age into those payable on males and females respectively. Upon this point the Board of Trade Returns give no information, and the omission is, I think, an important one. So different is known to be the mortality among males and females, and so different the reserves required, that it seems just as desirable—indeed, necessary—to have the means of ascertaining what annuities are payable on each sex separately as it is that we should know what basis of valuation has been adopted. The difficulty I was met with at this point, I endeavoured to remove by making private inquiries in professional circles; but the information I gleaned was in some measure conflicting, and afforded me no tangible ground for making any fixed scale of distribution. I was thus led to assume that the annuities in force at each age, on males and females respectively, were in the proportions which the sexes were found to bear to the total number in Mr. A. J. Finlaison's recent investigation into the experience of Government Annuitants, which shows there to have been 19,859 females to 10,929 males. Upon further reflection this course, though perhaps open to

* Mr. Sprague suggests that the word "extrapolation" should here be used in place of the more familiar word "interpolation," the latter strictly denoting the process of supplying terms *between* any given points in a series.

TABLE II.
Containing particulars of the MODEL OFFICE.

Age last birthday.	Annuities in course of payment Un-graduated.	Annuities (Male and Female) Graduated.	Annuities (Females) Graduated.	Annuities (Males) Graduated.	Logarithms (Male and Female) Annuities.	Log. Annuits. Females, (constant addend., '80958.)	Log. Annuits. Males, (constant addend., '55020.)
	£	£	£	£			
50	1,208	2,239	1,444	795	·35005	·15963	·90025
1	2,212	2,438	1,572	866	·38703	·19661	·93723
2	1,715	2,795	1,803	992	·44638	·25596	·99658
3	3,687	3,246	2,094	1,152	·51135	·32093	·06155
4	3,569	3,710	2,393	1,317	·56937	·37895	·11957
5	5,604	4,134	2,667	1,467	·61637	·42595	·16657
6	5,766	4,544	2,931	1,613	·65744	·46702	·20764
7	3,106	4,984	3,215	1,769	·69758	·50716	·24778
8	4,883	5,495	3,544	1,951	·73997	·54955	·29017
9	4,667	6,116	3,945	2,171	·78647	·59605	·33667
60	6,899	6,872	4,433	2,439	·83708	·64666	·38728
1	7,617	7,704	4,969	2,735	·88672	·69630	·43692
2	9,005	8,500	5,483	3,017	·92942	·73900	·47962
3	11,294	9,185	5,925	3,260	·96308	·77266	·51328
4	9,880	9,726	6,274	3,452	·98793	·79751	·53813
5	9,777	10,078	6,501	3,577	·00337	·81295	·55357
6	10,836	10,204	6,582	3,622	·00877	·81835	·55897
7	12,226	10,215	6,589	3,626	·00924	·81882	·55944
8	8,793	10,150	6,547	3,603	·00647	·81605	·55667
9	9,833	10,006	6,454	3,552	·00026	·80984	·55046
70	10,124	9,821	6,335	3,486	·99216	·80174	·54236
1	9,838	9,684	6,246	3,438	·98605	·79563	·53625
2	8,220	9,516	6,138	3,378	·97845	·78803	·52865
3	10,959	9,276	5,983	3,293	·96736	·77694	·51756
4	9,679	9,008	5,810	3,198	·95463	·76421	·50483
5	7,520	8,667	5,590	3,077	·93787	·74745	·48807
6	9,310	8,204	5,292	2,912	·91403	·72361	·46423
7	8,105	7,653	4,936	2,717	·88383	·69341	·43403
8	7,949	7,100	4,580	2,520	·85126	·66084	·40146
9	4,324	6,488	4,185	2,303	·81211	·62169	·36231
80	7,343	5,897	3,804	2,093	·77063	·58021	·32083
1	3,877	5,341	3,445	1,896	·72762	·53720	·27782
2	5,468	4,831	3,116	1,715	·68404	·49362	·23424
3	4,194	4,298	2,772	1,526	·63327	·44285	·18347
4	4,384	3,800	2,451	1,349	·57978	·38936	·12998
5	2,675	3,280	2,116	1,164	·51587	·32545	·06607
6	3,386	2,767	1,785	982	·44201	·25159	·99221
7	1,955	2,268	1,463	805	·35564	·16522	·90584
8	1,330	1,829	1,180	649	·26221	·07179	·81241
9	1,066	1,439	928	511	·15806	·96764	·70826
90	780	1,129	728	401	·05269	·86227	·60289
1	802	878	566	312	·94349	·75307	·49369
2	1,078	677	437	240	·83059	·64017	·38079
3	224	513	331	182	·71012	·51970	·26032
4	235	378	244	134	·57749	·38707	·12769
5	198	263	170	93	·41996	·22954	·97016
6	83	181	117	64	·25768	·06726	·80788
7	...	124	80	44	·09342	·90300	·64362
8	62	94	61	33	·97313	·78271	·52333
Total	257,745	257,745	166,254	91,491			

objection, seems to be as safe and probable an assumption as could have been made under the circumstances. Table II. gives the ungraduated and the graduated amounts of annuities payable on male and female lives, separately and combined.

The next process was that of valuing the annuities at each age, as given in Table II., by the mortality tables commonly used for such purpose. In practice, annuities are almost invariably apportionable up to the day of death, and are generally payable by half-yearly or other instalments. It seemed to me, however, that the conditions of the case would fairly be met by assuming that the correct annuity values could be represented by the expression $\frac{1}{2} + a_x$. Any extreme refinement on this point would have led to a great expenditure of time, while, for purposes of comparison, the expression adopted will, no doubt, serve every needful purpose.

As will be seen in Tables VII., VIII., IX., and the Summary Table, X., I have laid under contribution the following tables:—Carlisle, Davies's Equitable, De Parcieux', English (No. 3), the Government Annuitants' for 1829, 1860, and 1883, and the H^M Table. In addition to these I have constructed a new table from facts supplied in the Institute of Actuaries' Mortality Experience. This table contains the experience of the entrants at age 50 and upwards found in the H^M observations. My intention here was to ascertain whether the increase which the H^M rate of mortality exhibits at the older ages was caused by the adverse operation of the selection exercised by those who had originally come under observation at the earlier ages. In annuity tables a rate of mortality is required free from the influence of withdrawals; and it seemed likely that, as the number of discontinuances, occurring among those who had entered at age 50 and upwards, would in all probability form but a small proportion of the total discontinuances, the effect of the withdrawals upon the rate of mortality would be minimised, and thus a table suitable for the valuation of Annuity Contracts might be secured. The materials for the construction of the required table were ready at hand in the Institute Mortality Experience—a valuable source which has supplied the data for many similar inquiries, and which will, no doubt, be turned to still further account in times to come. Beyond stating that the table was deduced by two different processes—firstly, by collating the primary observations in the usual classes of “died,” “discontinued,”

“existing,” and “entered ;” and, secondly, by collecting together, under present ages, the suitable “exposed to risk” and deaths, from the subsidiary tables given in the Institute Experience—and that the two results were made to agree, I may perhaps be absolved from giving further particulars with respect to the construction of the table. When in their rough form, the results were graduated according to Mr. Woolhouse’s method, the precise form employed being that suggested by Mr. Ackland, to the great merits of which I wish to bear my humble testimony. I was supported in my intention of framing this table by the reflection that even should it not prove itself of much practical utility in regard to the subject for which it was actually designed, the labour expended upon its construction would, nevertheless, be far from profitless. Any new combination of the H^M observations is bound to possess intrinsic interest ; and in the present instance I was induced to think that a comparison of the new table with one based on the experience of the entrants before 50, and with the H^M Table itself, would, in a certain degree, throw an instructive light upon the effect of the discontinuances upon the rate of mortality. In order that it may be seen what relation the derived tables bear to the parent table, I append Table III., in which are given the rates of mortality and the annuity values, at 3 per cent. interest, according to the three bases. [*See next page.*]

As would be imagined, the table reflecting the experience of entrants after 50 shows a lighter rate of mortality than that of the H^M Table, throughout,—with the exception only of one or two points at the extreme end of the table, where the irregularity of the observations turns the scale in the opposite direction. On the other hand, the rate of mortality arising from those who entered before 50 is (subject to the same reservations) in excess of the H^M mortality. The difference, however, in the three bases of mortality is scarcely as great as might have been expected ; and in the annuity values, the variation, excepting at the first few ages after 50, is not very material. On referring to the reserves brought out by the Model Office in the case of the H^M Table and this new table, it is seen that the liability according to the latter is little more than $1\frac{1}{2}$ per cent. in excess of that required by the H^M Table, while the difference in this direction between the H^M reserves and those of the standard table—namely, the Government Annuitants’ Experience, 1883, males and females combined,—is approximately

TABLE III.

*Comparison of Rates of Mortality and Annuity Values (at 3 % Interest) by
H^M Table, Entrants after 50, and Entrants before 50.*

Age.	RATE OF MORTALITY.			ANNUITY VALUES AT 3 % INTEREST.			Age.
	* Entrants after 50.	H ^M Table.	* Entrants before 50.	* Entrants after 50.	H ^M Table.	* Entrants before 50.	
50	·00623	·01595	·01704	14·612	13·896	13·688	50
51	·00754	·01667	·01742	14·147	13·545	13·338	51
52	·00952	·01755	·01830	13·682	13·188	12·982	52
53	·01236	·01860	·02012	13·228	12·826	12·620	53
54	·01492	·01973	·02063	12·795	12·462	12·265	54
55	·01719	·02103	·02212	12·379	12·094	11·899	55
56	·01914	·02245	·02379	11·974	11·724	11·533	56
57	·02110	·02399	·02535	11·573	11·353	11·169	57
58	·02255	·02563	·02725	11·178	10·981	10·803	58
59	·02450	·02754	·02932	10·778	10·608	10·439	59
60	·02682	·02968	·03153	10·381	10·236	10·077	60
61	·02933	·03204	·03393	9·987	9·866	9·717	61
62	·03224	·03464	·03649	9·597	9·498	9·360	62
63	·03560	·03749	·03904	9·214	9·134	9·006	63
64	·03901	·04041	·04180	8·841	8·774	8·653	64
65	·04229	·04343	·04435	8·476	8·418	8·301	65
66	·04596	·04657	·04747	8·116	8·064	7·947	66
67	·04934	·04989	·05069	7·762	7·712	7·593	67
68	·05273	·05323	·05405	7·410	7·360	7·239	68
69	·05691	·05734	·05727	7·057	7·007	6·882	69
70	·06254	·06219	·06238	6·707	6·657	6·519	70
71	·06857	·06805	·06674	6·369	6·311	6·161	71
72	·07599	·07494	·07388	6·043	5·975	5·800	72
73	·08419	·08286	·08087	5·737	5·653	5·450	73
74	·09176	·09120	·09062	5·452	5·348	5·108	74
75	·09800	·09836	·09968	5·183	5·061	4·785	75
76	·10437	·10637	·11193	4·918	4·782	4·474	76
77	·11062	·11469	·12208	4·656	4·512	4·189	77
78	·11692	·12321	·13627	4·392	4·249	3·915	78
79	·12574	·13306	·14867	4·123	3·992	3·668	79
80	·13738	·14465	·16273	3·858	3·742	3·438	80
81	·15073	·15804	·17672	3·606	3·507	3·229	81
82	·16448	·17135	·18988	3·374	3·290	3·040	82
83	·18076	·18585	·20133	3·159	3·089	2·865	83
84	·19392	·19888	·20810	2·971	2·908	2·695	84
85	·20497	·20989	·22688	2·797	2·739	2·505	85
86	·21611	·21965	·23232	2·623	2·570	2·337	86
87	·22587	·23123	·24726	2·447	2·393	2·136	87
88	·23315	·23930	·25828	2·256	2·206	1·923	88
89	·24956	·25320	·26349	2·029	1·987	1·670	89
90	·27635	·27945	·28279	1·786	1·740	1·336	90
91	·30259	·31274	·35697	1·542	1·487	·919	91
92	·34455	·35131	·51451	1·278	1·229	·471	92
93	·40000	·41578		1·008	·951		93
94	·46943	·50730		·730	·677		94
95	·57222	·63704		·415	·415		95
96							96
97							97
98							98

* Graduated by Mr. Woolhouse's Formula.

10 per cent. The view, therefore, seems confirmed that Annuitants and Assured Lives constitute two separate and distinct classes ; and it is probable that even could we eliminate entirely the effect of discontinuances from the H^M mortality, we should not, by this means, obtain an instrument appropriate for the valuation of Annuity Contracts.

Before proceeding with the subject immediately under discussion, I may be pardoned for referring briefly to these tables. The total facts in the H^M observations, from age 50 upwards, comprised 370,802 years of life exposed to risk, and 12,346 deaths. Of this experience the entrants before 50 contributed the larger proportion 254,989·5 years of life exposed—and 7420 deaths out of the total being referable to this body. The remaining observations, consisting of 115,812·5 years of life exposed and 4926 deaths, were contributed by the class of entrants after 50. These figures will show the extent of the basis on which the two new tables have been constructed.

In Table IV., which is here appended, the graduation of the deaths (entered before 50) is shown as an illustration of Mr. Ackland's columnar method of applying Mr. Woolhouse's formula, and of his device for arriving at the adjusted initial value.

The full process of adjusting the series of values given in Column (2) by Mr. Woolhouse's Formula is contained in the above. With the figures in evidence, the simplicity of the operation becomes obvious, as does also the difficulty of an error creeping into the arithmetical work at any one point and being perpetuated throughout the other stages. The summation of each column verifies the accuracy of the results ; and it will be observed that the sum of the final graduated series is equal to the sum of the ungraduated values. This is not an essential characteristic of a successful graduation Mr. Ackland, indeed, merely applied his scheme of folding back the extended values to the beginning of the table. As, however, it seemed more desirable in the present instance to restrict the experience to the period during which alone observations were recorded, I applied the process at the end of the table as well as the beginning.

In dealing with the annuities in course of payment, I employed Mr. J. Finlaison's Formula to reduce the series to a state of smoothness, the extended terms arising from the successive processes of

TABLE IV.

Showing process of Graduation of Deaths (Entered before 50).

Age.	Deaths Ungra- duated (Radix= 72,726).	First Summa- tion in Fives.	Second Summa- tion in Fives.	Third Summa- tion in Fives.			Summa- tion in Fifteens.			Modified values of preceding function for ex- tremities of Table.	Gradu- ated Deaths (Radix= 72,726).	Age.
x	d_x	Σ_5^1	Σ_5^2	Σ_5^3	$9\Sigma_5^1$	$\Sigma_5^3 + 9\Sigma_5^1$	Σ_{15}	$3\Sigma_{15}$	(7)-(9)		d'_x	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
..							1,225	3,675	— 3,675			..
..				1,225		1,225	2,530	7,590	— 6,365			..
..				4,980		4,980	3,791	11,373	— 6,393			..
..			1,225	12,526		12,526	5,022	15,066	— 2,540			..
..			8,755	25,094		25,094	6,433	19,299	+ 5,795			..
..		1,225	7,546	44,095	11,025	55,120	7,968	23,904	31,216			..
..		2,530	12,568	67,389	22,770	90,159	9,534	28,602	61,557			..
50	1,225	3,791	19,001	92,627	34,119	126,746	11,152	33,456	93,290	154,847	1,239	50
51	1,305	5,022	24,519	117,644	45,198	162,842	12,809	38,427	124,415	155,631	1,245	51
52	1,261	6,433	28,993	140,404	57,897	198,301	14,447	43,341	154,960	160,755	1,286	52
53	1,231	6,743	32,568	158,312	60,687	218,999	14,374	43,122	175,877	173,337	1,387	53
54	1,411	7,004	35,328	172,365	63,036	235,401	18,247	54,741	180,660	174,267	1,394	54
55	1,535	7,361	36,909	183,653	66,249	249,902	20,218	60,654	189,248	182,883	1,463	55
56	1,566	7,787	38,572	193,076	70,083	263,159	22,343	67,029	196,130	192,455	1,540	56
57	1,618	8,014	40,281	201,481	72,126	273,607	24,495	73,485	200,122		1,601	57
58	1,657	8,406	41,986	210,339	75,654	285,993	25,414	76,242	209,751		1,678	58
59	1,638	8,713	43,733	219,393	78,417	297,810	26,141	78,423	219,387		1,755	59
60	1,927	9,066	45,767	228,449	81,594	310,043	27,010	81,030	229,013		1,832	60
61	1,873	9,534	47,626	237,317	85,806	323,123	28,138	84,414	238,709		1,910	61
62	1,971	10,048	49,337	246,721	90,432	336,153	29,398	88,194	247,959		1,984	62
63	2,125	10,565	50,854	253,379	92,385	345,764	30,003	90,099	255,755		2,046	63
64	2,152	10,424	52,137	260,245	93,816	354,061	30,320	90,960	263,101		2,105	64
65	2,144	10,583	53,425	266,159	95,247	361,406	31,315	93,945	267,461		2,140	65
66	2,032	10,817	54,492	271,639	97,353	368,992	31,790	95,370	273,622		2,189	66
67	2,130	11,336	55,251	276,458	102,924	378,482	33,379	100,137	278,345		2,227	67
68	2,359	11,332	56,334	280,648	101,988	382,636	33,633	100,899	281,737		2,254	68
69	2,671	11,183	56,956	284,475	100,647	385,122	34,232	102,696	282,426		2,259	69
70	2,140	11,666	57,615	288,985	104,994	393,979	34,658	103,974	290,005		2,320	70
71	1,883	11,439	58,319	293,155	102,951	396,106	35,070	105,210	290,896		2,327	71
72	2,613	11,995	59,761	298,078	107,955	406,033	35,178	105,534	300,499		2,404	72
73	2,132	12,036	60,504	302,194	108,324	410,518	35,278	105,834	304,684		2,437	73
74	2,227	12,625	61,879	305,480	113,625	419,105	35,126	105,378	313,727		2,510	74
75	2,181	12,409	61,731	306,017	111,681	417,698	34,585	103,755	313,943		2,511	75
76	2,472	12,814	61,605	303,912	115,326	419,238	33,960	101,880	317,358		2,539	76
77	2,397	11,847	60,298	297,325	106,623	403,948	32,178	96,534	307,414		2,459	77
78	2,537	11,910	58,399	287,375	107,190	394,565	31,120	93,360	301,205		2,410	78
79	2,260	11,318	55,292	272,815	101,862	374,677	30,276	90,828	283,849		2,271	79
80	2,244	10,510	51,731	254,577	94,590	349,167	28,240	84,720	264,447		2,116	80
81	1,880	9,707	47,045	233,049	87,363	320,412	26,639	79,917	240,495		1,924	81
82	1,589	8,336	42,060	209,039	75,024	284,063	23,784	71,352	212,711		1,702	82
83	1,734	7,174	36,871	183,805	64,566	248,371	21,882	65,646	182,725		1,462	83
84	889	6,333	31,282	158,931	56,997	215,928	21,689	65,067	150,861		1,207	84
85	1,082	5,321	26,547	134,747	47,889	182,636	17,432	52,296	130,340		1,042	85
86	1,039	4,118	22,171	112,032	37,062	149,094	15,313	45,939	103,155		825	86
87	577	3,601	17,876	92,276	32,409	124,685	13,053	39,159	85,526	84,272	674	87
88	531	2,798	14,156	74,770	25,182	99,952	10,809	32,427	67,525	66,269	530	88
89	372	2,038	11,526	59,679	18,342	78,021	8,929	26,787	51,234	50,117	401	89
90	279	1,601	9,041	47,403	14,409	61,812	7,340	22,020	39,792	39,651	317	90
91	279	1,488	7,080	37,664	13,392	51,056	5,606	16,818	34,238	35,910	287	91
92	140	1,116	5,600	29,067	10,044	39,111	4,717	14,151	24,960	33,218	266	92
93	418	837	4,417	21,839	7,533	29,372	3,635	10,905	18,467	31,436	251	93
..		558	2,929	15,735	5,022	20,757	2,596	7,788	12,969			..
..		418	1,813	10,553	3,762	14,315	2,019	6,057	8,258			..
..			976	6,136		6,136	1,488	4,464	1,672			..
..			418	3,207		3,207	1,116	3,348	— 141			..
..				1,394		1,394	837	2,511	— 1,117			..
..				418		418	558	1,674	— 1,256			..
..							418	1,254	— 1,254			..
All ages	72,726	363,630	1,818,150	9,090,750	3,272,670	12,363,420	1,090,890	3,272,670	9,090,750		72,726	All ages

summation being folded back as before explained. Taking the figures in Table IV., and assuming that we needed to graduate the deaths there given by this mode, we see the greater part of the operation already performed: in Columns 3, 4, and 5, the three summations have been made, and there remains only to eliminate the superfluous terms and reduce the series to the radix by dividing by 125, or multiplying by .008. It may be of interest to see how the process works out, and also how the graduated result compares with the rough series and Mr. Woolhouse's graduation. The commencing and closing terms have been fused together, and the other step is given in the following short table, with, also, the other two series. [*See next page.*]

With this digression, for the length of which I should apologise, I may pass on to the subject of the relative reserves which the usual bases provide. As the standard of comparison by which the reserves should be measured, I have adopted the new Experience of Government Annuitants from 1808 to 1875, henceforth called the Table of 1883: with males and females combined in the proportions of 10,929 males to 19,859 females, as before mentioned. It formed a part of my original design to enter into some slight analytical investigation of the elements of these tables; but the necessity of following this course has been obviated by the close and exhaustive scrutiny to which the President of this Society, Mr. James Meikle, submitted the facts in the able Address delivered at the inaugural meeting of the present Session. It will be noticed that I have employed the "average" annuity values, irrespective of the periods for which the annuities have been in operation. In neglecting to avail myself of the analysed tables which Mr. Finlaison supplies in his report, I have no doubt sacrificed the opportunity of showing the exact effect which the employment of the new Government Tables, in their most perfect form, would exercise in the valuation of annuities. The difficulties in the path of the construction of a Model Office showing the annuities in force at durations, since purchase, of one to five years, proved to be considerable; and, though, through the public spirit of the North British and Mercantile Company, and the kindness of Mr. David Chisholm and Mr. Henry Cockburn, the Actuaries of the Office, I was supplied with most valuable particulars of the operations of the large annuity business of that Company, within a quinquennial intervaluation period, I found myself, I regret to say, unable in the time at my disposal to avail myself of this important information. I shall, however, still

TABLE V.
Deaths (entered before 50), Graduated by Mr. Finlaison's
Method, and Mr. Woolhouse's Method.

Age.	Σ_x^3 (see Table IV.) modified at beginning and end of Series.	Deaths Graduated by Finlaison's Method.	Deaths Graduated by Woolhouse's Formula.	Ungraduated Deaths.	Age.
(1)	(2)	(3)	(4)	(5)	
50	160,016	1,280	1,239	1,225	50
51	161,739	1,294	1,245	1,305	51
52	165,498	1,324	1,286	1,261	52
53	170,838	1,367	1,387	1,231	53
54	177,345	1,419	1,394	1,411	54
55	184,878	1,479	1,463	1,535	55
56	193,076	1,545	1,540	1,566	56
57	201,481	1,612	1,601	1,618	57
58	210,339	1,683	1,678	1,657	58
59	219,393	1,755	1,755	1,638	59
60	228,449	1,828	1,832	1,927	60
61	237,317	1,898	1,910	1,873	61
62	245,721	1,966	1,984	1,971	62
63	253,379	2,027	2,046	2,125	63
64	260,245	2,082	2,105	2,152	64
65	266,159	2,129	2,140	2,144	65
66	271,639	2,173	2,189	2,032	66
67	276,458	2,212	2,227	2,130	67
68	280,648	2,245	2,254	2,359	68
69	284,475	2,276	2,259	2,671	69
70	288,985	2,312	2,320	2,140	70
71	293,155	2,345	2,327	1,883	71
72	298,078	2,384	2,404	2,613	72
73	302,194	2,418	2,437	2,132	73
74	305,480	2,444	2,510	3,227	74
75	306,017	2,448	2,511	2,181	75
76	303,912	2,431	2,539	2,472	76
77	297,325	2,379	2,459	2,397	77
78	287,375	2,299	2,410	2,537	78
79	272,815	2,183	2,271	2,260	79
80	254,577	2,037	2,116	2,244	80
81	233,049	1,864	1,924	1,880	81
82	209,039	1,672	1,702	1,539	82
83	183,805	1,470	1,462	1,734	83
84	158,931	1,271	1,207	889	84
85	134,747	1,078	1,042	1,082	85
86	112,032	896	825	1,039	86
87	92,276	738	674	577	87
88	75,188	601	530	531	88
89	61,073	489	401	372	89
90	50,610	405	317	279	90
91	43,800	350	287	279	91
92	39,620	317	266	140	92
93	37,574	301	251	418	93
All ages	9,090,750	72,726	72,726	72,726	All ages

hope, on a future occasion, to take advantage of these facts, and to carry the work I have now undertaken one step further by the construction of a Model Office which will contain those special characteristics in which the present one is deficient.

The main results of the Government Experience of 1883 (confining ourselves to a consideration of the ages above 50), were to increase the annuity values on male lives in a proportion which commenced at 1·4 per cent. and grew larger at the older ages. At 80 the percentage of increase is given by Mr. Finlaison as 19·3 per cent. In regard to this increase, however, it should be borne in mind that the comparison is instituted between the complete annuity values according to the Government Table of 1860 and the complete annuities *at the date of purchase* as obtained from the new Experience. We shall not therefore find these conditions established in the same degree in the Model Office reserves. In the case of female annuitants the values were decreased. From ages 55 to 70 the ratio of decrease shows little change, at the former age it being 2·9 per cent., and at the latter 2·4 per cent. After age 75, however, the decrease per cent. is but fractional. In relation to this point, it may be interesting to append an extract from Mr. Finlaison's remarks :—

“ A diminished value of the life annuity to females is shown at all ages at which annuities are usually purchased. A conclusion of this nature was to be anticipated from the result of the observation of the mortality of female life annuitants published in the report of Mr. A. G. Finlaison in 1860. On that occasion the difference between the table by which the grant of life annuities is regulated, and the values of annuities shown by a table computed from the data then collected, was admitted to indicate that the vitality of female life annuitants had been too highly rated ; but it was then further stated that it was impossible to predict that another observation to be taken after the lapse of a sufficient interval of time would not point to a restoration of the vitality of the female sex to the proportions shown by the tables then in use for the grant of annuities, and which, in fact, remain in force until the present day.”

These features are observable in the tables embodying the reserves according to the assumed Model Office—Nos. VII., VIII., and IX., and Summary Table X. ; but the fact that the average annuity rates, instead of the analysed values, have been employed renders

the characteristics less apparent. Dealing first with annuities on female lives, the necessary reserves, according to the Experience up to 1875, are seen to be slightly less than those brought out by the Table of 1860, which circumstance is corroborative of the remarks of Mr. Finlaison quoted in the foregoing lines. With respect to the annuities on male lives, it will be apparent on reference to the summaries appended to the tables containing the Model Office reserves, that between the ages 50 and 70, the liability according to the new Experience is in excess of that obtained by using the Experience of 1860. It should, however, be remarked that this excess of reserve is counteracted at the subsequent stages of life,—the total liability by “the 1883 Tables” falling somewhat short of that of “the 1860 Tables.”

It will be instructive to compare the results which the three Government Tables, based on the experience of the lives of annuitants, bring out when applied to the valuation of the assumed basis; and the importance of the tables will amply justify the inquiry. The table which follows (No. VI.) contains a comparison of the valuations required by the three Experiences; and it is not a little curious to observe that, whereas our scientific investigations into the proper reserves required on Assurance Contracts point unmistakably in the direction of increased stringency, the reserves for annuities would seem to follow a different course.

TABLE VI.

Analysis of Reserves by Government Annuity Tables.

	Reserve at 3 per cent.	Reserve at 3½ per cent.	Reserve at 4 per cent.
	£	£	£
Government Table, 1829—Males, . .	2,043,609	1,978,281	1,916,982
Females, . .	2,405,970	2,321,459	2,238,004
Males & Females,	2,277,341	2,199,628	2,124,013
Government Table, 1860—Males, . .	2,056,083	1,989,560	1,926,919
Females, . .	2,258,668	2,173,546	2,102,655
Males & Females,	2,183,530	2,108,244	2,040,279
Government Table, 1883—Males, . .	2,048,911	1,980,841	1,920,967
Females, . .	2,239,984	2,162,675	2,091,045
Males & Females,	2,172,158	2,098,128	2,030,660

Reserves at Decennial Periods of Life (Male and Female).

	50-59	60-69	70-79	80-89	90-98	TOTAL.
	£	£	£	£	£	£
Govt. Table, { 3 p. ct.	548,178	962,467	604,317	154,262	8,117	2,277,341
1829— { 3½	522,297	928,553	589,105	151,719	8,044	2,199,628
M. and F. { 4	498,057	894,683	574,040	149,250	7,983	2,124,013
Govt. Table, { 3	533,684	922,329	571,205	146,999	9,313	2,183,530
1860— { 3½	509,035	888,698	556,657	144,618	9,236	2,108,244
M. and F. { 4	486,218	859,264	543,333	142,309	9,155	2,040,279
Govt. Table, { 3	537,789	926,000	559,601	139,658	9,110	2,172,158
1883— { 3½	511,969	893,640	545,990	137,499	9,030	2,098,128
M. and F. { 4	489,851	863,511	532,911	135,401	8,906	2,030,660

Government Annuity Tables of 1829 are seen from this statement (Table No. VI.) to bring out a liability $4\frac{1}{2}$ per cent. in excess of that required by the Tables of 1883, while the latter fall short of the liability by the Tables of 1860 by about $\frac{1}{2}$ per cent. only. Companies, therefore, that may have based their rates for annuities upon the first of the three Government Experiences, and who may change their basis to the newest tables, may, it would seem, find either that they can reduce their rates, or else that, assuming them to be maintained, there will be a larger margin for profit. Further than this, the comparison of the three tables appears to call for no special comment on this occasion.

Turning our attention to Tables VII., VIII., IX., and X., if we now examine seriatim the Model Office reserves, we shall find exhibited, in a most striking form, the financial effects of valuing the annuity contracts of the assumed Company by the ordinary tables in use for this purpose. The Carlisle Table, which we may consider, in the first instance, is seen to provide fairly strong reserves, the total liability representing about 96 per cent. of the standard reserve. At the oldest periods of life,—from 80 upwards—the Carlisle, indeed, requires the strongest reserve of any of the tables included in my scheme. From 50 to 70, however, the reserves fall considerably short of the standard; and this, it must be remembered, is the period at which, in any progressive Company, most of the annuities will be in force.

On the next table, Davies's Equitable (which used to be more frequently employed than is now the case) it is only necessary to point out, that, except at the ages in the first decennial period

after 50, the reserves are strong and closely approximate to the standard. On the whole, the valuation of the Model Office by this table comes out less than $1\frac{1}{2}$ per cent. in defect of the standard, the reserves on the two bases at each point after 60 being nearly coincident. It is, indeed, questionable whether, for the construction of premiums and the valuation of deferred annuities, this table is not one of the best that can be employed. The deficiency in the facts at the younger ages in the Government Experiences (among other considerations), perhaps, render them in a measure unsuitable for this purpose.

The English Table No. 3 is but little adopted in the valuation of annuities; and it will be seen from the Model Office figures that the liability established by it is the lowest of all tables I have laid under contribution,—the H^M Table even not excepted. This fact will probably be considered somewhat surprising, bearing in mind the comparatively high vitality which is generally attributed to the English Table.

The succeeding table, that of M. De Parcieux, though not adopted in this country, still, I believe, forms the basis for the valuation of annuities in France. There is no necessity, in the present inquiry, to examine and discuss the data from which this table was computed; but I may be permitted to remind the meeting that Francis Baily, in the appendix to his famous work on the *Doctrine of Life Annuities and Assurances* (page 6), gives it the following recommendation: "The observations made in Sweden seem to be the most correct for mankind *in general*; and those given by M. De Parcieux appear to be peculiarly adapted for a set of *life annuitants*." This endorsement may be placed on record as a point of historical interest; but things have greatly changed since the times of Francis Baily, and the table of M. De Parcieux can hardly, in these days, be considered the most suitable one for the purposes of annuity valuations, any more than can the Swedish Table now be regarded as the most fit and proper basis for determining the reserves on life assurance policies. The liability by De Parcieux' Table for the Model Office represents about $91\frac{1}{2}$ per cent. of the standard.

Following the latter table occur the results yielded by the three Government Tables, which having already been separately considered in detail, seem to call for no further comment. If we take males and females in the assumed proportions, the reserves by the tables

of 1829 and 1860 respectively, when compared with the standard, are shown to be 104·7 and 100·5 per cent. respectively.

The H^M reserves, which succeed, represent roughly 90·5 per cent. of the standard. The relatively small liability shown to be required by this table confirms the opinions which have been so generally entertained with respect to the unsuitability of the H^M Table for annuity estimates. It seems, however, a surprising fact (to which I have before adverted) that the H^M reserves should be found to exceed those brought out by the English Table No. 3.

The reserves obtained by the new table framed from the H^M data are deserving of some attention. They are especially noteworthy from the fact of their differing so little from the H^M figures. It may well have been supposed that the exclusion of the unfavourable influences of observations in which the effect of selection may be considered to have expended its force, would have had the result of greatly increasing the vitality of the remainder. This expectation is, however, unrealised, there being no very material difference in the annuity values of the two tables. The actual liability is seen to be about 92 per cent. of the standard reserve, the increased percentage over the H^M results being no more than $1\frac{1}{2}$ per cent.

With the consideration of the last-mentioned table, the comparison of relative reserves may be ended. Other tables,—such as the Seventeen Offices' Experience, the H^F Table, and the new American Tables—have, indeed, been employed in the valuation of annuities. The most important bases, however, and those most likely to come into use in the future, have, I think, been included in my scheme.

The utility of Model Offices for the purpose of ascertaining what financial result will follow from a change in a Company's basis of valuation has, before now, been called in question; and it has been urged that the conditions and character of any particular Company's business may vary so much from the Model Office, that estimates based on the latter may at times be misleading. But even if it is agreed that a Model Office does not afford us the means of arriving at an absolutely correct idea of the results brought about by an alteration in the valuation basis of a Company whose business is of a special class, there yet remain many ways in which a Model Office may perform a useful function. The relative stringency of the several tables adopted can, perhaps, in no other way, be demonstrated so clearly and graphically; while it is, moreover,

an undeniable fact that in some particular cases extremely close approximations have been obtained by the use of a Model Office.

With these remarks, and with a general apology for the incomplete and discursive manner in which I have handled the important subject of this Essay, I may be allowed to bring this paper to a close. So little would seem to be known of the position of the Annuity business of the country, that I am induced to believe that the particulars I have gathered together will be considered, if only to a limited extent, a serviceable and interesting contribution to our science. In the course of preparing my paper I have personally learned many useful and instructive facts ; and, if the result of my labours should yield an equal store of information to any fellow-students of our science, the end will amply justify the means, and the leisure I have devoted to the matter will have been most profitably employed.

TABLE VII.—Model Office Reserves.
INTEREST, 3 PER CENT.

Age period.	CARLISLE TABLE.	DAVIES'S EQUITABLE.	ENGLISH No. 3.	DE PARCIEUX.	GOVERNMENT ANNUITIES, 1829.		GOVERNMENT ANNUITIES, 1860.		GOVERNMENT ANNUITIES, 1883.		H _M TABLE.	NEW TABLE FROM H _M DATA.	Age period.
	Reserve.	Reserve.	Reserve.	Reserve.	MALES. — Reserve.	FEMALES. — Reserve.	MALES. — Reserve.	FEMALES. — Reserve.	MALES. — Reserve.	FEMALES. — Reserve.	Reserve.	Reserve.	
50-59	505,707	510,441	480,697	500,463	176,259	371,919	178,313	355,371	179,860	357,929	493,793	506,327	50-59
60-69	874,929	912,762	819,435	850,981	548,178 309,031	653,436	533,684 307,172	615,157	537,789 308,566	617,434	833,338	840,144	60-69
70-79	540,352	575,918	505,409	507,753	962,467 194,074	410,243	922,329 191,212	379,993	926,000 188,003	371,598	500,357	508,930	70-79
80-89	147,765	139,382	134,759	121,853	604,317 43,726	110,536	571,205 49,966	97,033	559,601 47,712	91,946	127,307	130,075	80-89
90-98	13,031	7,984	10,252	5,059	154,262 2,342	5,775	146,999 3,197	6,116	139,658 3,173	5,937	7,106	7,288	90-98
Total,	2,081,784	2,146,487	1,950,552	1,986,109	8,117		9,313		9,110		1,961,901	1,992,764	Total.
Ratio to Standard,	95·9	98·8	89·8	91·4	104·8		100·6		100		90·3	91·8	Ratio to Standard.

TABLE VIII.—Model Office Reserves.
INTEREST 3½ PER CENT.

Age period.	CARLISLE TABLE.	DAVIES'S EQUITABLE.	ENGLISH No. 3.	DE PARCIEUX.	GOVERNMENT ANNUITIES, 1829.		GOVERNMENT ANNUITIES, 1860.		GOVERNMENT ANNUITIES, 1883.		H _M TABLE.	NEW TABLE FROM H _M DATA.	Age period.
	Reserve.	Reserve.	Reserve.	Reserve.	MALES. — Reserve.	FEMALES. — Reserve.	MALES. — Reserve.	FEMALES. — Reserve.	MALES. — Reserve.	FEMALES. — Reserve.	Reserve.	Reserve.	
50-59	483,282	487,455	460,241	476,981	168,498	353,799	170,510	338,525	171,138	340,831	472,413	483,976	50-59
60-69	845,279	880,544	793,110	823,289	298,668	629,885	296,785	591,913	298,262	595,378	806,529	812,872	60-69
70-79	527,185	561,717	493,887	496,993	189,593	399,422	186,597	370,060	183,608	362,382	489,307	497,401	70-79
80-89	145,302	137,291	132,669	120,440	43,155	108,564	49,183	95,435	46,996	90,503	125,500	128,192	80-89
90-98	12,854	7,925	10,147	5,036	2,317	5,727	3,171	6,065	3,147	5,883	7,059	7,218	90-98
Total,	2,013,902	2,074,932	1,890,054	1,922,739	702,231	1,497,397	706,246	1,401,998	703,151	1,394,977	1,900,808	1,929,659	Total.
Ratio to Standard,	96·0	98·9	90·1	91·6	104·7	100·4	100·4	100·4	100	100	90·6	92·0	Ratio to Standard.

TABLE IX.—Model Office Reserves.

INTEREST, 4 PER CENT.

Age period.	CARLISLE TABLE.	DAVIES'S EQUITABLE.	ENGLISH No. 3.	DE PARCIEUX.	GOVERNMENT ANNUITIES, 1829.		GOVERNMENT ANNUITIES, 1860.		GOVERNMENT ANNUITIES, 1883.		H ^m TABLE.	NEW TABLE FROM H ^m DATA.	Age period.
	Reserve.	Reserve.	Reserve.	Reserve.	MALES. — Reserve.	FEMALES. — Reserve.	MALES. — Reserve.	FEMALES. — Reserve.	MALES. — Reserve.	FEMALES. — Reserve.	Reserve.	Reserve.	
50-59	462,430	465,989	441,246	458,263	161,304	336,753	163,134	323,084	164,501	325,350	452,778	463,796	50-59
60-69	817,393	850,654	768,277	797,151	498,057	605,767	486,218	572,191	489,851	574,949	781,233	787,142	60-69
70-79	514,642	548,145	482,858	486,159	894,683	388,713	859,264	361,092	863,511	353,583	478,530	486,361	70-79
80-89	142,790	135,230	130,679	118,868	574,040	106,654	543,333	93,885	532,991	89,104	123,744	126,374	80-89
90-98	12,682	7,868	10,045	5,015	149,250	5,679	142,309	6,011	135,401	5,788	7,015	7,188	90-98
					2,304	7,983	3,144	9,155	3,118	8,906			
Total,	1,949,937	2,007,886	1,833,105	1,865,456	680,447	1,443,566	684,016	1,356,263	681,886	1,348,774	1,843,300	1,870,861	Total.
					2,124,013		2,040,279		2,030,660				
Ratio to Standard,	96·0	98·9	90·3	91·8	104·6		100·4		100		90·7	92·1	Ratio to Standard.

TABLE X.

Reserves according to various Bases of Valuation, with ratio to Standard.
(GOVERNMENT ANNUITIES, 1883.—MALE AND FEMALE.)

BASIS OF VALUATION.					RESERVE BY MODEL OFFICE.	PERCENTAGE OF RESERVE BY GOVT. ANNS., 1883 (MALES AND FEMALES), AT		
						3 %	3½ %	4 %
					£			
Carlisle,	3 per cent.,	.	.	.	2,081,784	95·9	99·2	102·5
"	3½ "	.	.	.	2,013,902	92·7	96·0	99·2
"	4 "	.	.	.	1,949,937	89·8	92·9	96·0
Davies's Equitable,	3 per cent.,	.	.	.	2,146,487	98·8	102·3	105·6
"	3½ "	.	.	.	2,074,932	95·5	98·9	102·2
"	4 "	.	.	.	2,007,886	92·4	95·7	98·9
English No. 3,	3 per cent.,	.	.	.	1,950,552	89·8	93·0	96·1
"	3½ "	.	.	.	1,890,054	87·0	90·1	93·1
"	4 "	.	.	.	1,833,105	84·4	87·4	90·3
De Parcieux',	3 per cent.,	.	.	.	1,986,109	91·4	94·7	97·8
"	3½ "	.	.	.	1,922,739	88·5	91·6	94·7
"	4 "	.	.	.	1,865,456	85·9	88·9	91·8
Govt. Anns., 1829—Males,			3 p. ct.		2,043,609	94·1	97·4	100·7
"	"	"	3½ "		1,978,281	90·7	94·3	97·4
"	"	"	4 "		1,916,982	88·2	91·4	94·4
"	"	Females,	3 p. ct.		2,405,970	110·8	114·7	118·5
"	"	"	3½ "		2,321,459	106·9	110·6	116·9
"	"	"	4 "		2,238,004	103·0	106·7	110·2
"	"	Males & Females,	3 "		2,277,341	104·8	108·5	112·1
"	"	"	3½ "		2,199,628	101·3	104·7	108·3
"	"	"	4 "		2,124,013	97·8	101·3	104·6
Govt. Anns., 1860—Males,			3 p. ct.		2,056,083	94·7	98·0	101·3
"	"	"	3½ "		1,989,560	91·6	94·9	98·0
"	"	"	4 "		1,926,919	88·7	91·9	94·9
"	"	Females,	3 "		2,253,668	103·8	107·4	111·0
"	"	"	3½ "		2,173,546	100·1	103·6	107·0
"	"	"	4 "		2,102,655	96·8	100·2	103·5
"	"	Males & Females,	3 "		2,183,530	100·6	104·1	107·5
"	"	"	3½ "		2,108,244	97·0	100·4	103·8
"	"	"	4 "		2,040,279	93·9	97·2	100·4
Govt. Anns., 1883—Males,			3 p. ct.		2,048,911	94·3	97·7	100·9
"	"	"	3½ "		1,980,841	91·2	94·4	97·5
"	"	"	4 "		1,920,967	88·4	91·6	94·6
"	"	Females,	3 "		2,239,984	103·1	106·8	110·3
"	"	"	3½ "		2,162,675	99·6	103·1	106·5
"	"	"	4 "		2,091,045	96·3	99·7	102·9
"	"	Males & Females,	3 "		2,172,158	100·0	103·5	106·9
"	"	"	3½ "		2,098,128	96·4	100·0	103·3
"	"	"	4 "		2,030,660	93·5	96·8	100·0
H ^m Table, 3 per cent.,	.	.	.	.	1,961,901	90·3	93·5	96·6
"	3½ "	.	.	.	1,900,808	87·5	90·6	93·4
"	4 "	.	.	.	1,843,300	84·8	87·8	90·7
New Table from H ^m data, 3 per cent.,	.	.	.	.	1,992,764	91·8	95·0	98·2
"	"	"	3½ "	.	1,929,659	88·9	92·0	95·0
"	"	"	4 "	.	1,870,861	86·1	89·2	92·1