number scale could better serve the purpose of mankind than the decimal, with the single exception of the duodecimal. But the advantage of twelve as a base never becomes apparent until the arithmetic of a people has reached such a degree of development that a change from one system to the other would be attended with difficulties so great as to render it quite impracticable. Civilization is wedded to the decimal arithmetic; and though it may buy and sell by dozens and perform its astronomical calculations by sixties, it will always continue to use the arithmetic of tens in preference to any other. All other methods of computation give way, sooner or later, before the decimal; just as all other systems of weights and measures must ultimately yield and disappear before the metric system.

A MINOR REFORM.

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In the Hindu notation of numbers the higher units always precede the lower. This is in accord with the almost universal law, first observed by Hankel, that in the additive combination of numbers the larger precedes the smaller.* In numeration we observe the same law except for the nine numbers between ten and twenty. Instead of saying ty-one, ty-two, ty-three, ty-four, ty-five, ty-six, ty-seven, ty-eight, ty-nine, we say eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, respectively. Why should we say thirteen when we do not say three-twenty or three-thirty?

The matter looks worse when it is observed that in numeration the smaller units before the larger always signify multiplication with the exception of the numeration of the nine numbers mentioned above. Thirteen means three-ten just as thirty does. Should such ambiguity be tolerated when the remedy is so simple? If we should adopt nine words like those suggested above we would have a rational, consistent numeration at least up to 10,000.

The difficulties in the way of adopting these nine number words instead of those in vogue are slight when compared with those encountered in the change of the base of notation, or even in the

* Cf. Cantor *Geschichte der Mathematik*, 1894, p. 14.

change from the ordinary system of weights and measures to the metric system. It would involve nothing more than the learning of nine closely related words. No tables would have to be reconstructed nor would any books become useless by their adoption.

While the difficulties in the way of this minor reform seem slight compared with those in the way of other reforms just mentioned, it must be admitted that the advantages resulting from its adoption would also be very much smaller. The question is whether the advantages are sufficient to justify the slight difficulties. Does consistency and uniformity appeal sufficiently to our ethical nature to pay attention to this matter which lies at the threshold of the education of every child?

As Taylor has aptly remarked, "The case is not uncommon of high civilization bearing evident traces of the rudeness of its origin in ancient barbaric life." Do we want to wipe away another one of these traces or are we anxious to be continually reminded of our past? If the change is to be made it requires agitators and adherents in large numbers. It seems to be merely a question as to whether the time is ripe for this work; for, as culture advances more refined elements of progress receive attention.

The English speaking people could perfect their numeration with more ease than the German, since in the latter language the smaller precedes the larger in additive combination up to 100. For instance, the Germans say three and twenty, four and ninety instead of twenty-three, ninety-four, respectively. They would therefore have to employ eighty-one new number words whereas we would need only nine to effect this reform. The French, on the contrary, use the words ten-seven, ten-eight, ten-nine for seventeen, eighteen, nineteen, respectively, but they employ some other irregular number words, such as sixty-sixteen for seventy-six, four-twenty-thirteen for ninety-three.

It seems that reform is also desirable in the numeration of large numbers. The fact that we should have separate names for each order of units up to the third and then employ separate names for every third order thereafter can scarcely be justified. However, these numbers occur seldom and do not enter very largely into the education of the child, and hence reform along this line is not so imperative.