XIV.—*Apparatus for Measuring Strain and Applying Stress, with an Account of some Experiments on the Behaviour of Iron and Steel under Stress.* By E. G. COKER, D.Sc. Communicated by Dr C. G. KNOTT. (With Eight Plates.)

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I.—INTRODUCTION.

The behaviour of metals under stress has long been the subject of investigation, both by mathematicians and physicists, so that the laws of strength are tolerably complete. Owing to the importance of iron and steel in construction, these materials have been subjected to very extensive tests, particularly in simple tension and compression.

Numerous tests of cylindrical iron and steel bars in torsion are also available, the bulk of these being tests to destruction of samples of material used in actual machines and structures designed by engineers. In such tests scientific accuracy is not of much importance, the chief consideration being the obtaining of sufficient data for use in design. The most accurate torsional work upon iron and steel has been the work of physicists, and nearly all their investigations have been conducted upon specimens of very small sectional area; the reasons for this, no doubt, being that such specimens in the form of wires are easily obtainable, and of great uniformity in size and quality, while large test pieces are costly to prepare, and, moreover, cause considerable difficulty in testing, because of the magnitude of the forces involved. Owing to the mode of manufacture, the physical properties of wire often differ to a considerable extent from turned

specimens of iron and steel. These differences may be caused by the hardening effect of the drawing, minute cracks in the wires, want of roundness, and the like. It therefore appeared probable that experiments on the lines indicated by physicists would be of some service, and it was with this idea that the investigation was commenced.

The chief difficulty in the accurate investigation of the torsional properties of metal bars lies in the lack of suitable apparatus for the work; and after reviewing the chief machines available for measuring strain and applying torque—to all of which there seemed some objection—it was resolved to design and construct special appliances for the work.

Attention was first directed to the design and construction of a self-contained instrument for measuring strains, which should be sufficiently accurate to measure strains of one second of arc; and after some experiments an instrument was constructed which satisfied these conditions.* A modification of this was used in the work of this paper, and is described in Section II.

In most machines for applying torque, the construction is such that the weigh-lever can only be used for torsion in one direction, and the ends of the specimen are fixed, so that it is impossible, for instance, to apply a bending moment and torque, or a tension and torque, together.

A machine was therefore constructed to allow of torque in either direction, and also permit of the application of a uniform bending moment and a pure torque, to give a combined stress. A separate device was constructed for giving the combined stresses of tension and torsion.

II.—DESCRIPTION OF THE APPARATUS.

1. *Instrument for Measuring Strains.*

In making measurements of small strains it is a great advantage to use an instrument which will read directly, and which is self-contained and wholly supported on the specimen under test, thereby avoiding external scales, the positions of which with respect to the specimen may be changed by a disturbing element, such as slipping of the grips, applied bending moment, and the like. In order to meet these conditions, an instrument was designed for the purpose of these experiments, and is shown in sectional elevation by fig. 1, and in side elevation by fig. 2. It consists of a graduated circle *A* mounted upon a chuck plate *B*, provided with three centring screws adjustable by hand. Upon the Vernier plate *J*, an arm *O* carries an extension *K*, upon which is secured a frame *X*, carrying a thick wire *P*. The movement of the wire is observed by a reading microscope carried in the sleeve *R* of an arm *S* mounted upon the short cylinder *C*, which latter is gripped upon the test bar by screws *L*.

The reading microscope has an eye-piece *T* provided with a glass scale, and a right-

* COKER, "On Instruments for Measuring Small Torsional Strains," *Phil. Mag.*, December 1899.

angled prism is interposed between this and the objective W , so that readings can be easily taken. The tube Q is free to slide or rotate in its guide R , but, in order to readily focus the wire, this latter is carried in a frame X , pivoted upon the Vernier plate J , and adjusted by a screw V .

The microscope arm S is secured to the cylinder C by a divided collar, the two halves of which are pivoted on one side, and the free ends clamped by screws.

If it is desirable to turn the telescope round or to release it altogether, the screw may be thrown out of engagement. Readings are taken from one edge of the thick wire, and as this edge is very distinct, it fatigues the eye much less than a spider line or scratch upon glass, which latter have the further disadvantage of being of appreciable thickness. Fig. 5 shows the appearance of the field of view of the reading microscope and the wire P .

No appreciable error is caused by the fact that the divisions upon the glass scale of the microscope are linear measurements, while the movement of the wire is in a circle. For if ABC be the path of the wire and AC the chord, then the error is the difference between the arc ABC and its chord AC when the angle α is a small quantity of the first order.

$$\begin{aligned} \text{I.e. } \Delta &= ra - 2r \sin \frac{\alpha}{2} \\ &= ra - 2r \left(\frac{\alpha}{2} - \frac{\alpha^3}{24} + \frac{\alpha^5}{240} - \text{etc.} \right) \\ &= r \left(\frac{\alpha^3}{24} - \frac{\alpha^5}{240} + \text{etc.} \right) \end{aligned}$$

a small quantity of the third order, and therefore negligible. In practice this is shown to be the case, as no difference can be observed between the parallelism of the wire P and scale for any position of the former. The reading of the microscope scale may therefore be taken as directly proportional to the angular displacement, and the calibration is effected by moving the wire through a definite angle and noting the equivalent reading of the micrometer eye-piece.

It is essential that the graduated circle be set accurately upon the bar, with its plane perpendicular thereto and its centre coinciding with the longitudinal axis of the bar. An arrangement was devised to effect this, consisting of a pair of divided collars a , the halves pivoted together at b , and secured by nuts c . The collars are wedge-shaped, in radial section to engage with correspondingly wide-angled grooves, upon the chuck plate and cylinder only the angled sides being in contact, so that the collars are readily fixed or freed when required. The lower halves d of the divided collars are connected by one or more distance pieces e , so that when the former grip their respective grooves each piece has one degree of freedom with respect to the clamp, and

this is sufficiently suppressed by the frictional grip of the collars, thereby causing the parts to act as one rigid whole for setting the instrument on the bar.

Both main pieces are chucked by set-screws, and with a little experience this can be effected as accurately as by self-centring chucks.*

In nearly all machines for applying torque some bending is also present, and it is therefore necessary to eliminate any possible errors due to this cause.

If the bar is bent in the plane containing its centre line and the observation wire, it has the effect of causing new parts of the wire to come opposite the scale, but no error in reading is caused thereby. If, however, the bar is bent in a plane at right angles to this, the effect of the bending will be read as an addition to, or subtraction from the twist. Bending in any other plane may be resolved into components in these two planes, and it is therefore only necessary to eliminate the error due to bending in a plane perpendicular to the plane of the paper.

The error may be got rid of by using two reading microscopes set opposite to one another, and a mean reading taken, but as this doubles the labour of observation it is inconvenient. Another plan is to arrange the wire mid-way between the sections gripped by the set-screws: then if

$$\begin{aligned} 2a & \text{ be the length under measurement,} \\ \theta & = \text{angle of bending at first section,} \\ \phi & = \text{angle of bending at second section,} \\ & \text{error in reading becomes } a (\sin \theta + \sin \phi); \end{aligned}$$

and if θ and ϕ are equal and opposite the error vanishes. A specimen stressed by two equal and opposite couples applied at its ends bends into the arc of a circle, and fulfils the necessary condition for the equality of θ and ϕ , and in the application of torque this condition has been fulfilled. As a matter of precaution the observation wire is always set in the plane of bending.

In order to test the accuracy of the instrument, torsion tests were made (I.) with no bending, (II.) with bending moment of known amount. As an example the following may be quoted:—

Turned bar of rivet steel	.	.	Torsion arm = 15·00.
Diameter 0·662	.	.	Calibration.
Length under test	.	.	1 min. = 54·4 divisions.

* *Loc. cit. ante.*

TABLE I.

Torque in inch lbs.	No Bending Moment.		Bending Moment 480 inch lbs.		Bending Moment 800 inch lbs.		Bending Moment 1120 inch lbs.	
	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ
0	0		0		0		0	
7.5	51	-51	51	-51	50	-50	50	-50
15.0	102	-51	101	-50	101	-51	100	-50
22.5	153	-51	152	-51	150	-49	150	-50
30.0	204	-51	203	-51	200	-50	201	-51
37.5	256	-52	254	-51	251	-51	252	-51
45.0	307	-51	306	-52	301	-50	303	-51
52.5	357	-50	356	-51	352	-51	354	-51
60.0	408	-51	406	-50	404	-52	404	-50
67.5	459	-51	457	-51	455	-51	453	-49
75.0	509	-50	507	-50	505	-50	503	-50
0	2		1		1		2	

The differences caused by bending will be discussed in Section XI.

2. Machine for applying Torque and Bending.

The apparatus used for applying twist in either direction, and for the combined stresses of twist and bending, is shown in side elevation by fig. 6, in plan by fig. 7, and in sectional end elevation by fig. 8. The machine was specially constructed for the work of this paper, and consists essentially of two similar and equal castings *A*, bored axially to receive double-coned spindles *B*, the outer ends of which project through the castings and are secured by nuts *C*. At the weigh-lever end the cone is secured to the casting by studs *D*, but at the other end, in order to take up the twist upon the specimen, the cone is gripped partly by the back-nut *C*, and partly by a plate *E* pressed against its face by studs.

Each casting is bored at right angles to the axis to receive arms *F G*, of which the former are used for hanging weights therefrom to give the torque, while the latter one, *G*, carries a link *H*, having an adjusting screw *I*, and nut *J*, whereby the weigh-levers *F* can always be brought to the horizontal position, the final adjustment being made with a sensitive level *K*; while the other carries a balance weight *L*. The ends of the

specimen *M* under test are secured in grips *NN* upon the projecting ends of the cones *B*.

In order to obtain a pure torque and a pure bending moment, both acting at the same time, each casting is supported by a ring (fig. 9) encircling the spindles *B*, and furnished with friction rollers *P*, running in grooves in the spindles *B*. The rings have bearings *Q*, turning in stirrups *R*, these latter being hung from a horizontal bar *S*, by adjustable vertical hangers *T*. With this arrangement we get a pure torque of a known amount throughout the specimen.

Bending Moment.—Into the outer ends of the nuts *C* are screwed projecting arms *V*, of known length and carrying weights at their ends. These put a bending couple upon the specimen without shear, the arm of the couple being the distance of the weight from the hanger *T*. With this arrangement simple twist and simple bending, or any combination thereof, can be applied to a specimen with ease. The specimen is free to take up its own position of equilibrium, since it is imperfectly constrained—(the specimen can be easily rocked about even when fully loaded, but always comes back to the first position after a few oscillations)—and the conditions of stress are accurately known.

Corrections: Twist.—The results of tests on the friction of the roller bearing show that the friction is so small a quantity as not to introduce any sensible error.

Bending.—The friction error is that due to the stirrups embracing the bearings *Q*, which latter were made large purposely. No experiments were made in which the bending moment varied during the experiment; consequently it was sufficient in each case to calculate the error due to friction for the particular load applied, and to make the small corrections necessary. This has been done in all cases.

3. *Apparatus for applying Torque and Tension.*

The apparatus for applying torque and tension is shown in general elevation by fig. 11, and in plan by fig. 12. A detailed section of some of the parts with the measuring instrument fixed to the specimen is shown by fig. 13.

The specimen *A* was screwed into a turned piece *B*, having a slotted hole above, and tapped to receive a screw *C*, centred in a corresponding depression in a piece of tool steel *D* resting upon a plate *E*, which latter was carried by four bolts *F*, depending from a cast-iron beam *G* mounted upon two pillars *H*. Below, the specimen was screwed into a turned piece *J*, carrying a sleeve and pulley *L*, the lower end of the piece being fitted to receive a nut *M* and hanger *N* for weights. The torque was applied by weights attached to fine bands made of clock spring steel, which latter were attached to the pulley at convenient points, and passed over guide-pulleys *O* mounted on ball-bearings. The applied torque was balanced above by a double-ended lever *P*, keyed to the piece *B*, so that its axis passed through the point of suspension, and the lever being furnished with screws at its outer ends, these latter pressed equally against the pillars *H*. The

weights used for applying the torque were made by Oertling, or were copies therefrom, and the twelve 200-pound weights for applying the tension were standard weights forming part of the equipment of the 100-ton Buckton testing machine in the laboratory. The method of suspension ensures that all the tension load is evenly distributed in the section of the specimen, and there is no correction for friction, as the load is a dead-weight one.

In applying the torque a small correction must be made for the friction of the pulleys. This was determined as follows :—

The pulley was first balanced by winding lead strips round its arms until it would stand in any position, or when rotated by a smart pull, continue to revolve several minutes. Next, equal weights, A B, were attached to the spring steel tape passing over the pulley with the ends vertical, and the additional mass required to just start the pulley in one direction was determined. The weights were then reversed, and the additional mass required was again determined, the mean value of the two being taken.

Let T_1 be the tension on one side.

T_2 „ „ the other side.

T_3 = tension horizontally.

We have

$$\frac{T_2}{T_1} = e^{\mu\pi}$$

$$\frac{T_3}{T_1} = e^{\mu\frac{\pi}{2}}$$

very approximately ; from which we get by a simple transformation,

$$T_3 = \sqrt{T_1 T_2}.$$

This value was calculated throughout the range. As an example the following numbers may be quoted for the left-hand pulley :—

TABLE II.

T_1 pounds.	$T_3 = \sqrt{T_1 T_2}$	T_1 pounds.	$T_3 = \sqrt{T_1 T_2}$
2	2.0078	12	12.0118
4	4.008	14	14.012
6	6.0088	16	16.013
8	8.0095	18	18.014
10	10.0113	20	20.015

The correction for the other pulley was slightly less.

III.—METHOD OF EXPERIMENTING.

The diameter of the bar to be tested was first ascertained by a micrometer caliper, the mean of several readings being taken. The measuring instrument was then applied, and the calibration value of the readings ascertained. The bar was then placed in the testing machine, and the balance weights adjusted to give zero torque. If bending moment had to be applied, this was effected before the final adjustment of the reading microscope, care being taken to bring the wire midway between the two sets of clamping screws, and also to set the wire in the most favourable position for taking accurate readings, viz., the plane in which bending takes place. Unless readings are taken at equal intervals of time, the time effect of a stress will show itself, and it is therefore very necessary that the separate loadings be at equal intervals. It was found that the most convenient interval was one and a half minutes, this being necessary to bring the weigh-beam back to its zero position; and all readings were taken with this interval, except where otherwise stated.

IV.—THE FORM OF THE STRESS-STRAIN CURVE.

Before taking up the detailed examination of the relation of stress to strain, it is of interest to consider the stress-strain curve as a whole.

Fig. 15 shows the general nature of the stress-strain curve for a wrought-iron or steel bar of circular section when subjected to a gradually increasing torque. Starting from no torque, and gradually increasing the load upon the bar, the relation between stress to strain is found to be a linear one until near the point *a*, when the defect from linearity is first noticed in the gradual creeping up of the readings, the whole twist upon the bar being at the rate of from 1° to 2° per inch of length.

At *a* the yield-point occurs, and there is a large increase in the strain, with no increase in the loading. The material has also changed from a nearly perfectly elastic to a semi-plastic condition, and the bar when released from load will no longer go back to zero, but shows a very considerable set. The material has also hardened by the process, and the curve rises at first quickly to a point *c*, and then more slowly until fracture occurs at *d*; the strain then being generally considerably more than one hundred-fold the strain within the perfectly elastic condition.

V.—THE FORM OF THE CURVE AT THE YIELD-POINT.

The first sign of deviation from the linear law indicates the failure of the elastic state at the fibres most severely strained—viz., the outer ones—and a semi-plastic condition is entered upon, which, as the loading proceeds, extends inwardly until a more or less uniform shear stress is established throughout the section. The passage from the one

state to the other in a solid specimen requires a certain range of stress, so that the diagram at the point *a* exhibits a well-marked rounding.

If we call q_o the maximum shear at the outer surface,

$$\begin{aligned} r_o &= \text{radius of the specimen.} \\ r_p &= \text{radius to which plasticity extends.} \end{aligned}$$

Then the resistance of the bar to torque is the sum of the resistances due to (1) the still elastic core, (2) the semi-plastic shell, and may be represented by

$$T = 2\pi q_o \frac{1}{r_o} \int_o^{r_p} r^3 dr + 2\pi q_o \int_{r_p}^{r_o} r^2 dr$$

where r = any radius

$$= \frac{\pi}{2} q_o \frac{r_o^4}{r_o} + 2\pi q_o \left(\frac{r_o^3}{3} - \frac{r_p^3}{3} \right)$$

up to the point where perfect elasticity prevails $r_o = r_p$ and

$$T = \frac{\pi}{2} q_o r_o^3;$$

but when the specimen is wholly plastic

$$r_p = 0,$$

and we get

$$T = \frac{2}{3} \pi q_o r_o^3,$$

which is $\frac{4}{3}$ of the value at the elastic limit.

It would therefore appear that if the bar changes from the elastic to the plastic condition at the yield-point, the maximum torque will be four thirds of that value at which the first-marked deviation from perfect elasticity occurs.

The result of experiment shows a fair agreement with this conclusion. In the example of a wrought-iron specimen quoted in the next section, Table III., col. I., the first-marked deviation occurs below 375 inch pounds, while the material failed at 525 inch pounds, giving a ratio of 1.4.

Taking another case for the steel specimen quoted in the same section, Table IV., col. I., the first deviation occurred below 675 inch pounds, and failure took place at 870 inch pounds, corresponding roughly to a ratio of 1.29, which is very close to $\frac{4}{3}$.

Having regard to the difficulty of observing exactly the first sign of failure, it seems probable that the conditions assumed are not far from the truth.

VI.—RECOVERY OF ELASTICITY WITH TIME.

If a bar of iron or steel be subjected to a torque causing a permanent strain in it, the condition of the bar becomes quite different; it no longer obeys HOOKE'S law, and the strain for a given stress is now greater than before the increase, being more marked at the higher loads. As an example we may take that of a turned wrought-iron bar

of length between centres of 4.00 inches; diameter 0.472; calibration value of readings 1 min. = 12.85 divisions. The following results were obtained:—

TABLE III.

Column I.

Torque in inch lbs.	Reading.	Δ
0	0	243
75	243	243
150	486	244
225	730	242
300	972	254
375	1226	268
450	1484	140
480	1624	160
510	1784	
525	Went off scale	

The load was now removed, and immediately afterwards re-applied, with the following result:—

TABLE III.—*continued.**Column II.*

Torque in inch lbs.	Reading.	Δ
0	0	245
75	245	255
150	500	262
225	762	269
300	1031	289
375	1320	249
300	1071	249
225	822	259
150	563	265
75	298	272
0	26	

It will be seen at once that the bar exhibits quite different qualities from that shown before. The stress is now no longer proportional to the strain, and the curve showing the relation between the two no longer returns upon itself, but forms a looped figure.

Similar results are obtainable from bars subjected to tensional stress.* †

If the bar be tested again after a short interval of time, recovery will be found to be very marked. At the end of one hour a test of the bar gave the results shown by column III., there being a marked falling off of the increments at the higher loads. Thus the strain caused by increasing the torque from 300 inch pounds to 375 inch pounds now caused only 272 units of strain, instead of 289; and similarly at the end of three hours we find a further decrease to 268 units. The recovery of the bar was tested at suitable intervals of time, as shown in the annexed table, the effect becoming less apparent as the time increased; but practically perfect recovery was reached at the end of two days, and very little change was noticeable after this time.

TABLE III.—*continued.*

Column III. (One hour afterwards.)

Torque in inch lbs.	Reading.	Δ
0	0	
75	246	246
150	498	252
225	753	255
300	1019	266
375	1291	272
300	1043	248
225	793	250
150	536	257
75	276	260
0	7	269

The results may be shown graphically by direct plotting, but it is more convenient to adopt the plan of subtracting from each reading a number proportional to the torque, and plot the new set of readings thus obtained. The method is due to Prof. EWING, and is used in fig. 16, the diminution in the case being 200 units of scale reading for

* EWING, "On the Measurement of Small Strains in the Testing of Materials and Structures." *Proc. Royal Society*, May 1898.

† MUIR, "On the Recovery of Iron from Overstrain." *Phil. Trans.*, 1899.

TABLE III.—*continued.*

Torque in inch lbs.	Col. IV.		Col. V.		Col. VI.		Col. VII.		Col. VIII.		Col. IX.		Col. X.		Col. XI.	
	3 hours after.	Δ	6 hours after.	Δ	12 hours after.	Δ	1 day after.	Δ	2 days after.	Δ	2 days after.	Δ	5 days after.	Δ	6 days after.	Δ
	Reading.		Reading.		Reading.		Reading.		Reading.		Reading.		Reading.		Reading.	
0	0	245	0	246	0	245	0	245	0	243	0	245	0	245	0	244
75	245	246	246	244	245	243	245	243	243	245	245	247	245	244	244	245
150	491	252	490	249	488	245	488	246	488	247	492	246	489	245	489	244
225	743	256	739	256	733	247	734	244	735	245	738	245	734	246	733	246
300	999	265	995	260	980	253	978	249	980	246	983	244	979	246	979	246
375	1264	245	1255	245	1233	242	1227	244	1226	247	1127	245	1225	246	1225	247
300	1019	249	1010	248	991	246	983	243	979	243	972	244	979	245	978	245
225	770	254	762	250	745	247	740	243	736	247	728	247	734	245	733	245
150	516	254	512	255	498	249	497	246	489	245	481	247	489	246	488	245
75	252	259	257	256	249	248	251	246	244	245	234	244	243	243	243	243
0	-7		1		1		0		-1		-10		0		0	

a torque of 75 inch pounds. A time-recovery curve, fig. 17, has been plotted, the ordinates of which correspond to the reading under a torque of 375 inch pounds and the abscissæ are times; this curve shows in a marked manner how rapid is the recovery at first.

As a means of comparison with the last bar, a steel bar was now tested, the specimen being classed as machinery steel, *i.e.*, semi-mild. Length under test 4.00; diameter 0.425; calibration 1 min. = 12.85 divisions.

The following results were obtained :—

Torque in inch lbs.	Reading.	Difference.
0	0	
75	385	385
150	770	385
225	1157	387
300	1543	386
375	1930	387
450	2323	393
525	2710	387
600	3103	393
675	3503	400
750	3920	420
780	4231	311
810	4675	444
825	4929	254
840	5309	380
855	6009	700
870	Went off scale	

The load was then removed, and again applied by increments of 75 inch pounds until a limit of 750 inch pounds was reached, the load being afterwards reduced by 75 inch pounds to nothing. Tests were made at intervals of time, as recorded in Table IV., which latter shows that the recovery is much slower in the former case; and even at the end of nineteen days the specimen showed signs of the initial overstrain.

The curves, fig. 18, showing the results of the experiments were plotted by the indirect method previously described, 350 units being subtracted for an increment of

TABLE IV.

Torque in inch lbs.	Col. I.	Col. II.	Col. III.	Col. IV.	Col. V.	Col. VI.	Col. VII.	Col. VIII.	Col. IX.
	Immediately after I.	Reading. Δ	1 hour after I.	3 hours after I.	6 hours after I.	1 day after I.	3 days after I.	6 days after I.	19 days after I.
	Reading. Δ								
0	0	395	0	392	0	390	0	390	0
75	385	395	395	392	390	390	390	389	389
150	770	792	790	784	783	778	777	780	778
225	1157	1202	1195	1187	1184	1175	1170	1168	1167
300	1543	1612	1611	1593	1590	1579	1564	1557	1555
375	1930	2026	2023	2007	1997	1988	1968	1957	1944
450	2320	2448	2441	2429	2412	2407	2380	2360	2337
525	2707	2860	2859	2844	2829	2822	2795	2770	2728
600	3100	3302	3284	3268	3244	3242	3216	3189	3128
675	3500	3746	3706	3609	3674	3669	3643	3616	3533
750	3920	4206	4166	4137	4108	4103	4077	4038	3940
675		384	3768	3738	3715	3706	3694	3643	3549
600		406	398	399	393	398	392	398	390
525		3416	3370	3339	3315	3308	3292	3245	3159
450		407	406	403	404	401	401	400	393
375		3009	2964	2936	2911	2907	2891	2845	2766
300		2509	2548	2528	2503	2497	2485	2446	2374
225		2192	2135	2117	2092	2087	2080	2041	1981
150		1777	1724	1702	1679	1675	1672	1638	1587
75		1359	1305	1286	1266	1261	1260	1229	1191
0		935	889	864	840	837	839	814	793
		497	445	433	414	415	416	407	393
		52		3	-14	-9	-1	-11	-4

75 inch pounds of torque. The rate of recovery is also shown by fig. 19, corresponding to fig. 17 of the former case. The readings at 750 inch pounds are plotted as ordinates, and the times as abscissæ. The difference between these latter curves is very apparent. From the diagrams it is apparent that the bar recovers very rapidly at first like the wrought iron; but this rate of recovery soon slackens and becomes less and less apparent as the time increases, and unless a very considerable time is given the recovery does not become complete (*cp.* Table XV., col. X.).

VII.—THE POSITION OF THE YIELD-POINT AS AFFECTED BY PREVIOUS STRESS.

The effect of a previous stress upon the properties of a bar has been explained in Section VI., and it remains to point out that overstrain in one direction has a very considerable influence upon the yield-point or curve, separating the elastic from the plastic stage; in fact it disappears, but gradually reappears again—the recovery in the case of the iron being practically complete in two days, while for the steel nineteen days effects partial recovery only. In both cases, if sufficient time be given, the yield-curve will assume a definite position above the last position, and this rise is augmented by every overstrain. As an example, we may take that of a wrought-iron bar, having a length under test of 4·00 inches; diameter 0·420 inches; calibration 1 min. = 12·8 divisions.

The bar was subjected to stress extending beyond the yield-point, and afterwards left to rest for a minimum period of $1\frac{1}{2}$ days, when the load was repeated. Eight tests were made, and each time there was a perceptible rise in the yield-curve. The observations were plotted with each curve spaced 1000 divisions from its neighbour. The curves are given in fig. 20, and require no further explanation.

VIII.—TWIST IN ALTERNATELY OPPOSITE DIRECTIONS.

It has long been a common assumption that the limits of elasticity for a bar subjected to torsion lie equally distant from the position of no torque, and this is no doubt true for a specimen not previously strained.

Apparently the first theoretical discussion of the problem is that by JAMES THOMSON,* and in his original paper he makes the further assumption “that the limits of elasticity in a substance which has already been strained beyond its limits of elasticity are equal on the two sides of the shape which it has when in equilibrium without disturbing force.” This note, added in October 1877, goes on to say: “A supposition which may be true or may not be true. Experiment is urgently needed to test it, for its truth or falseness is a matter of much importance in the theory of elasticity.”

The paper further points out that these assumptions lead to the important result that if a wire be overstrained, its strength to resist torsion in the original direction is twice that in the other direction.

* *Cambridge and Dublin Mathematical Journal*, 1848; and article “Elasticity,” *Enc. Brit.*

was reached. The return curve was less so, but as soon as torsion was applied in the negative direction the linearity disappeared, and the strains, though irregular, became greater and greater as the torque increased. The material finally gave way under a torque of about 1100 inch pounds. The torque was now reversed, and the stress-strain curve became approximately linear until the zero torque was reached, from which point the curve began to bend over to the left, until a torque of 1175 inch pounds was reached.

In order to roughly test the behaviour of the specimen still further, the applied torque was continued, but no strain measurements were taken.

Each complete cycle produced a hardening effect on the bar, widening its limits of endurance each time until a final limit of 1750 inch-pounds of torque was reached after fourteen reversals of the stress. The bar was now cracked in several places along the minute seams of impurities, and further experiment seemed useless.

This experiment demonstrates that the limits of elasticity do not remain in their original positions, and, further, it shows that stress carried beyond the elastic limit in one direction reduces the other limit to zero. The conclusion derived from the theory above, that the bar is twice as strong to resist torsion in the original direction as in the other, is also not borne out by the experiment.

A second bar of wrought iron was next examined in the same manner, only four cycles being performed, of which the first two are shown in fig. 23 and the last two in fig. 24. These curves exhibit the same general properties as the one described above. It is evident from figs. 23 and 24 that, after the first reversal of stress, there is no perceivable yield-point, all such critical points being absent. The commonly received idea

that raising the elastic limit in one direction lowers it in the contrary direction does not hold good here, since all critical points vanish.

The further development of the idea that the distance apart of the limits is a constant, appears to have no physical basis for the torsion of iron.

IX.—THE INFLUENCE OF TENSION ON TORSION.

1. *Tension within the Elastic Limit, and Torsion within the Elastic Limit.*

Among the notable experiments made upon the influence of tension upon torsion are those of M'FARLANE upon steel pianoforte wire.*

From the article it does not appear that any experiments were made to ascertain whether tension within the elastic limit has any influence upon torsion within the elastic limit, but in any case it was thought worth while to make the experiments, as specimens of much larger diameter could be dealt with.

The first specimen tried had the comparatively large diameter of $\frac{3}{4}$ inch, and the maximum tension load which could be applied was 3000 pounds. Repeated experiments failed to show any difference in the torsional properties of the bar, whether loaded or unloaded.

A second bar was then prepared, having a diameter of $\frac{1}{2}$ an inch, and the experiment was repeated with tension loads varying from 200 to 3000 pounds, the latter corresponding to a stress of 15,300 pounds per square inch. The diameter of the pulley was 41·62 inches, so that a weight of one pound in each pan corresponded to a torque of 41·62 inch pounds.

The length of the specimen was 8 inches, and the calibration value gave one division of the scale = 16·85 seconds.

The following Table gives a summary of the results obtained :—

TABLE V.

Tension Load on Specimen. Lbs.	Mean Value of Reading corresponding to 1 lb. in each pair = 41·62 inch lbs.		Mean of Columns 2 and 3.
	Torque increasing.	Torque diminishing.	
200	59·00	58·98	58·99
600	59·00	58·74	58·87
1000	58·56	58·70	58·63
2000	58·59	58·70	58·65
3000	58·75	58·65	58·70

* *Enc. Brit.* Art. "Elasticity."

As will be seen from the last column, the values obtained differ very little, in no case varying more than one-half of one per cent.

The above experiments were carried out for me during the latter part of 1898 and the beginning of 1899 by Mr COLPITTS—then a student in the Civil Engineering Department of the University. As a test of the accuracy of these results, a third bar was prepared, having a diameter of 0.375 inch; length 8.00. A new objective was fitted to the measuring instrument, rendering it much more sensitive. The calibration value gave 1 minute of arc = 62.4 divisions of the scale. This necessitated low torques, in order to prevent the observation wire from passing out of the field of view. A new pulley was used, having a diameter of 20.82 inches, and weighing with hangers, etc., 120 pounds.

A test was first made with no tension load beyond that of the pulley, and immediately afterwards a load of 2400 pounds was applied, corresponding to an increased load of 21,700 pounds per square inch. The readings obtained are shown in Table VI.

Note.—Considerable difficulty was experienced in making accurate readings when the tensional load was applied, as the time of vibration of the heavy weights with respect to the specimen was so large that any accidental motion due to the putting on or taking off the weights was very difficult to damp out.

A deep four-armed vane was attached to the hanger and dipped into a water-trough on the floor; this effected a great improvement, but did not wholly counteract the vibration. The readings obtained in Table VI. show a slight diminution when a tensional load is applied, but owing to the difficulties of observation mentioned above, the author feels he cannot lay much stress upon them. The observations were repeated with very nearly the same result. As will be seen in the next section, bending affects the angular distortion in the same manner.

TABLE VI.

Load in each Pan in lbs.	No Tension except Weight of Pulley.		Tension, 2400 lbs. + Weight of Pulley.	
	Reading.	Difference.	Reading.	Difference.
0.4	0		0	
0.5	196	196	194	194
0.6	391	195	389	195
0.7	586	195	582	193
0.8	783	197	777	195
0.9	979	196	973	196
1.0	1174	195	1167	194

2. *Effect of Tensional Stress on the Yield-point.*

The only experiments upon the yield-point appear to be those of M'FARLANE.* These showed that a tension lowered the yield-point. Reasoning from this result, Lord KELVIN concludes that a compression stress would raise it, but no experiments appear to have been made to verify this conclusion.

In order to examine the effect of tension at or about the yield-point, a bar of wrought iron was taken and cut into two parts; one specimen was turned truly parallel to a convenient diameter (0.424 inches), and the second was made exactly the same size. Both specimens were tested and found to be perfectly cylindrical, as far as could be ascertained by a micrometer gauge.

The first specimen was then tested in the ordinary way, with the result shown in column I., Table VII. The first noticeable deviation occurred when each pan was loaded with a weight of 16 pounds—corresponding to a torque of 333 inch-pounds, the maximum torque being 385 inch pounds.

The second bar was then stressed; but before the tension load was applied a preliminary reading was taken—to see whether the readings agreed with those from the first specimen, and, as will be seen (col. II.), the agreement is very close.

The specimen was now loaded with an additional 2400 pounds—corresponding to an increase of stress of 17,900 pounds per square inch—and a torque applied by increments. As shown by column III., a slight deviation was noticed at 333 inch-pounds, and failure was accomplished by a torque of 360 inch-pounds.

This result shows in a marked way the lowering of the yield-point by tension, and confirms M'FARLANE's experiments.

* *Loc. cit. ante.*

TABLE VII.

Diameter, 0.424". Length, 4.00". 12.8 divisions of scale = 1 min. of arc.

Load in each Pan. Lbs.	Column I.		Column II.		Column III.	
	No Tension.		No Tension.		Load in each Pan. Lbs.	Tension 2400 lbs.
	Reading.	Δ	Reading.	Δ		Reading. Δ
2	0		0		2	0
3	157		155	155	3	156
4	312		312	157	4	311
5	469		468	156	5	467
6	625		625	157	6	624
7	781		781	156	7	782
8	935		937	156	8	938
9	1090		1094	157	9	1094
10	1247		1250	157	10	1251
11	1403		1407	157	11	1406
12	1559		1564	157	12	1564
13	1716				13	1722
14	1872				14	1878
15	2029				15	2034
16	2188				15.5	2115
17	2356				16.0	2221
17.5	2476				16.5	2341
18	3426				17.0	2813
18.2	3826				17.3	Went off scale
18.5	Went off scale					

The results of Table VII. are plotted and are shown on fig. 25.

3. Tension beyond the Elastic Limit.

A machinery steel bar turned to a mean diameter of 0.537, with a length of 8.00 under test, was chosen. The bar was first placed in the torsion machine, and gave the results under test shown by col. I., Table VIII. The mean twist for 75 inch pounds

was 278 divisions. The specimen was then set in the tension grips of the Buckton Testing Machine, and a gradually increased load applied, until a permanent set of 0·18 inches was produced. Immediately after, a fresh test in torsion was made, the results obtained being shown by col. II. The results are also plotted in fig. 26, from which it will be seen that the effect of the tensional overstrain has entirely altered the properties of the material under torsion. The strain is no longer proportional to the stress; the deviation being even more marked than in the case of specimens upset by a previous overstrain by torsion.

TABLE VIII.—*Steel Specimen.*

Diameter, 0·537". Length, 8·00". 12·75 divisions of scale = 1 min. of arc.

Column I. No Tension.			Column II. Specimen permanently lengthened 0·18 inch.		
Torque in inch lbs.	Reading.	Δ	Torque in inch lbs.	Reading.	Δ
0	0	278	0	0	299
75	278	279	75	299	301
150	557	278	150	600	321
225	835	277	225	921	339
300	1112	277	300	1260	364
375	1390		375	1624	406
			450	2030	484
			525	2514	612
			600	3126	292
			525	2834	298
			450	2536	300
			375	2236	300
			300	1936	305
			225	1631	310
			150	1321	311
			75	1010	310
			0	700	

X.—EFFECT OF TORSION ON TENSION.

The effect of torsion upon the properties of a bar subjected to tensional stress was only examined below the elastic limit.

The measuring instrument used was of the Ewing type,* each unit of extension representing $\frac{1}{50000}$ inch upon an 8-inch length; diameter of bar, 0.498 inch.

A series of tests were made, beginning with no tension and increasing by equal increments until the yield-point of the material was reached. The results are recorded in Table IX., and it will be seen that no difference was observable, whether the bar was twisted or not, provided the elasticity of the bar remained unimpaired.

TABLE IX.

Loads. Lbs.	No Torsion.		Torque of 141 inch lbs.		Torque of 282 inch lbs.		Torque of 423 inch lbs.	
	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ
200	0		0		0		0	
		15		15		16		15
400	15		15		16		15	
		14		15		15		16
600	29		30		31		31	
		14		14		15		14
800	43		44		46		45	
		16		15		15		15
1000	59		59		61		60	
		14		15		14		15
1200	73		74		75		75	
		14		15		14		15
1400	87		89		89		90	
		15		14		14		14
1600	102		103		103		104	
		14		15		14		14
1800	116		118		117		118	
		15		15		14		16
2000	131		133		131		134	
		15		14		15		13
2200	146		147		146		147	
		15		14		14		15
2400	161		161		160		162	

XI.—EFFECT OF BENDING ON TORSION.

One of the most interesting cases of stress which occurs in practice is that of torsion combined with bending, a subject which has received little or no attention from the experimental side. The apparatus described in Section II. enables uniform twist and uniform bending to be applied to a bar in any proportions, and the torsional strain to be accurately measured without reference to any external body; so that the bar can

* On Measurements of Small Strains in the Testing of Materials and Structures. By J. A. EWING, F.R.S. *Proc. Royal Society*, May 1895.

assume its own position of equilibrium without affecting the readings of the measuring instrument.

Attention has been directed to the influence of bending on the torsional properties of a bar.

1. *Bending within the Elastic Limit, and Torsion within the Elastic Limit.*

In a previous section it has been shown that the effect of a tension produced little or no effect upon the torsional properties of a bar while in the elastic state. It might be expected, therefore, that a bending action which results in a varying tension and compression upon the longitudinal fibres would have but little effect upon the strain. As an example we may take the case quoted in Section II. of a rivet steel bar, in which an increase in the bending moment caused a slight diminution of the strain per unit torque.

Similar decrements were found in every case of the same type; as, for example, in the case of a semi-mild steel bar of diameter 0.869 inch, and of length 8.00 inches under test.

The unit reading corresponds to $\frac{1}{54}$ minute of arc.

TABLE X.

Torque in lbs.	No Bending Moment.		640 inch lbs.	
	Reading.	Difference.	Reading.	Difference.
0	0		0	
75	166	166	163	163
150	332	166	327	164
225	498	166	493	166
300	663	165	659	166
375	830	167	824	165

2. *Bending beyond the Elastic Limit.*

A more interesting case was that of a steel bar in which the bending moment was gradually increased until a permanent set was given to the bar in the plane of bending. The readings obtained are shown in Table XI., and a summary of the results in Table XII. It will be noticed that when the bar is bent the true value of the torque is given by its apparent value multiplied by $\cos \theta$, where θ = angle of bending of the weigh-lever about its axis.

The results show that the slight diminution in the readings within the elastic limit is followed by a much greater rise when the yield-point is reached.

TABLE XI.

Diameter of bar, 0.511". Length, 8.00". 54.7 divisions = 1 min. of arc.

Torque in lbs.	No Bending Moment.		Bending Moment 224 inch lbs.		Bending Moment 377 inch lbs.		Bending Moment 529 inch lbs.		Bending Moment 752 inch lbs. *		Bending Moment 922 inch lbs.		Bending Moment 1092 inch lbs.		Bending Moment 1432 inch lbs.		Bending Moment 1772 inch lbs.	
	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ
0	0	137	0	137	0	135	0	135	0	135	0	135	0	136	0	138	0	139
7.5	137	137	137	136	135	136	135	135	135	136	135	136	136	137	138	139	139	140
15.0	274	136	273	136	271	136	271	136	271	136	271	137	273	137	277	138	279	140
22.5	410	136	409	135	408	136	407	135	407	135	408	137	410	136	415	137	419	141
30.0	546	136	544	136	544	135	542	134	542	134	543	137	546	137	552	138	560	140
37.5	682	136	680	137	679	135	677	135	676	135	680	137	683	137	690	138	702	140
45.0	818	136	817	135	814	136	812	135	811	137	817	137	820	138	828	139	842	141
52.5	954	137	952	136	950	136	947	136	948	136	954	136	958	137	967	138	983	140
60.0	1091		1088		1086		1083		1084		1090		1095		1105		1123	

* Slight permanent set in the bar due to bending.

TABLE XII.

Diameter of bar, 0.511". Length, 8.00". 54.7 divisions = 1 min. of arc.

Bending Moment.	Readings for Torque of 60 inch lbs.	L. of Bending θ at Weigh-lever.	Cos θ .	Corrected Reading = Reading $\times$ cos θ .
0	1091	0°	1.000	1091
224	1088	30'	.999	1088
337	1086	48'	.999	1086
529	1083	1° 54'	.999	1083
752*	1084	2° 24'	.999	1084
922	1090	3° 10'	.998	1088
1092	1095	3° 20'	.998	1092
1432	1105	4° 30'	.997	1102
1772	1123	7° 0'	.993	1115

* Permanent set due to bending.

3. *Effect of Bending upon the Yield-point.*

The experiment was performed in a similar manner to that in Section IX. Two bars cut from the same rod were turned up to exactly the same size. One of them was tested in the ordinary manner, and the other was subjected to bending moment below the elastic limit, and then twisted beyond the yield-point. Each bar was 0.489 inch in diameter, the length under test being 4 inches, and 12.85 divisions on the scale corresponded to 1 minute of arc. The readings are recorded in Table XIII., in which column I. refers to the first specimen, and the remaining columns to the second specimen. The readings in column II. were made to check the correctness of the setting of the measuring instrument. The readings show a remarkable lowering of the yield-point for a bending moment of 668 inch pounds; the reason for which was not at first apparent, until it was noticed that the specimen took a permanent set, the ends being bent to a considerable degree.

At first sight this might appear to be a mere time effect; but in the author's opinion the probable cause was the increase of stress, due to the torque applied later. Appar-

ently the maximum stress due to bending was of itself insufficient to cause yield, but the application of a further torque caused the principal stresses to assume the values

$$p_1 = \frac{p_n}{2} + \sqrt{\frac{p_n^2}{4} + q^2}$$

$$p_2 = \frac{p_n}{2} - \sqrt{\frac{p_n^2}{4} + q^2}$$

where p_n = normal stress due to bending.

q = shear stress due to applied torque.

If we adopt RANKINE's theory of maximum stress, then p_1 in this case passed the working limit of the material, and a set resulted.

On the maximum strain theory of ST VENANT,

if e_1 = principal strain,

m = POISSON'S ratio,

then

$$\begin{aligned} Ee_1 &= p_1 - \frac{p_2}{m} \\ &= \frac{m-1}{2m} p_n + \frac{m+1}{2m} \sqrt{p_n^2 + 4q^2}; \end{aligned}$$

and since m has a value between 3 and 4 for steel, it is clear that the addition of a shear stress q would cause an increase in the value of e , which, if below the limit before, might increase sufficiently to cause failure.

The relation of stress to strain after the permanent set is clearly shown by a further test indicated in the table, column IV. There is now considerable hysteresis in the relation of stress to strain.

TABLE XIII.—*Combined Bending and Twist.*

Bar I.			Bar II.				
Col. I.			Col. II.	Col. III.		Col. IV.	
Torque in inch lbs.	No Bending Moment.		No Bending Moment.	Torque in inch lbs.	Bending Moment 668 inch lbs.	Torque in inch lbs.	3 hours after last Test.
	Reading.	Δ	Reading. Δ		Reading. Δ		Reading. Δ
0	0		0	0	0	0	0
75	195	195	196	75	196	75	201
150	390	195	196	150	196	150	202
225	587	197	193	225	198	225	207
300	784	197	197	300	215	300	208
375	980	196		375	765	375	266
		125			1570		194
420	1105	44		After 2 minutes, went off scale entirely		300	890
435	1149	42				225	689
450	1191	42				150	486
465	1233	42				75	284
480	1276	43					208
495	1322	46				0	76
		46					
510	1368	50					
525	1418	50					
540	1468	48					
555	1516	62					
570	1578	98					
585	1676	700					
605	2376	1109					
615	3476						

The author has not been able to find any other experiments bearing upon the position of the yield-point as affected by bending. The yield-point is, however, known to be lowered by tension, as mentioned previously.

A case which bears considerable resemblance to the case of permanent set last quoted is one by M'FARLANE,* who has shown that if a wire is twisted to nearly its limit of torsional elasticity, an increase in pull will cause the torque to give the wire a permanent set. This latter case can be easily explained in the same manner as the one described above.

XII.—EFFECT OF ANNEALING.

It has long been known that iron and mild steel stressed beyond the limit of elasticity regain their elastic properties when heated to a red heat and allowed to cool slowly. The process may be repeated many times without apparently changing the elastic properties of the material. The yield-point, however, is found to alter in position as the annealings proceed. In a particular case † a mild steel bar, which in an ordinary test would give an extension of 25 per cent. upon a ten-inch length, and a yield-point of about 18 tons per square inch, was stretched approximately $\frac{1}{4}$ inch, and annealed in the ordinary manner after each operation. Throughout the experiment the bar appeared to recover its elastic properties after each annealing, and finally broke with a total extension of approximately 100 per cent. The yield-point remained fairly constant, except at the end, when it experienced a rise.

Copper treated in the same manner has been drawn out by the author to considerably more than double its length in this way without causing fracture. Remarkable advances in our knowledge of annealing have been recently obtained by MUIR,‡ acting upon a suggestion of Professor EWING.

MUIR has shown that comparatively low temperatures, such as boiling water, will restore a strained bar to its elastic condition. The yield-point, however, alters during the process, and is always higher than in the original condition.

After a few applications of stress, followed by heating in boiling water, or even water at 50° C., the bar fractures, with a total extension not very different from a bar stressed to breaking without special treatment. The annealing at low temperatures, therefore, appears to be less complete than that at a high temperature.

In order to discover what effect a temperature of 100° C. would exert on a bar overstrained by a torque, the steel bar which had been used for experiments on the recovery of elasticity with time (see Section VI., Table IV.) was selected, and after being boiled for fifteen minutes was overstrained, giving results (col. X., Table XIV.) practically identical with those of col. I., Table IV., for the first part of the curve. As in practice it is troublesome to get exactly the same calibration value for each setting of the instrument, this latter (stripped of the reading microscope and wire holder) remained on the bar during the heating, and the labour of comparing readings whose unit values differed by

* Art. "Elasticity," *Enc. Brit.*, Part 21.

† COKER, "Note on the Endurance of Steel Bars subjected to Repetitions of Tensional Stress," *Proc. Inst. C.E.*, 1899.

‡ "The Recovery of Iron from Overstrain." By JAMES MUIR. *Phil. Trans.*, 1899.

a small amount was thereby avoided. Each stress operation causing overstrain was succeeded by a heating in water at 100° C. for fifteen minutes, and in all the bar was stressed eight times. The readings obtained are given in columns X.-XVII. in Table XIV., and are plotted in the ordinary manner, fig. 29, the curves being spaced 100 units apart for convenience. As might be expected, the curves show a general agreement with those obtained by MUIR, having regard to the fact that the stress is non-uniform.

TABLE XIV.

Column X.			Column XI.			Column XII.			Column XIII.		
Torque in inch lbs.	Reading.	Δ	Torque in inch lbs.	Reading.	Δ	Torque in inch lbs.	Reading.	Δ	Torque in inch lbs.	Reading.	Δ
0	0		0	0		0	0		0	0	
		385			390			389			386
75	385		75	390		75	389		75	386	
		385			390			387			389
150	770		150	780		150	776		150	775	
		388			389			389			389
225	1158		225	1169		225	1165		225	1164	
		387			389			389			388
300	1545		300	1558		300	1554		300	1552	
		389			388			387			389
350	1934		375	1946		375	1941		375	1941	
		392			391			389			389
450	2326		450	2337		450	2330		450	2330	
		389			390			388			392
525	2715		525	2727		525	2718		525	2722	
		395			388			389			388
600	3110		600	3115		600	3107		600	3110	
		400			392			389			389
675	3510		675	3507		675	3496		675	3499	
		404			390			386			394
750	3914		750	3897		750	3882		750	3893	
		426			394			391			402
825	4340		825	4291		825	4273		825	4295	
		463			410			398			402
900	4803		900	4701		900	4671		900	4697	
					617			419			414
930	Went off scale		975	5318		975	5090		975	5111	
					260			303			475
			990	5578		1025	5393		1050	5586	
			1015	Went off scale		1050	Went off scale		1080	Went off scale	

TABLE XIV.—*continued.*

Column XIV.		Column XV.		Column XVI.		Column XVII.	
Torque in inch lbs.	Reading. Δ	Torque in inch lbs.	Reading. Δ	Torque in inch lbs.	Reading. Δ	Torque in inch lbs.	Reading. Δ
0	0	0	0	0	0	0	0
	386		387		387		388
75	386	75	387	75	387	75	388
	387		387		387		389
150	773	150	774	150	773	150	777
	384		386		387		391
225	1157	225	1160	225	1159	225	1168
	386		387		385		389
300	1543	300	1547	300	1544	300	1557
	388		386		386		389
375	1931	375	1933	375	1930	375	1946
	387		387		387		389
450	2318	450	2320	450	2317	450	2335
	388		388		387		390
525	2706	525	2708	525	2704	525	2725
	386		391		389		393
600	3092	600	3099	600	3093	600	3118
	389		397		389		395
675	3481	675	3496	675	3482	675	3513
	388		395		392		396
750	3869	750	3891	750	3874	750	3909
	392		402		394		401
825	4261	825	4293	825	4268	825	4310
	392		412		398		409
900	4653	900	4705	900	4666	900	4719
	403		425		410		410
975	5056	975	5130	975	5076	975	5129
	430		445		437		411
1050	5486	1050	5575	1050	5513	1050	5540
	197		522				458
1080	5683	1125	6097	1105	Went off scale	1125	5998
1110	Went off scale	1155	Went off scale			1200	Went off scale

In conclusion the author desires to express his thanks to Prof. BOVEY, Dean of the Faculty of Applied Science, McGill University, who placed the resources of the Testing Laboratory of the Civil Engineering Department at his disposal, and also to Mr WIRTHYCOMBE, Mechanical Superintendent, who gave much help in the preparation of the apparatus.

MR E. G. COKER on an "Apparatus for measuring Strain and applying Stress."—PLATE I.

Instrument for measuring angular distortion upon the specimen.

Apparatus for applying the combined stresses of Bending and Torsion.

Mr E. G. Coker on an "Apparatus for measuring Strain and applying Stress."—PLATE II.

Apparatus for applying the combined stresses of Tension and Torsion.

Instrument for measuring angular distortion, with the clamp in position for setting the pieces correctly on the specimen.

DR. COKER: APPARATUS FOR MEASURING STRAIN AND APPLYING STRESS.—PLATE III.

Fig 2
End Elevation

Fig 5

Sectional Side Elevation

Fig 4

Fig 3

DR. COKER: APPARATUS FOR MEASURING STRAIN AND APPLYING STRESS.—PLATE IV.

DR. COKER: APPARATUS FOR MEASURING STRAIN AND APPLYING STRESS.—PLATE V

DR. COKER: APPARATUS FOR MEASURING STRAIN AND APPLYING STRESS.—PLATE VI.

DR. COKER: APPARATUS FOR MEASURING STRAIN AND APPLYING STRESS.—PLATE VII.

DR. COKER: APPARATUS FOR MEASURING STRAIN AND APPLYING STRESS.—PLATE VIII.