EPICYCLIC GEARING IN AUTOMOBILE PRACTICE.

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EPICYCLIC GEARING IN AUTOMOBILE PRACTICE.

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PART I.

NOTES ON DESIGN.

THE automobile engineer occasionally desires to avail himself of the advantages offered by epicyclic gearing under certain circumstances. When such occasion arises, the designer usually has in mind a certain ratio which he desires to adopt, and he is faced with the task of arranging his gears, and deciding upon his driving and driven members to give the required result. This in itself is not a simple problem, and is further complicated when two ratios are required with one set of gears. The many excellent methods given in text-books and the like for analysing existing gears are not as a rule convenient for use in a reverse direction, i.e., to obtain the proportions of the gears to suit a given ratio, and the author will attempt now to present the problem from this view-point.

There are, of course, a large number of possible combinations of epicyclic trains, but the author will restrict himself to the three most commonly used in motor car practice, namely, the simple epicyclic, the compound epicyclic, and the reverted train in which the internally toothed ring is replaced by a gear wheel in alinement with the first or sun wheel.

Simple Epicyclic Train.

A diagram of this gear is shown in Fig. 1, in which C represents the sun pinion, D the planet pinions, A the cage or member carrying the planet pinions D, and B the internally toothed ring. The gears are indicated by their pitch diameters, and it is this diameter which is referred to in all cases when a gear is mentioned by letter.

There are two methods of analysing an epicyclic gear train to which the author would draw attention. The first is given in "The Theory of Machines," by R. F. McKay, and the second

can be found in "Machinery's Handbook" and other works of reference. Both methods have similar characteristics, and can be used for any type of epicyclic gearing, no matter how complex. They also possess the advantage that they can be easily memorised. The six possible combinations of stationary, driving, and driven members are given in Table I., and are numbered 1 to 6. It will be obvious that case 1 is the inverse of case 2 since the stationary member is the same, and this is also true of cases 3 and 4, and also 5 and 6. The velocity-ratio of the first case will now be investigated by both methods.

Analysis of Simple Epicyclic Gearing.

1st Method. Case 1. Stationary member. Ring B.
Driving member. Sun pinion C.
Driven member. Cage A.

FIG. 1.

Since the relative motion between the component parts is the same, no matter which element is assumed fixed, it can be written down at sight by choosing judiciously the fixed element. By holding the cage A the gear resolves itself at once into a simple train, and by revolving any one of the other members the behaviour of the remaining elements can at once be noted. This gives us row (a), Table II.

These values in row (a) can have a constant added to each of them, or subtracted from them, or they can each be multiplied by a constant without altering their relative values, and they can therefore be manipulated until they show the conditions of the problem, namely, B stationary, C driving, A driven.

By adding C/B to row (a), row (b) is obtained, and the relation between A and C can at once be seen, and also the fact that they

TABLE I.

Case.	(a)	(b)	(c)	(d)	(e)	(f)
	Stationary Member.	Driving Member.	Driven Member.	Ratio.	Substituting K	Value of K
(1)	Ring B	Sun Pinion C	Cage A	$A_r = \frac{C}{B+C}$	$A_r = \frac{1}{2(K+1)}$	$K = \frac{1}{2A_r} - 1$
(2)	Ring B	Cage A	Sun Pinion C	$C_r = 1 + \frac{B}{C}$	$C_r = 2(K+1)$	$K = \frac{C_r}{2} - 1$
(3)	Cage A	Sun Pinion C	Ring B	$B_r = -\frac{C}{B}$	$B_r = -\left(\frac{1}{2K+1}\right)$	$K = -\frac{1}{2}\left(\frac{1}{B_r} + 1\right)$
(4)	Cage A	Ring B	Sun Pinion C	$C_r = -\frac{B}{C}$	$C_r = -(2K+1)$	$K = -\frac{1}{2}(C_r + 1)$
(5)	Sun Pinion C	Cage A	Ring B	$B_r = 1 + \frac{C}{B}$	$B_r = 1 + \frac{1}{(2K+1)}$	$K = \frac{1}{2}\left\{\frac{1}{(B_r-1)} - 1\right\}$
(6)	Sun Pinion C	Ring B	Cage A	$A_r = \frac{B}{B+C}$	$A_r = 1 - \frac{1}{2(K+1)}$	$K = \frac{1}{2}\left\{\frac{1}{(1-A_r)} - 2\right\}$

In using column (f) give A_r , B_r , C_r the correct sign as indicated in column (d).

both revolve in the same direction. If it is required to know the revolutions of A to one of C it is known that:—

$$\begin{aligned} \text{When C revolves } 1 + \frac{C}{B}, \text{ A revolves } \frac{C}{B} \\ \therefore \text{ „ C „ 1 „ „ } \frac{C}{B} \times \frac{B}{B+C} \\ = \frac{C}{B+C} \end{aligned}$$

The rules governing this method are therefore quite simple.

1st. Assume the cage fixed, rotate one of the other members, and write down the resulting velocity-ratios.

TABLE II.

	A	B	C	D
(a) Cage A Fixed C given +1 turn	0	$-\frac{C}{D} \cdot \frac{D}{B}$ $= -\frac{C}{B}$	+ 1	$-\frac{C}{D}$
(b) Add $+\frac{C}{B}$ to (a).....	$+\frac{C}{B}$	0	$1 + \frac{C}{B}$	$\frac{C}{B} - \frac{C}{D}$
(c) Reducing C to 1	$\frac{C}{B+C}$	0	1	$\frac{C(D-B)}{D(B+C)}$

2nd. Manipulate the results obtained so that the required member is fixed.

2nd method. Case 1, as before.

In this method each element is given a turn in the same direction, usually positive. The cage is then assumed to be held, and the element which is fixed is given a negative turn, the results of this negative turn on the other elements being noted under the appropriate headings. These two results are then added algebraically, the answer giving the required velocity-ratio. Taking case 1 again as an example, each element is given a positive turn, resulting in row (a), Table III.

The cage A is then assumed to be held, and B the stationary member rotated -1, thus giving row (b), Table III. Adding the

two gives row (c), from which it is seen that if C rotates $1 + (B/C)$ times, then A rotates once in the same direction.

If C rotates once, then A revolves the inverse of C, this being $C/(B + C)$ as obtained by the first method.

It is sometimes stated that in using this method it is the driving member which is to be assumed stationary when obtaining row (b). It is important, however, to notice that in all cases the cage A must be taken as held, otherwise the whole simplicity of the Table is lost. In those cases where A is the driving member, the required ratio is given directly to one revolution of the cage, and when A is driven, as in the example above, it is quite a simple matter to invert if necessary. It will be seen, therefore, that in those cases where A is the stationary member, this method cannot be used, but it is really not necessary, since the required

TABLE III.

	A	B	C	D
(a) Each element + 1....	+ 1	+ 1	+ 1	+ 1
(b) A held B - 1.....	0	- 1	$+ \frac{B}{D} \cdot \frac{D}{C}$	$- \frac{B}{D}$
(c) (a) + (b).....	+ 1	0	$1 + \frac{B}{C}$	$1 - \frac{B}{D}$

relative velocities can be written down at sight from the diagram. It is of the greatest importance, in both methods, to affix the correct sign of rotation to the results obtained.

As stated before, both methods are very similar, and perhaps the only advantage possessed by the first method is that all six cases can be written down in one Table, while by the second method it is better to have a separate Table for each case.

The other cases are dealt with in a similar manner, and will result by either method in column (d), Table I., where A_r , B_r , C_r are the revolutions of A, B and C to one revolution of the driving member, a minus sign indicating that the driven member revolves in the opposite direction to the driving member. For purposes of reference the full workings of each case are given in Appendix A.

Turning now to Fig. 1, it will be seen that if the proportions of two of the elements are decided upon, then the size of the third is necessarily fixed. Thus $C + 2D = B$.

$$\text{Let } \frac{D}{C} = K$$

$$\text{Then } 2K + 1 = \frac{B}{C}$$

$$\text{And } B = C(2K + 1)$$

Substituting in the first case, Table I., column (d)

$$A_r = \frac{C}{C(2K + 1) + C}$$

$$\therefore A_r = \frac{1}{2(K + 1)}$$

$$\text{And } K = \frac{1}{2A_r} - 1$$

The other cases are dealt with in a like manner by substituting the above value of B in each of the remaining equations, resulting in columns (e) and (f), Table I.

In these two columns equations are now given from which can be found the velocity-ratio corresponding to a given proportion between the sun pinion and the planet pinion, or alternatively the diameters of these pinions necessary to give a certain velocity-ratio. Having found the relative proportions of the sun and planet pinions, it is an easy matter to find the diameter of the ring B from the preceding formula, $B = C + 2D$.

In Fig. 2 are plotted the equations given in row (f), Table I. From this graph can be read directly the gear-ratio resulting from any particular arrangement of the epicyclic gear for a given size of planet pinion, and it also shows clearly the limiting ratio for any particular combination. For example, it can be seen at once that there is only one arrangement of the elements which will give a ratio of exactly 2 to 1; this is case 4, in which the cage is stationary, a negative drive resulting in which the sun pinion (driven) revolves in the opposite direction to the ring (driving). A further use for the chart is in those cases where it is desired that one member shall always be driving, the other members being alternately held and driven, thus obtaining two velocity-ratios. A line drawn parallel to the base line through the selected value of K will cut the "ratio lines" at certain points, and lines dropped vertically from these points on to the base line will give the corresponding ratios. The chart can, of course,

also be used in the reverse direction; working from one ratio a

Fig. 2.

line is projected upwards from the base until it cuts the correct

"ratio line." and another drawn through this point parallel to the base will give the other gear-ratio and the necessary value of K to obtain these results. As an example, let it be supposed that the cage is to be the driving member in both cases, and that K is to be 0.75. A line drawn through $K = \frac{3}{4}$ cuts "ratio lines" for cases 2 and 5 at $R = 1.4$ and $R = 3.5$, and these are the two ratios possible under the conditions given when the various elements have the proportions:— $C = 1$, $D = \frac{3}{4}$, and $B = C + 2D = 2\frac{1}{2}$. Since the pitch is constant throughout, these figures are also proportional to the number of teeth in each gear, the actual pitch and number of teeth being selected to suit.

The range of Fig. 2 greatly exceeds proportions which can be used in practice; actually, the planet pinion will never be

FIG. 3.

more than twice the diameter of the sun pinion except in special cases, and therefore all the ratios possible in practice are represented on the graph between lines drawn through $K = 2$ and $K = 0$.

Compound Epicyclic Train. (a) With Internally Toothed Ring.

The analysis of compound epicyclic gearing is carried out in exactly the same way as already described for the simple epicyclic gear. An actual example will perhaps make this perfectly clear, a reference to Fig. 3 indicating the notation employed. In this diagram A is the cage carrying the planet pinions D_1 and D_2 , which are locked together so that they revolve as one but are free to rotate upon a spindle incorporated in A, the pinion D_1 engages with the sun pinion C, and D_2 with the ring B.

TABLE IV.

Case.	(a) Stationary Member.	(b) Driving Member.	(c) Driven Member.	(d) Ratio.	(e) Conversion Formula.	(f) Substituting G and K in (d).	(g) Value of K.
(1)	Ring B	Sun Pinion C	Cage A	$A_r = \frac{CD_2}{CD_2 + BD_1}$	$(1 - (6))$ $\left(\frac{1}{1 - (4)}\right)$	$A_r = \frac{G}{(G + 1)(K + 1)}$	$K = \frac{G}{A_r (G + 1)} - 1$
(2)	Ring B	Cage A	Sun Pinion C	$C_r = 1 + \frac{B}{C} \cdot \frac{D_1}{D_2}$	$\left(\frac{1}{1 - (6)}\right)$ $(1 - (4))$	$C_r = \frac{K(G + 1) + (G + 1)}{G}$	$K = \frac{G(C_r - 1) - 1}{(G + 1)}$
(3)	Cage A	Sun Pinion C	Ring B	$B_r = -\frac{C}{B} \cdot \frac{D_2}{D_1}$	$(1 - (5))$ $\left(\frac{1}{1 - (2)}\right)$	$B_r = -\left(\frac{G}{1 + K(G + 1)}\right)$	$K = -\left(\frac{G}{B_r (G + 1)} + \frac{1}{(G + 1)}\right)$
(4)	Cage A	Ring B	Sun Pinion C	$C_r = -\frac{B}{C} \cdot \frac{D_1}{D_2}$	$(1 - (2))$ $\left(\frac{1}{1 - (5)}\right)$	$C_r = -\left(\frac{1 + K(G + 1)}{G}\right)$	$K = -\left(\frac{GC_r + 1}{G + 1}\right)$
(5)	Sun Pinion C	Cage A	Ring B	$B_r = 1 + \frac{C}{B} \cdot \frac{D_2}{D_1}$	$\left(\frac{1}{1 - (1)}\right)$ $(1 - (3))$	$B_r = 1 + \frac{G}{1 + K(G + 1)}$	$K = \frac{G}{(B_r - 1)(G + 1)} - \frac{1}{(G + 1)}$
(6)	Sun Pinion C	Ring B	Cage A	$A_r = \frac{BD_1}{BD_1 + CD_2}$	$(1 - (1))$ $\left(\frac{1}{1 - (3)}\right)$	$A_r = \frac{1 + K(G + 1)}{(G + 1)(K + 1)}$	$K = \frac{A_r (G + 1) - 1}{(G + 1)(1 - A_r)}$

In using Col. (g) give A_r , B_r , C_r , the correct sign as indicated in Col. (d).

As before, six different combinations of the train are possible, counting inversions, and these six cases are set out in columns (a), (b) and (c), Table IV.

Analysis of Compound Epicyclic Gearing, with Internally Toothed Ring.

1st method. Case 1. Stationary member. Ring B.
Driving member. Sun pinion C.
Driven member. Cage A.

It will be seen, therefore, that when C revolves once A rotates $\frac{D_2C}{D_2C + D_1B}$ in the same direction.

2nd method. Case 1, as before.

This method gives, as it should do, exactly the same result.

The other cases are dealt with in a similar manner, the results being given in column (d), Table IV., and for purposes of reference the full workings are shown in Appendix B.

In the case of a compound epicyclic gear train of the type under consideration, there are two proportions governing the size of gear, that of D_1 to C (Fig. 3) and D_2 to B, or alternatively, D_2 to D_1 . The first, that of D_1 to C, has been called K, so that $D_1/C = K$ as before. For the second proportion, the author has selected D_2 to D_1 , and has introduced a factor $G = D_2/D_1$. With the aid of these two factors the third member can be fixed, since:—

$$D_1 = CK$$

$$D_2 = D_1G = CKG$$

$$B = (C + D_1 + D_2) = (C + CK + CKG) = C(1 + K + KG)$$

Turning now to Case 1, column (d) (Table IV.), it is seen that

$$A_r = \frac{CD_2}{CD_2 + BD_1}$$

and substituting the foregoing values of D_1 , D_2 and B this becomes

$$A_r = \frac{C^2KG}{C^2KG + C^2K(1 + K + KG)} = \frac{G}{G + 1 + K + KG}$$

$$\text{And } A_r = \frac{G}{(K + 1)(G + 1)}$$

From which

$$K = \frac{G}{A_r(G + 1)} - 1$$

The other cases are dealt with in the same way, the results being given in columns (f) and (g), Table IV.

An examination of column (d), Table IV., reveals the interesting

fact that there is a simple relation existing between the various cases. The most obvious, perhaps, is that between Cases 2 and 4,

TABLE V.

	A	B	C	$D_1 \& D_2$
(a) Fixing A revolve B once.	0	+ 1	$-\frac{B}{D_2} \cdot \frac{D_1}{C}$	$+\frac{B}{D_2}$
(b) Add - 1 to row (a)	- 1	0	$-1 - \frac{D_1}{D_2} \cdot \frac{B}{C}$	$-1 + \frac{B}{D_2}$
(c) Reducing C to 1 ..	$\frac{D_2 C}{D_2 C + D_1 B}$	0	1	

TABLE VI.

	A	B	C	$D_1 \& D_2$
(a) Each element positive, turn.	+ 1	+ 1	+ 1	+ 1
(b) A held, B negative, turn.	0	- 1	$+\frac{B}{D_2} \cdot \frac{D_1}{C}$	$-\frac{B}{D_2}$
(c) Adding rows (a) and (b).	+ 1	0	$1 + \frac{D_1}{D_2} \cdot \frac{B}{C}$	$1 - \frac{B}{D_2}$
(d) Reducing C to 1 ..	$\frac{D_2 C}{D_2 C + D_1 B}$	0	1	

where it will be seen that Case 2 = (1 - Case 4) and conversely, Case 4 = (1 - Case 2). The conversion formulæ for the remaining

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cases are given in column (e), Table IV., where the encircled figures refer to cases. It is clear, then, that having worked out the velocity-ratio for a particular train, it is a simple matter to convert this result to the corresponding ratio for any other combination, and further, that if K and G are plotted graphically with their corresponding ratios, one chart will accommodate all the different combinations. Such a chart is shown in Fig. 4, and gives the various relations between K and G necessary for a given ratio. A minus sign before a velocity-ratio indicates that

FIG. 4.

the driven member revolves in the opposite direction to the driver. The chart is self-explanatory, but an example will be taken in order to make its use perfectly clear. Taking $K = 1$ and $G = \frac{1}{2}$ it is found that the curve upon which these values intersect, if followed round until it reaches the Tables, shows that the ratios under the conditions named at the head are $1/6$, 6, $-1/5$, -5 , $1 \frac{1}{5}$ or $5/6$, and the proportions of the gear to obtain these results are $C = 1$, $D_1 = CK = 1$, $D_2 = D_1G = \frac{1}{2}$, and $B = (C + D_1 + D_2) = 2 \frac{1}{2}$. The chart can, of course, be used to solve existing gear trains, but in this connection it should be noted that K and G must be

calculated on pitch diameters and not on numbers of teeth, unless the pitch is constant throughout.

FIG. 5.

Compound Epicyclic Train.—Reverted Train.

This arrangement is shown diagrammatically in Fig. 5, from which it will be seen that the ring has been replaced by a pinion

FIG. 6.

on the same centre as the first. The velocity-ratios can be found by the methods already described, and are given for each case in column (d), Table VII., while Appendix C. shows in detail

TABLE VII.

	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Case.	Stationary Member.	Driving Member.	Driven Member.	Ratio.	Conversion Formula.	Substituting G and K in (d).	Value of K.
(1)	Wheel B	Sun Pinion C	Cage A	$A_r = \frac{CD_2}{CD_2 - BD_1}$	$(1 - (6))$ $\left(\frac{1}{1 - (4)}\right)$	$A_r = \frac{G}{(G - 1)(K + 1)}$	$K = \frac{1}{A_r \left(1 - \frac{1}{G}\right)} - 1$
(2)	Wheel B	Cage A	Sun Pinion C	$C_r = 1 - \frac{B}{C} \cdot \frac{D_1}{D_2}$	$(1 - (4))$ $\left(\frac{1}{1 - (6)}\right)$	$C_r = \frac{(G - 1)(K + 1)}{G}$	$K = \frac{C_r}{\left(1 - \frac{1}{G}\right)} - 1$
(3)	Cage A	Sun Pinion C	Wheel B	$B_r = \frac{C}{B} \cdot \frac{D_2}{D_1}$	$(1 - (5))$ $\left(\frac{1}{1 - (2)}\right)$	$B_r = \frac{G}{1 + K - KG}$	$K = \frac{G}{B_r(1 - G)} - \frac{1}{(1 - G)}$
(4)	Cage A	Wheel B	Sun Pinion C	$C_r = \frac{B}{C} \cdot \frac{D_1}{D_2}$	$(1 - (2))$ $\left(\frac{1}{1 - (5)}\right)$	$C_r = \frac{1 + K - KG}{G}$	$K = \frac{GC_r - 1}{(1 - G)}$
(5)	Sun Pinion C	Cage A	Wheel B	$B_r = 1 - \frac{CD_2}{BD_1}$	$(1 - (3))$ $\left(\frac{1}{1 - (1)}\right)$	$B_r = \frac{K(1 - G) - (G - 1)}{K(1 - G) + 1}$	$K = \frac{G}{(1 - B_r)(1 - G)} - \frac{1}{(1 - G)}$
(6)	Sun Pinion C	Wheel B	Cage A	$A_r = \frac{BD_1}{BD_1 - CD_2}$	$(1 - (1))$ $\left(\frac{1}{1 - (3)}\right)$	$A_r = \frac{K(1 - G) + 1}{K(1 - G) - (G - 1)}$	$K = \frac{A_r G}{(A_r - 1)(1 - G)} - \frac{1}{(1 - G)}$

In using Column (g) give A_r , B_r , C_r the correct sign. Cases (1), (2), (5), (6) can have both positive and negative drives for the same value of K, by varying G. Cases (3) and (4) are always positive.

how these results were obtained. There is, again, a simple relationship (shown in column (e), Table VII.) between each case, so that a graph showing the variations of G and K for a given ratio will be common to all the various combinations, and this chart is shown in Fig. 6. As before, $K = D_1/C$ and $G = D_2/D_1$, and it will be obvious from Fig. 5 that $B = (C + D_1 - D_2)$.

The proportions of the gear train can be written in terms of G and K , thus:—

$$D_1 = CK_1$$

$$D_2 = D_1G = CKG$$

$$B = (C + D_1 - D_2) = C(1 + K - KG).$$

These values can now be substituted in column (d), Table VII.,

TABLE VIII.

K	Limiting value of G.
$\frac{1}{4}$	5
$\frac{1}{2}$	3
1	2
2	$1\frac{1}{2}$
3	$1\frac{1}{3}$
4	$1\frac{1}{4}$
5	$1\frac{1}{5}$
6	$1\frac{1}{6}$

and will result in columns (f) and (g), from which the graph, Fig. 6, has been plotted.

There is, of course, already a limit to the proportions of the gear train when $K=0$, but it should be noted also that in this case G reaches its limit when B becomes 0. The limiting value of G in respect to K can therefore be found as follows: $B + D_2 = C + D_1$.

Limit occurs when $B=0$

$$\begin{aligned} \text{and } D_2 &= C + D_1 \\ &= CKG = C + CK \\ &= KG = 1 + K \end{aligned}$$

$$\text{and } G = \frac{1 + K}{K} = 1 + \frac{1}{K}$$

Therefore G cannot exceed $1 + (1/K)$, and these values of G from $K = \frac{1}{4}$ to $K = 6$ are given in Table VIII. and are shown graphically in Fig. 6 by the curve referred to as "Limiting Ratio."

The chart, Fig. 6, can be used for selecting or determining velocity-ratios in the way already described for Fig. 4, and calls for no further comment beyond pointing out an interesting feature of cases 1, 2, 5 and 6. It will be noticed that after $G = 1$ is passed the direction of rotation of the driven member is reversed. In cases 1 and 2 for values above $G = 1$ the driven member rotates

FIG. 7.

in the same direction as the driving member, below $G = 1$ its direction is opposite to that of the driving member; in cases 5 and 6 these conditions are reversed, i.e., above $G = 1$ the rotation of the driven element is negative, below $G = 1$ it is positive. Remembering that $G = D_2/D_1$ this action is best described by reference to Figs. 7 and 8, the conditions being as in case 1. Fig. 7 represents an arrangement in which G is greater than 1, and upon examining the end view it will be seen that if C is revolved in the direction of the arrow then the resultant pressure on the radius of D_1 is in the direction of the arrow Y . Since the point X can be regarded

as fixed, then the resultant motion of the axis of D_1 and D_2 must be in the direction of the arrow Z , which is the same as the rotation of C .

In Fig. 8 is shown the position when G is less than 1. Referring to the end view, it will be seen that if C is rotated in the direction of the arrow, then the resultant motion on the radius of D_1 is in the direction of the arrow Y as before, but in this case the fixed point X is between Y and the axis of D_1 , and D_2 , which therefore revolves in a direction opposite to C as shown by the arrow Z . Arising out of this change of direction, it will be noted that when

FIG. 8.

G approximates closely to 1 very high velocity-ratios can be obtained. As an example, let $G = 1.01$ and $K = 1$.

Taking case 1, i.e., B stationary, C driving, from column (f), Table VII., it is seen that

$$\begin{aligned}
 A_r &= \frac{G}{(G - 1)(K + 1)} \\
 &= \frac{\frac{101}{100}}{\frac{1}{100} \times 2} = \frac{101}{100} \times 50 = 50.5 : 1
 \end{aligned}$$

As a further example, let $K = \frac{1}{2}$ and $G = 1.001$ then for case 1,

$$A_r = \frac{G}{(G-1)(K+1)}$$

$$= \frac{\frac{1001}{1000}}{\frac{1}{1000} \times \frac{3}{2}} = \frac{1001}{1000} \times \frac{2000}{3} = \underline{\underline{667\frac{1}{3}:1}}$$

As before, G is the ratio between the pitch diameters of D_1 and D_2 , Fig. 5.

These examples are, of course, extreme cases, but they are given in order to show the theoretical possibilities of this type of epicyclic gearing.

In concluding this portion of the paper, it may be as well to point out that Figs. 4 and 6 cannot claim to show a complete range of possible ratios. They are intended to act merely as a guide,

FIG. 9.

and if the desired ratio does not happen to be one of those given in the graphs, a close approximation to G and K can be made by interpolation, when a few minutes spent in plotting the portion of the curve required will enable suitable values of G and K to be selected.

PART II.

Application to Motor Car Design.

The only use of epicyclic gearing which is at all general in automobile work is its application to the back-axle differential. This is usually a bevel gear train of a type not mentioned in the previous notes, since its use is restricted to this one instance. The general arrangement of a differential gear is too well known to need a detailed description, and its design presents no difficulties beyond that of deciding upon the pitch and number of teeth it is desired to employ. The action of the gear can be analysed by the methods already described.

Fig. 9 shows a differential gear in diagrammatic form where A comprises the cage carrying the planet pinions D which gear with the equal wheels B and C.

The procedure described under Method 1 has to be varied somewhat in this instance, but the alteration is a matter of detail only, the underlying principle being the same. It is clear, from an inspection of Fig. 9, that if A is held, one turn of the wheel C will cause the wheel B to turn once in the opposite direction, and row (a), Table IX., can therefore be written down, giving C "X" turns in a positive direction.

The whole mechanism is then given Y revolutions (row (b)), and the addition of rows (a) and (b) will result in row (c). From an inspection of this row it will be seen that A is the mean of

TABLE IX.

	A	B	C
(a) Hold A, C + X ,.....	0	- X	+ X
(b) All elements + Y.....	Y	+ Y	+ Y
(c) Add (a) and (b).....	Y	Y - X	Y + X

the number of revolutions of B and C. Thus, if X is zero, then B and C revolve at the same speed and in the same direction as A, while if one wheel runs X revolutions per minute faster, the mechanism permits the other to run X revolutions per minute slower, the speed of A remaining constant. The spur gear type can be analysed in the same way.

It is in the differential gear that the epicyclic train has so far held its own, but there are signs that in this case it may have to give way to some other form of gearing, or that at any rate its range of action may be limited. It possesses advantages in that it is compact, simple, and cheap to manufacture, but if it is to work properly, the resistances offered to the wheels C and B should be the same. It frequently happens, however, that, due to a bump, one road wheel leaves the ground, in which case, assuming

the wheel remains suspended, "X" (Table IX.) increases rapidly, the speed of the other road wheel falling in proportion until in the limit when $X = Y$, the speed of the wheel in the air becomes twice that of the cage, the other wheel being stationary. When the wheel returns to earth the conditions are reversed, "X" changing sign and falling to zero, until the drive becomes normal again.

Actually, of course, the wheel would not remain in the air long enough for this to happen. A part of the power released when one wheel leaves the ground will cause additional equal pressures on the teeth of the differential wheels B and C, and therefore will develop additional equal torques in the respective axle driving-shafts. The torque in the axle shaft connected to the wheel in the air is absorbed in accelerating its rotational speed ($Y + X$), whereas the torque in the other axle shaft will be absorbed in overcoming the resistance to rotation of the wheel which is still in contact with the ground. If this resistance is less than the available torque the car will slightly gain speed, and if it is greater the car will lose speed, but in general the speed change will be negligible and $(Y - X)$ may be regarded as constant. When the wheel again comes to the ground Y returns to approximately its original value, and X decreases to zero.

In the case of a propeller shaft brake, if the adhesion of each wheel is the same, and their radii are equal, the retardation on each wheel will be the same, and there will be no tendency for reverse rotation when the brake is applied, but if the wheel diameters differ, or if one wheel has a greater coefficient of adhesion than the other, then if the cage A is locked, the small wheel, or the wheel with the smaller coefficient of friction will be accelerated backwards.

The attention of designers has been for some time directed towards the elimination of these defects, but up to the present the bevel or spur gear epicyclic train is still conventional practice.

Epicyclic Trains for Gear-Boxes.

At first sight epicyclic gearing seems to be ideal for the change-speed mechanism of a motor car. The advantages are obvious; gears are always in engagement, and the different velocity-ratios are obtained by means of brake-bands or clutches operating on the members which have to remain stationary. The engagement of a gear, therefore, becomes a simple matter, demanding no special experience or particular "knack" to ensure a quiet change such

as is required by the usual "unmechanical" sliding-gear type of box. These advantages, however, are offset by the inherent complexity of the gear when more than two velocity-ratios are required, as an examination of the problem will show that considerable ingenuity is necessary to keep the number of parts within reasonable limits. Upon consideration it will be seen that although different members can conveniently be held stationary when required by means of suitably arranged brake-drums, it will greatly simplify the design if either the driving or the driven member is the same throughout. This condition implies that another train of gears will have to be introduced, as a glance at graphs Figs. 2, 4 and 6 shows that only two different ratios can be obtained when using the same driving or driven member. The same graphs also show

FIG. 10.

that it is advisable to have a separate reverse train, since although it is possible to obtain two drives having opposite directions of rotation and using the same driving member, the velocity-ratios involved are not convenient for use in practice.

From this it is apparent, then, that if the gear-box is to be reasonably simple, a reverse and three forward speeds only are available. For these reasons, perhaps, the epicyclic gear-box has not found favour amongst designers in this country, there being, as far as the author is aware, only one British car employing this type of change-speed gearing.

The Ford Gear-Box.

It may be of interest to examine two epicyclic gear-boxes which have given satisfactory service over long periods.

The Ford gear is shown in diagrammatic form in Fig. 10. Two

forward speeds and a reverse are provided, and since the second speed is a direct drive the reverse and one forward speed only are obtained by epicyclic gearing. Referring to Fig. 10, A is the cage connected to the crankshaft and carrying the three planet pinions D_1 , D_2 and D_3 , these engaging with the gears B, C_1 , C_3 ; B is connected to the propeller shaft, and the second speed is obtained directly by clutching B to E, an extension of the clutch shaft. For the first speed, gears B, D_2 , D_1 and C_1 are employed, A is, of course, the driving member, C_1 is held stationary by means of a brake-band acting on a drum, and B is the driven member. This corresponds to case 5, Fig. 6. The proportions of the gears have been scaled from a small size illustration, and from this it appears that $D_1/C_1 = K$ is approximately $-1\frac{1}{2}$, and $D_2/D_1 = G$ is $5/6$. Reference to Fig. 6, case 5, will show that this ratio is in the neighbourhood of $1/3$ to 1, thus the engine makes three revolutions to one of the propeller shaft. The reverse train consists of gears C_3 , D_3 , D_2 and B. The driving member is again A, the stationary element being C_3 . In this case K is approximately $\frac{3}{4}$ and G is $7/6$, and from Fig. 6, case 5, it is seen that this gives a ratio of $-1/3$, the reverse, therefore, being approximately equal to the first speed. An estimate of the number of parts shows that there are at least nine gear wheels, two brake-drums, and three shafts necessary to obtain the two speeds and reverse. A third brake-drum is fitted, but this is not part of the gear-box as such, being solely used for reducing the car speed.

The Lanchester Gear-Box.

The foregoing example was selected as it illustrates one of the simplest forms of epicyclic gear-box, and an examination of the Lanchester gear may be of interest as showing its possibilities when applied to high-class automobile practice. The author has been unable to find a later description of this gear-box than that given in Vol. I. of the "Automobile Engineer," and from the illustration given on page 85 the diagram Fig. 11 has been prepared. It will be seen that the gear consists of three simple epicyclic trains, two of them, however, being inter-connected. These two trains provide the first and second speeds, the third speed is direct drive, and the reverse is obtained by means of the left-hand train, which consists of gears C_1 , D_1 , the internally toothed ring B_1 and the cage A_1 . This last is capable of being held stationary when the reverse is required, and C_1 , rotating with

the crankshaft, becomes the driving member, turning B_1 in the reverse direction, as can be seen by inspecting graph Fig. 2 for the conditions specified. The proportions of the gears have been roughly scaled from the illustration in the "Automobile Engineer" and a value of $K=4$ deduced, and reference to Fig. 2, case 3, shows the ratio to be approximately -0.11 ; actually for this value of K it is $-1/9$. The first-speed train consists of the gears C_2 , D_2 , the internally toothed ring B_2 , and the cage A_2 , which is the driven member. To obtain the first speed, B_2 is held stationary, and C_2 , revolving with the crankshaft, becomes the driving member. These conditions correspond to case 1, Fig. 2, and estimating, as before, the value of K to be $\frac{3}{4}$, the ratio is seen to be about 0.3 ; actually for this value of K it is $2/7$, as can be proved by an inspection of the graph for case 2, which is the inverse of case 1.

FIG. 11.

For the next speed, the gears C_3 , D_3 , the ring B_3 , and the cage A_3 are used in addition to the first speed train. For the second speed, C_2 is again the driving member, the stationary member being the sun pinion C_3 of the second train, and the driven member B_3 . It will be noticed that B_3 is connected to, and revolves with, A_2 , and that A_3 is really part of B_2 .

As the analysis of these trains is rather interesting, it is set out in full beneath Fig. 11, and the rules given under the "1st Method" described previously have been followed. Obviously, the first thing to do is to discover the relative motion of the elements. This is best accomplished by considering the trains separately, holding the cages A_2 , A_3 and turning one of the other elements in each train. This results in row (a), Fig. 11. Now, considering the first train for the present, imagine the ring B_2 to be stationary, and by adding C_2/B_2 to row (a) for this train the corresponding motions of C_2 and A_2 are obtained as in row (b). But B_2 is not stationary, it has rotational motion imparted to it

by the second train, this rotation being the same as that given to A_3 . By adding 1 to row (a) for the second train, the conditions are obtained in row (b) where the second sun pinion C_3 is stationary, and it is seen that for a certain rotation of B_3 , A_3 revolves once in the same direction. By substituting this value of A_3 for B_2 the two trains can be compared, since they satisfy the conditions previously set forth. To bring B_2 to this value, it is necessary to add 1 to row (b) of the first train, and the result is seen in row (c). It is now clear that since C_2 is revolving, C_3 is stationary and B_2 and A_3 are rotating at equal speeds, A_2 is of necessity equal to B_3 . It therefore follows that this design of epicyclic gearing can only work when C_2/B_2 equals C_3/B_3 . These results can be brought to a more convenient form by reducing the rotation of C_2 to 1, and this can be done by multiplying row (c) throughout by $B_2/(2 B_2 + C_2)$, thus obtaining row (d). The conditions necessary for the second speed have now been complied with; C_2 has rotated once, C_3 is stationary, B_2 is being driven by the second train, and the corresponding rotation of A_2 can now be obtained.

From the illustration previously mentioned, the proportions of the gears appear to be approximately as follows:—

$$\begin{array}{ll} C_2 = 4 & C_3 = 3\frac{1}{2} \\ D_2 = 3 & D_3 = 2\frac{2}{3} \\ B_2 = 10 & B_3 = 8 \end{array}$$

On substituting these values in row (d) it is seen that the second speed ratio is roughly $7/12$.

An estimate of the number of pieces in the gear shows that there are at least nine gear wheels, three internally toothed rings, and three brake-drums. This, no doubt, explains the charge of complexity usually levelled against epicyclic gear boxes, but an examination will show that in the main the parts are comparatively simple, and the great advantage of obtaining a noiseless and mechanically correct gear must considerably offset the disadvantage of using a few extra parts.

Engine Starters, &c.

Another direction in which epicyclic gearing offers possibilities is in connection with the gear reduction for the engine starter. A good example of this is the Scott Dynamotor, illustrated and described in "The Autocar" for November 1st, 1913. From the

diagram of this gear given in Fig. 12, it will be seen that it is of the reverted type. When used for starting the engine, the sun pinion C (Fig. 12) becomes the driving member of the train and B the driven member. The speed of A increases as the engine picks up the drive, until it reaches the same speed as the shaft of C, to which it then becomes clutched by means of a free-wheel device, and the epicyclic gearing goes out of action. Thus, a low gear is provided for engine starting, and a higher driven gear for the rotation of the electric motor as a dynamo when the engine has taken up the work. An estimate of the proportions of the gears indicates values of $K=2$ and $G=\frac{1}{2}$. From the equation given in row (f), case 3, Table VII., the velocity-ratio is seen to be about $\frac{1}{4}$ to 1, or, putting it the other way round, the armature shaft revolves at four times the speed of the driven wheel. It will be understood, of course, that this is only the epicyclic gearing

FIG. 12.

reduction; between this and the engine is a further reduction of approximately $3\frac{1}{2}$ to 1.

The principal uses of epicyclic gearing in automobile practice have now been briefly touched upon. There remains, however, one further example of the use of this gear in motor car design, and that is in steering-box reduction. So far, this has been confined to comparatively light cars with short wheel bases, and one of the reasons for this is apparent upon inspection of Fig. 2. Bearing in mind that the usual steering box ratio is in the neighbourhood of 8 to 1, it will be seen that to obtain this with simple epicyclic gearing, which is the type usually employed for this purpose, a planet pinion three times the diameter of the sun pinion will be required. A gear of this size would, of course, have a clumsy appearance, and would seem to be out of all proportion to the work required of it. Other factors enter into the restriction

of the use of epicyclic gearing in this direction, one being that to take full advantage of it the steering-box requires to be close up to the front axle in order that a connection can be taken direct on to the steering cross rod.

The study of epicyclic gearing is fascinating because of the element of uncertainty about it. Its movements cannot very well be pictured graphically, and it is only by the aid of pencil and paper that its action can be analysed.

The epicyclic gear-box is acclaimed by many as being the cure for all change-speed troubles, and this is undoubtedly true, but an inspection of the Lanchester gear-box will show that a tremendous amount of thought had to be expended before such a compact arrangement could be arrived at for providing three epicyclic gear ratios. This type of gear also calls for careful and unusual machining operations, since it will be obvious that a cage which has to carry three pinions engaging accurately with a common sun pinion and internally toothed ring needs to be very accurately produced. These difficulties, however, are not insurmountable with modern machine-shop methods, and in the author's opinion the greatest difficulty that the epicyclic gear-box has to face is the question of weight. If it can be designed together with its change-speed mechanism to closely approximate to the weight of the conventional type of box and change-speed gear providing the same number of speeds, then the epicyclic gear-box will have gone a long way towards establishing its supremacy.

APPENDIX "A."—SEE TABLE I.

Case.	1st Method.					2nd Method.			
	A	B	C	D		A	B	C	D
(1)	(a) Hold A, C, + 1.....	0	$-\frac{C}{D} \cdot \frac{D}{B}$	$+\frac{1}{1+\frac{C}{B}}$	$\frac{C}{D}$ $-\frac{C}{D}$	+1	+1	$+\frac{1}{B} \cdot \frac{D}{C}$	$+\frac{1}{B} \cdot \frac{D}{D}$
	(b) Add $\frac{C}{B}$ to (a)	$\frac{C}{B}$	0	$1+\frac{C}{B}$	(b) Hold A, B, - 1.....	0	- 1	$+\frac{1}{D} \cdot \frac{D}{C}$	$-\frac{1}{D}$
	(c) Reducing C to 1	$\frac{C}{B+C}$	0	1	(c) (a) + (b)	+1	0	$1+\frac{B}{C}$	$\frac{B}{1-\frac{D}{C}}$
(2)	(d) Reducing A, row (b) to 1	1	0	$1+\frac{B}{C}$	(d) Reducing C to 1	$\frac{C}{B+C}$	0	1	
(3)	(e) Given in row (a)	0	$-\frac{C}{B}$	$+\frac{1}{1+\frac{C}{B}}$	(e) Given in row (c)	1	0	$1+\frac{B}{C}$	
(4)	(f) Reducing B, row (a) to 1	0	+1	$-\frac{B}{C}$	(f) By inspection of diagram.	0	$-\frac{C}{B}$	$+\frac{1}{-\frac{C}{B}}$	$\frac{B}{-\frac{C}{D}}$
(5)	(g) Row (a) - 1	- 1	$-\frac{C}{B}$	0	(g) By inspection of diagram.	0	+1	$-\frac{B}{C}$	$+\frac{B}{+\frac{D}{C}}$
	(h) Changing signs.....	+1	$1+\frac{C}{B}$	0	(h) Each element + 1.....	+1	$+\frac{1}{C}$	$+\frac{1}{+\frac{C}{B}}$	$+\frac{1}{+\frac{D}{C}}$
	(i) Reducing B, row (h) to 1	$\frac{B}{B+C}$	+1	0	(i) Hold A, C, - 1.....	0	$+\frac{1}{B}$	- 1	$+\frac{1}{+\frac{D}{C}}$
(6)	(j) Reducing B, row (h) to 1	$\frac{B}{B+C}$	+1	0	(k) (h) + (j)	+1	$1+\frac{1}{B}$	0	$1+\frac{1}{+\frac{D}{C}}$
					(l) Reducing B, row (k) to 1	$\frac{B}{B+C}$	+1	0	

APPENDIX "B."—SEE TABLE IV.

Case.	1st Method.					2nd Method.				
		A	B	C	D_1 & D_2		A	B	C	D_1 & D_2
(1)	(a) Hold A, +1	0	+1	$-\frac{B}{D_2} \cdot \frac{D_1}{C}$	$\frac{B}{D_2} + \frac{B}{D_2}$	(a) Each element +1....	+1	+1	+1	+1
	(b) Add -1 to row (a) ..	-1	0	$-1 - \frac{D_1}{D_2} \cdot \frac{B}{C} \cdot \frac{D_2}{D_2} - 1$		(b) Hold A, B, -1	0	-1	$+\frac{B}{D_2} \cdot \frac{D_1}{C}$	$+\frac{B}{D_2} - \frac{B}{D_2}$
	(c) Reduce C to 1	$\frac{D_2 C}{D_2 C + D_1 B}$	0	1		(c) (a) + (b)	+1	0	$1 + \frac{D_1}{D_2} \cdot \frac{B}{C}$	$1 - \frac{B}{D_2}$
						(d) Reduce C to 1	$\frac{D_2 C}{D_2 C + D_1 B}$	0	1	
(2)	(d) Given in row (b)	+1	0	$1 + \frac{D_1}{D_2} \cdot \frac{B}{C}$		(e) Given in row (e)	+1	0	$1 + \frac{D_1}{D_2} \cdot \frac{B}{C}$	
(3)	(e) Reduce C, row (a) to 1	0	$-\frac{D_2}{D_1} \cdot \frac{C}{B}$	1		(f) From inspection	0	$-\frac{C}{D_1} \cdot \frac{D_2}{B}$	+1	$-\frac{C}{D_1}$
(4)	(f) Given in row (a)	0	+1	$-\frac{D_1}{D_2} \cdot \frac{B}{C}$		(g) From inspection ...	0	+1	$-\frac{B}{D_2} \cdot \frac{D_1}{C}$	$+\frac{B}{D_2}$
(5)	(g) Add -1 to row (c) ..	$\frac{D_1 B}{-D_2 C + D_1 B}$	-1	0		(h) Each element +1 ..	+1	+1	+1	+1
	(h) Reduce A, row (g) to 1	1	$1 + \frac{D_2 C}{D_1 B}$	0		(k) Hold A, C, -1	0	$+\frac{C}{D_1} \cdot \frac{D_2}{B}$	-1	$+\frac{C}{D_1}$
(6)						(l) (h) + (k)	+1	$1 + \frac{D_2 C}{D_1 B}$	0	$1 + \frac{C}{D_1}$
	(k) Given in row (g)	$\frac{D_1 B}{D_1 B + D_2 C}$	1	0		(m) Reduce B, row (l) to 1	$\frac{D_1 B}{D_1 B + D_2 C}$	1	0	

APPENDIX "C"—SEE TABLE VII.

Case.	1st Method.					2nd Method.				
	A	B	C	D_1 & D_2		A	B	C	D_1 & D_2	
(1)	(a) Hold A, C, + 1	$\frac{C}{+D_1} \times \frac{D_2}{B}$	+ 1	$-\frac{C}{D_1}$	(a) Each element + 1 ..	+ 1	+ 1	+ 1	+ 1	
	(b) Add $-\frac{C}{D_1} \times \frac{D_2}{B}$ to (a)	0	$1 - \frac{C}{D_1} \cdot \frac{D_2}{B}$		(b) Hold A, B, - 1	0	- 1	$-\frac{B}{D_2} \cdot \frac{D_1}{C}$	$+\frac{B}{D_2}$	
	(c) Reduce C to 1	$\frac{CD_2}{CD_2 - BD_1}$	1		(c) (a) + (b)	+ 1	0	$1 - \frac{B}{D_2} \cdot \frac{D_1}{C}$		
					(d) Reduce C to 1	$\frac{CD_2}{CD_2 - BD_1}$	0	1		
(2)	(d) Reduce A, row (b) to 1	+ 1	$\frac{BD_1}{1 - \frac{CD_2}{BD_1}}$		(e) Given in row (c)	+ 1	0	$1 - \frac{BD_1}{CD_2}$		
(3)	(e) Given in row (a)	0	$+\frac{CD_2}{BD_1}$		(f) From inspection.....	0	$+\frac{C}{D_1} \cdot \frac{D_2}{B}$	+ 1	$\frac{C}{-D_1}$	
(4)	(f) Reduce B, row (e) to 1.	0	$\frac{BD_1}{CD_2}$		(g) From inspection	0	+ 1	$+\frac{B}{D_2} \cdot \frac{D_1}{C}$	$\frac{B}{-D_2}$	
(5)	(g) Add - 1 to row (a) ..	- 1	$-\frac{1}{1 + \frac{CD_2}{BD_1}}$		(h) Each element + 1 ..	+ 1	+ 1	+ 1	+ 1	
	Row (g) =	1	$1 - \frac{CD_2}{BD_1}$		(k) Hold A, C, - 1	0	$-\frac{C}{D_1} \cdot \frac{D_2}{B}$	- 1	$+\frac{C}{D_1}$	
(6)					(l) (h) + (k)	+ 1	$\frac{CD_2}{1 - \frac{CD_2}{BD_1}}$	0		
	(h) Reduce B, row (g) to 1	$\frac{BD_1}{BD_1 - CD_2}$	1	0	(m) Reduce R, row (l) to 1	$\frac{BD_1}{BD_1 - CD_2}$	1	0		