LXXXV.—On the heat of vapours and on astronomical refractions

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To cite this article: John William Lubbock Esq. Treas. R.S. F.R.A.S. F.L.S. (1840) LXXXV.—On the heat of vapours and on astronomical refractions , Philosophical Magazine Series 3, 16:106, 562-569, DOI: [10.1080/14786444008650093](https://doi.org/10.1080/14786444008650093)

To link to this article: <http://dx.doi.org/10.1080/14786444008650093>

Published online: 01 Jun 2009.

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appears to open a field for discovery, which experimental industry cannot fail to cultivate with immediate success.

Note.—On the evening of the meeting at which my investigations were presented to the Society, my friend, Dr. Bache of the Girard College, gave an account of the investigations of Professor Ettingshausen of Vienna, in reference to the improvement of the magneto-electric machine, some of the results of which he had witnessed at the University of Vienna about a year since. No published account of these experiments has yet reached this country, but it appears that Professor Ettingshausen had been led to suspect the development of a current in the metal of the keeper of the magneto-electric machine, which diminished the effect of the current in the coil about the keeper, and hence to separate the coil from the keeper by a ring of wood of some thickness, and afterwards, to prevent entirely the circulation of currents in the keeper, by dividing it into segments, and separating them by a non-conducting material. I am not aware of the result of this last device, nor whether the mechanical difficulties in its execution were fully overcome. It gives me pleasure to learn that the improvements, which I have merely suggested as deductions from the principles of the interference of induced currents (76), should be in accordance with the experimental conclusions of the above-named philosopher.

LXXXV.—*On the Heat of Vapours and on Astronomical Refractions.* By JOHN WILLIAM LUBBOCK, Esq., Treas. R.S. F.R.A.S. and F.L.S., Vice-Chancellor of the University of London, &c.

[Continued from p. 514.]

ON THE STEAM-ENGINE.

THE law which connects the pressure and the temperature of steam having been unknown, various empirical rules have been given. As, however, the expressions which arise are not in a convenient form for the calculations which are required in order to ascertain the *duty* which steam-engines are capable of performing, or to solve other problems of the same nature, M. de Pambour*, in his work on that subject, has employed another expression, viz.

$$\mu = \frac{1}{g} = \frac{1}{n + qp},$$

in which g is the density of steam, p the pressure, and n and q constants. According to my expression

* *Théorie de la Machine à Vapeur*, p. 111.

$$\frac{1}{\rho} = \frac{(1 - E)}{\frac{1}{p^\gamma} - E p}$$

The pressure being reckoned in atmospheres, and the density of steam corresponding to the pressure of one atmosphere (or 14·706 lbs. per square inch) being unity. If we take the density of water for unity, then as the volume of steam at the pressure of one atmosphere is 1700 times greater than that of the same weight of water,

$$\mu = - \frac{K}{\frac{1}{p^\gamma} - E p} \quad K = 1700 (E - 1)$$

$$\log K = 2\cdot4765041 \quad \frac{1}{\gamma} = 1\cdot0134 \quad E = 1\cdot17602.$$

If we suppose that a certain volume of water represented by S be transformed into vapour at the pressure p , and that M is the absolute volume of vapour which results, we shall have

$$\frac{M}{S} = \mu = \frac{[0\cdot4109002]}{p} \left(\frac{1}{\alpha} + \theta \right).$$

If afterwards the same volume of water is transformed into vapour at the pressure p' , and that the absolute volume which the resulting vapour occupies be called M' , we shall have

$$\frac{M'}{S} = \mu'$$

$$\frac{M}{M'} = \frac{\frac{1}{p'^\gamma} - E p}{\frac{1}{p^\gamma} - E p}$$

“ Soit* P la pression totale de la vapeur dans la chaudière, et p' la pression qu'aura cette vapeur à son arrivée dans le cylindre, pression qui sera toujours moindre que P , excepté dans un cas particulier que nous traiterons plus loin. La vapeur pénétrera donc dans le cylindre à la pression p' , et elle continuera d'affluer avec cette pression et de produire un effet correspondant, jusqu'à ce que la communication entre la chaudière et le cylindre soit interceptée. Alors il cessera d'arriver de la vapeur nouvelle dans le cylindre, mais celle qui y est déjà parvenue, commencera à se dilater pendant le reste de la course du piston, en produisant par sa détente une certaine quantité de travail, qui s'ajoutera à celle déjà produite pendant la période d'admission de la vapeur.

* The reasoning here is taken from M. de Pambour's work.

" P étant la pression de la vapeur dans la chaudière, et p' la pression qu'elle prendra à son arrivée dans le cylindre avant la détente, soit π la pression de cette vapeur en un point quelconque de la détente. Soit en même temps l la longueur totale de la course du piston, l' la portion parcourue au moment où a commencé la détente, et λ celle qui correspond au point où la vapeur a acquis la pression π . Enfin, soit encore a l'aire du piston, et c la liberté du cylindre, c'est-à-dire l'espace libre qui existe à chaque bout du cylindre, au-delà de la portion parcourue par le piston, et qui se remplit nécessairement de vapeur à chaque course; cet espace, y compris les passages aboutissants, étant représenté par une longueur équivalente du cylindre.

"Si l'on prend le piston au moment où la longueur de course parcourue est λ , et la pression π , on verra que si le piston parcourt, en outre, un espace élémentaire $d\lambda$, le travail élémentaire produit dans ce mouvement sera $\pi a d\lambda$. Mais en même temps, le volume $a(l' + c)$ occupé par la vapeur avant la détente sera devenu $a(\lambda + c)$." Hence,

$$\frac{M}{M'} = \frac{p'^{\frac{1}{\gamma}} - E p'}{\frac{1}{\pi \gamma} - E \pi} = \frac{\lambda + c}{l' + c}.$$

$$\lambda + c = (l' + c) \frac{(p'^{\frac{1}{\gamma}} - E p')}{(\frac{1}{\pi \gamma} - E \pi)}$$

The elementary work produced $= \pi a d\lambda$.

$$\begin{aligned} \int \pi a d\lambda &= \pi a (\lambda + c) - \int a (\lambda + c) d\pi \\ &= \pi a (\lambda + c) - \int a (l' + c) (p'^{\frac{1}{\gamma}} - E p') \frac{d\pi}{(\frac{1}{\pi \gamma} - E \pi)} \\ &= \pi a (\lambda + c) - a (l' + c) (p'^{\frac{1}{\gamma}} - E p') \int \frac{d\pi}{\frac{1}{\pi \gamma} - E \pi} \\ &= \pi a (\lambda + c) \\ &\quad + a (l' + c) \frac{(p'^{\frac{1}{\gamma}} - E p') \gamma}{E (\gamma - 1)} \log (1 - E \pi^{\frac{\gamma-1}{\gamma}}) + \text{const.} \end{aligned}$$

This integral is to be taken from $\lambda = l'$ to $\lambda' = l$, let $\pi = p$ when $\lambda = l$, when $\lambda = l'$, $\pi = p'$.

$$\int \pi a \, d\lambda = p a (l + c) - p' a (l' + c) \\ + a (l' + c) \frac{(p'^{\frac{1}{\gamma}} - E p') \gamma}{E(\gamma - 1)} \log \left\{ \frac{1 - E p^{\frac{\gamma-1}{\gamma}}}{1 - E p'^{\frac{\gamma-1}{\gamma}}} \right\}$$

for the values of the constants E, γ . See p. 511.

To this must be added the work effected during the course of the piston through l' , which is $p' a l$, and if R is the total pressure exerted upon unity of surface of the piston

$$p a (l + c) - p' a c \\ + a (l' + c) (p'^{\frac{1}{\gamma}} - E p') \frac{\gamma}{E(\gamma - 1)} \log \left\{ \frac{1 - E p^{\frac{\gamma-1}{\gamma}}}{1 - E p'^{\frac{\gamma-1}{\gamma}}} \right\} \\ = a R l \quad (A)^*$$

$R = (1 + \delta) r + p'' + f$, f is the friction of the machine not loaded, δ the increase of this friction due to unity of the charge r , p'' the pressure on the surface of the piston, representing the atmospheric pressure when the machine works without condensation, and otherwise the pressure of condensation in the cylinder.

If S denote the volume of water converted into vapour by the boiler in unity of time, this volume in the cylinder becomes

$$- \frac{S K}{p'^{\frac{1}{\gamma}} - E p'}$$

K being the same constant as in p. 563.

It is evident, according to the reasoning of M. de Pambour, in p. 125. of his work, that if v denote the velocity of the piston

$$\frac{S K}{p'^{\frac{1}{\gamma}} - E p'} = - v a \frac{l' + c}{l} \quad (B.)$$

* This equation is equivalent to the equation (A) of M. de Pambour, p. 123, which may be put into the form

$$p a (l + c) - p' a c + \frac{l S}{q v} \text{Nap. log} \left\{ \frac{l + c}{l' + c} \right\} = a R l.$$

or
$$p a (l + c) - p' a c + [6.9505960] \frac{l S}{v} \log \left\{ \frac{l + c}{l' + c} \right\} = a R l,$$

the pressure being reckoned in lbs. per square inch.

$$p'^{\frac{1}{\gamma}} - E p' = - \frac{l S K}{a v (l' + c)} = \frac{K}{\mu'},$$

Similarly,

$$p^{\frac{1}{\gamma}} - E p = - \frac{l S K}{a v (l + c)} = \frac{K}{\mu}$$

$$\frac{1}{\mu} = \frac{l S}{a v (l + c)} \qquad \frac{1}{\mu'} = \frac{l S}{a v (l' + c)}$$

$$p a (l + c) - p' a c$$

$$- \frac{l S K \gamma}{v E (1 - \gamma)} \text{ Nap. log } \left\{ \frac{1 - E p'^{\frac{\gamma-1}{\gamma}}}{1 - E p^{\frac{\gamma-1}{\gamma}}} \right\} = a R l$$

$$p a (l + c) - p' a c - (p'' + f) a l$$

$$- [4.6411966] \frac{l S}{v} \log^* \left\{ \frac{1 - E p'^{\frac{\gamma-1}{\gamma}}}{1 - E p^{\frac{\gamma-1}{\gamma}}} \right\} = a l (1 + \delta) r$$

$$\log \left\{ \frac{1 - E p'^{\frac{\gamma-1}{\gamma}}}{1 - E p^{\frac{\gamma-1}{\gamma}}} \right\} = \log \left\{ \frac{l + c}{l' + c} \right\} - \frac{1}{\gamma} \log \left(\frac{p'}{p} \right).$$

If the machine work without expansion,

$$p = p', \qquad \log \left\{ \frac{1 - E p'^{\frac{\gamma-1}{\gamma}}}{1 - E p^{\frac{\gamma-1}{\gamma}}} \right\} = 0,$$

$$r = \frac{p - (p'' + f)}{1 + \delta}$$

The data upon questions relating to the steam-engine are the quantities a , l , l' , S , and v , and it is evident that from these quantities the quantities μ and μ' may at once be found by an easy arithmetical operation; from these the following table will give the corresponding pressures p and p' , and these pressures being introduced into equation A, the value of $a r$ may be easily found.

* Log. of Briggs, the pressure being reckoned in atmospheres, the log. of the constant is [7.9670537], the pressure being reckoned in lbs. per square foot.

Table showing the volume (compared with that of water at 212°),
and the temperature of steam.

Pressure in lbs. per square inch. $14\cdot706 \times p$.	Temperature. Fahrenheit. τ		Volume. $\mu^\dagger$	Pressure in lbs. per square inch. $14\cdot706 \times p$.	Temperature. Fahrenheit. τ		Volume. μ
	Air*. Therm.	Merc. Therm.			Air. Therm.	Merc. Therm.	
1	101 ^o ·5	101 ^o ·5	20816	56	287 ^o ·6	289 ^o ·5	498
2	126·0	126·0	10871	57	288·7	290·7	490
3	141·4	141·4	7442	58	289·8	291·8	482
4	153·0	153·0	5691	59	290·9	293·0	474
5	162·2	162·2	4622	60	292·0	294·1	467
6	170·1	170·1	3902	61	293·0	295·1	460
7	176·8	176·8	3381	62	294·0	296·2	453
8	182·8	182·8	2986	63	295·0	297·2	447
9	188·3	188·3	2678	64	296·0	298·2	440
10	193·2	193·2	2429	65	297·0	299·2	434
11	197·8	197·8	2223	66	298·0	300·3	428
12	202·0	202·0	2051	67	299·0	301·3	422
13	205·9	205·9	1905	68	300·0	302·3	417
14	209·5	209·5	1778	69	301·0	303·3	411
14·706	212·0	212·0	1700	70	301·9	304·2	406
15	212·9	212·9	1669	71	302·8	305·2	401
16	216·3	216·4	1572	72	303·7	306·1	396
17	219·3	219·5	1487	73	304·6	307·0	391
18	222·3	222·6	1410	74	305·5	308·0	386
19	225·1	225·5	1342	75	306·4	308·9	381
20	227·9	228·3	1280	76	307·3	309·8	377
21	230·4	230·9	1224	77	308·2	310·7	372
22	232·9	233·5	1172	78	309·1	311·6	368
23	235·2	235·8	1125	79	310·0	312·6	364
24	237·6	238·3	1082	80	310·9	313·5	359
25	239·8	240·6	1042	81	311·8	314·5	355
26	242·0	242·8	1005	82	312·7	315·4	351
27	244·0	244·9	971	83	313·5	316·3	348
28	246·1	247·0	939	84	314·3	317·1	344
29	248·0	249·0	909	85	315·1	317·9	340
30	250·0	251·0	881	86	315·9	318·7	337
31	251·8	252·8	855	87	316·7	319·6	333
32	253·7	254·8	831	88	317·5	320·4	330
33	255·4	256·5	808	89	318·3	321·2	326
34	257·2	258·3	786	90	319·1	322·0	323
35	258·9	260·1	765	91	319·9	322·9	320
36	260·6	261·8	746	92	320·7	323·7	317
37	262·2	263·5	727	93	321·5	324·5	313
38	263·8	265·1	709	94	322·3	325·3	310
39	265·3	266·6	693	95	323·0	326·0	307
40	266·8	268·2	677	96	323·7	326·8	305
41	268·2	269·6	662	97	324·4	327·5	302
42	269·7	271·2	647	98	325·1	328·3	299
43	271·1	272·6	633	99	325·8	329·0	296
44	272·5	274·1	620	100	326·5	329·7	293
45	273·8	275·4	608	105	330·0	333·3	281
46	275·2	276·8	596	120	339·7	343·4	249
47	276·5	278·2	584	135	348·4	352·4	223
48	277·8	279·5	573	150	356·5	360·8	203
49	279·1	280·8	562	165	363·9	368·4	186
50	280·4	282·1	552	180	370·7	375·5	172
51	281·7	283·5	542	195	377·0	382·0	160
52	282·9	284·7	532	210	383·3	388·4	150
53	284·1	286·0	523	225	389·0	394·4	141
54	285·2	287·1	514	240	394·5	400·1	133
55	286·4	288·3	506				

$$* \tau = - \frac{[2\cdot0651059]}{p^{0\cdot184} - 1\cdot17602} - 448^\circ \quad \dagger \mu = \frac{[0\cdot4109002] \left(\frac{1}{\alpha} + \theta \right)}{p^{0\cdot184} - 1\cdot17602}.$$

The following example will serve to show in what manner the table was calculated.

Ex.—Calculation of the temperature and volume of steam for the pressure of 180 lbs. per square inch.

$$\log 180 = 2.2552725$$

$$\log 14.706 = 1.1674946$$

$$\log p = 1.0877779 \times .0134 = .01457622386 = \log 1.03413$$

$$1.17602$$

$$1.03413$$

$$2.0651059$$

$$\log .14189 = 9.1519518$$

$$2.9131541 = \log 818.7$$

$$448.0$$

$$\text{Temp. Fahr. Air Therm.} = 370.7$$

$$4.8$$

$$\text{Temp. Fahr. Merc. Therm.} = 375.5$$

$$0.4109002$$

$$\log \frac{1}{p} = 8.9122221$$

$$2.9131541$$

$$2.2362764 = \log 172 \quad \mu = 172$$

The following data are taken from M. de Pambour's work on the Steam Engine, p. 238.

$$\begin{array}{llll} p' = .25 \text{ l} & v = 250 & f = .5 \text{ (lb. per square inch)} & \delta = .14 \\ l = 10 \text{ ft.} & a = 12.566 \text{ sq. ft.} & & S = .927 \text{ cub. ft.} \\ p'' = 4 \text{ lbs. per square inch.} & c = .05 \text{ l} & & \end{array}$$

$$\log v = 2.3979400$$

$$\log v = 2.3979400$$

$$\log a = 1.0991971$$

$$\log a = 1.0991971$$

$$\log .30 = 9.4771213$$

$$\log 1.05 = 0.0211893$$

$$2.9742584$$

$$3.5183264$$

$$\log S = 9.9670797$$

$$\log S = 9.9670797$$

$$3.0071787 = \log 1016.6$$

$$3.5512467 = \log 3558.3$$

$$\mu' = 1016.6$$

$$\mu = 3558.3$$

Hence by the table $p' = 25.686$, $p = 6.627$ reckoned in lbs. per square inch.

$$\begin{aligned} p' &= 1.7466, & p &= .4506, & p'' + f &= .3060 \text{ in atmospheres.} \\ \log 1.7466 &= .2422019, & \log .4506 &= 9.6538224 \\ .2422019 \times .0134 &= .00324550546 \end{aligned}$$

$$9.9967545$$

$$\log E = 0.0704184$$

$$0.0671729 = \log 1.16727$$

$$0.3461776 \times .0134 = .00463877984$$

$$.0704184$$

$$.0750571 = \log 1.18865$$

$$\log .18865 = 9.2756568$$

$$\log .16727 = 9.2234181$$

$$0.0522387$$

$$\log p = 9.6538224$$

$$\log a = 1.0991971$$

$$\log (l + c) = 1.0211893$$

$$1.7742088$$

$$59.457$$

$$\log p' = 0.2422019$$

$$\log a = 1.0991971$$

$$\log c = 9.6989700$$

$$1.0403690$$

$$10.974$$

$$38.451$$

$$49.425$$

$$\log (p'' + f) = 9.4857179$$

$$\log a = 1.0991971$$

$$\log l = 1.0000000$$

$$1.5849150$$

$$38.451$$

$$4.6411966$$

$$\log l = 1.0000000$$

$$\log S = 9.9671554$$

$$8.7179923$$

$$4.3263443$$

$$\log v = 2.3979400$$

$$1.9284043$$

$$84.802$$

$$59.457$$

$$144.259$$

$$49.425$$

$$94.834$$

$$\log 94.834 = 1.9769641$$

$$\log 14.706 = 1.1674946$$

$$\log 144 = 2.1583625$$

$$5.3028212$$

$$\log l (1 + \delta) = 1.0569049$$

$$4.2459163$$

$$ar = 17616$$

$ar = 17616$ expressed in lbs., M. de Pambour finds $ar = 17337$.

[To be continued.]