Philosophical Magazine Series 5

ISSN: 1941-5982 (Print) 1941-5990 (Online) Journal homepage: <http://www.tandfonline.com/loi/tphm16>

XXVIII. Note on comparing capacities

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To cite this article: E.C. Rimington (1887) XXVIII. Note on comparing capacities , Philosophical Magazine Series 5, 24:148, 238-244, DOI: [10.1080/14786448708628088](https://doi.org/10.1080/14786448708628088)

To link to this article: <http://dx.doi.org/10.1080/14786448708628088>

Published online: 29 Apr 2009.

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and we have an additional equation,

$$-X\dot{x}_0 = v/C,$$

whence

$$vX = cX^2\dot{x}_0.$$

Thus the subsequent investigation is the same as that of the last section if we replace L by CX^2 , and the condition of zero-deflection is

$$aMb + d^2 = CX^2Rb^2.$$

XXVIII. *Note on Comparing Capacities.*

By E. C. RIMINGTON*.

LET two condensers, of capacities K_1 and K_2 , be connected to the arms A and D of a Wheatstone-bridge, as shown in woodcut; and let

$$AC = BD,$$

so that there is a balance for permanent currents.

Let $x+y$, x , z , and w be the cyclic currents at some moment after the battery-circuit is broken, and when the condensers are discharging; and let q_1 be the charge on K_1 and q_2 that on K_2 at this moment. Then we have

$$(A + B + G)(x + y) - Gx - Az = 0 \dagger,$$

$$(C + D + G)x - G(x + y) - Dw = 0;$$

also

$$z = \frac{dq_1}{dt} \quad \text{and} \quad w = \frac{dq_2}{dt}.$$

Therefore

$$(A + B)x + (A + B + G)y = A \frac{dq_1}{dt},$$

$$(C + D)x - Gy = D \frac{dq_2}{dt}.$$

* Communicated by the Physical Society: read June 25, 1887.

† The self-induction of the galvanometer need not be taken into account, as the current in it commences and ends at zero. The arms A, B, C, and D are supposed to possess no self-induction.

Hence

$$y = \frac{\begin{vmatrix} A \frac{dq_1}{dt}, & A+B \\ D \frac{dq_2}{dt}, & C+D \end{vmatrix}}{\begin{vmatrix} A+B+G, & A+B \\ -G, & C+D \end{vmatrix}},$$

or

$$\Delta y = A(C+D) \frac{dq_1}{dt} - (A+B)D \frac{dq_2}{dt}, \quad \dots \quad (1)$$

where

$$\Delta = G(A+B) + (A+B+G)(C+D).$$

Integrating,

$$\Delta \int_0^\infty y dt = A(C+D) \int_{K_1 A x_0}^0 dq_1 - (A+B)D \int_{K_2 D x_0}^0 dq_2,$$

where x_0 is the permanent current through A and D when the battery is on.

But $\int_0^\infty y dt$ is the quantity which passes through the galvanometer, $= q$.

Therefore

$$\Delta q = x_0 \{ K_2 D^2 (A+B) - K_1 A^2 (C+D) \}.$$

If x_1 is the permanent current in B and C ,

$$x_0 = x_1 \frac{C}{D};$$

and since $AC=BD$, we have

$$q = x_1 BD \frac{K_2 C - K_1 B}{G(B+C) + B(C+D)}, \quad \dots \quad (2)$$

which may be written

$$q = x_1 \frac{K_2 C - K_1 B}{\frac{BD}{G(B+C)} + \frac{C}{D} + 1}.$$

If now A and D be made infinite, which will obviate the necessity of first adjusting for a permanent balance,

$$q = x_1 (K_2 C - K_1 B).$$

Also

$$x_1 = \frac{E}{\rho + B + C},$$

where ρ is the internal resistance and E the E.M.F. of the testing battery.

$$q = \frac{E}{\rho + B + C}(K_2 C - K_1 B).$$

If $q = 0$,

$$\frac{K_1}{K_2} = \frac{C}{B}.$$

If a key be also put in the galvanometer-circuit, and the battery-key be first depressed and then, after a certain interval of time, the galvanometer-key, we have Gott's method of comparing capacities. This allows the condenser longer time to charge; and if the galvanometer when its key is depressed shows no throw, it indicates that the potentials v_1 and v_2 are equal, and that therefore

$$\frac{K_1}{K_2} = \frac{C}{B},$$

as before.

If, however, either of the condensers has an appreciable leakage, the result will be false by this method.

Suppose the condenser K_2 has an insulation-resistance R , that of K_1 being infinite. On depressing the battery-key, as the condensers are in series, they take initial charges each equal to Q .

$$Q = (V_1 - v_1)K_1 = (v_1 - V_2)K_2 = (V_1 - V_2) \frac{K_1 K_2}{K_1 + K_2}.$$

Let an interval of time t elapse before the galvanometer-key is depressed, and at the end of it let Q_1 and Q_2 be the charges on K_1 and K_2 respectively, and v the value of v_1 . Then

$$Q_2 = Q e^{-\frac{t}{K_2 R}}, \text{ where } e = 2.718,$$

$$Q_2 = (V_1 - V_2) \frac{K_1 K_2}{K_1 + K_2} e^{-\frac{t}{K_2 R}}$$

and

$$\begin{aligned} Q_1 &= (V_1 - v)K_1 = \{(V_1 - V_2) - (v - V_2)K_1 \\ &= \left\{ (V_1 - V_2) - \frac{Q_2}{K_2} \right\} K_1, \end{aligned}$$

$$Q_2 = (V_1 - V_2) \left(1 - \frac{K_1}{K_1 + K_2} e^{-\frac{t}{K_2 R}} \right) K_1.$$

Now

$$(v - V_2) = \frac{Q_2}{K_2} = (V_1 - V_2) \frac{K_1}{K_1 + K_2} e^{-\frac{t}{K_2 R}}$$

and

$$(V_1 - v) = \frac{Q_1}{K_1} = (V_1 - V_2) \left(1 - \frac{K_1}{K_1 + K_2} e^{-\frac{t}{K_2 R}}\right).$$

Hence

$$\frac{v - V_2}{V_1 - v} = \frac{K_1 e^{-\frac{t}{K_2 R}}}{K_1 (1 - e^{-\frac{t}{K_2 R}}) + K_2}.$$

Also

$$\frac{v_2 - V_2}{V_1 - v_2} = \frac{C}{B}.$$

If, therefore, $v = v_2$, or there appears to be a balance,

$$\frac{C}{B} = \frac{K_1 e^{-\frac{t}{K_2 R}}}{K_1 (1 - e^{-\frac{t}{K_2 R}}) + K_2}$$

or

$$K_1 = \frac{K_2 C}{(B + C) e^{-\frac{t}{K_2 R}} - C}.$$

So that, unless t be made very small compared to $K_2 R$, we cannot compare K_1 and K_2 unless we know R and t . In the case of no leakage the conditions will be the same as if there were no key in the galvanometer circuit.

To find the best conditions for the test:—Taking equation (2), and substituting for x_1 its value

$$\frac{\frac{E}{(A + D)(B + C)}}{\rho + \frac{A + D + B + C}{A + D + B + C}} \times \frac{D}{C + D},$$

which equals $\frac{ED}{\rho(C + D) + D(B + C)}$, since $AC = BD$, we obtain

$$q = \frac{EBD^2(K_2 C - K_1 B)}{\{\rho(C + D) + D(B + C)\} \{G(B + C) + B(C + D)\}}.$$

If $q = 0$, $K_2 C = K_1 B$.

Suppose a fractional error p to be made in the value of C
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Then

$$q = \frac{EBD^2pK_2C}{\{\rho(C+D) + D(B+C)\}\{G(B+C) + B(C+D)\}}$$

$$= \frac{EpK_2}{\left(\frac{\rho}{D} + \frac{\rho}{C} + \frac{B+C}{C}\right)\left(\frac{G}{D} + \frac{GC}{BD} + \frac{C+D}{D}\right)}.$$

Now the throw is proportional to $q\sqrt{G}$ for a galvanometer whose coil-volume is constant. Therefore the throw is proportional to

$$\frac{EpK_2}{\left(\frac{\rho}{D} + \frac{\rho}{C} + \frac{B+C}{C}\right)\left(\frac{\sqrt{G}}{D} + \frac{C\sqrt{G}}{BD} + \frac{C+D}{D\sqrt{G}}\right)},$$

and this is to be a maximum ; which it obviously is when

$$\sqrt{G} \cdot \frac{B+C}{BD} = \frac{C+D}{D} \frac{1}{\sqrt{G}}, \quad \text{or} \quad G = \frac{B(C+D)}{B+C}.$$

If A and D are infinite, the higher the resistance of the galvanometer the better. Also, in this case,

$$q = \frac{EpK_2}{\frac{\rho}{C} + \frac{B}{C} + 1} = \frac{EpK_2}{\frac{\rho}{C} + \frac{K_2}{K_1} + 1}.$$

Here q is largest when C is largest.

To find the conditions under which a telephone may replace the galvanometer :—

In equation (1) we have

$$\Delta y = A(C+D) \frac{dq_1}{dt} - (A+B)D \frac{dq_2}{dt}.$$

Now, in order to employ a telephone, y must always be zero. Therefore

$$\frac{A}{A+B} \frac{dq_1}{dt} = \frac{D}{C+D} \frac{dq_2}{dt}.$$

But

$$\frac{A}{A+B} = \frac{D}{C+D}, \quad \text{since } AC = BD.$$

Hence

$$\frac{dq_1}{dt} = \frac{dq_2}{dt}.$$

Now, since there is no current in the galvanometer, its resistance may be anything : let it be zero. Then K_1 discharges through $\frac{AB}{A+B}$, and K_2 through $\frac{CD}{A+D}$.

Consequently

and
$$q_1 = x_0 K_1 A e^{-\frac{t}{K_1 \frac{AB}{A+B}}},$$

Hence
$$q_2 = x_0 K_2 D e^{-\frac{t}{K_2 \frac{CD}{C+D}}}.$$

$$\begin{aligned} \frac{dq_1}{dt} &= -x_0 K_1 A \cdot \frac{A+B}{K_1 AB} e^{-\frac{t}{K_1 \frac{AB}{A+B}}} \\ &= -x_0 \frac{A+B}{B} e^{-\frac{t}{K_1 \frac{AB}{A+B}}}, \end{aligned}$$

and

$$\frac{dq_2}{dt} = -x_0 \frac{C+D}{C} e^{-\frac{t}{K_2 \frac{CD}{C+D}}}.$$

Therefore

$$\frac{A+B}{B} e^{-\frac{t}{K_1 \frac{AB}{A+B}}} = \frac{C+D}{C} e^{-\frac{t}{K_2 \frac{CD}{C+D}}},$$

and

$$\frac{A+B}{B} = \frac{C+D}{D}.$$

Therefore

$$K_1 \frac{AB}{A+B} = K_2 \frac{CD}{C+D}.$$

and, since $AC=BD$,

$$\begin{aligned} K_1 B &= K_2 C, \\ \frac{K_1}{K_2} &= \frac{C}{B}. \end{aligned}$$

If there is self-induction in the four conductors, let L_1 be the coefficient of self-induction of A, L_2 of D, L_3 of B, and L_4 of C. Then

$$(A+B+G)y + (A+B)x + L_1 \frac{d(x+y)}{dt} + L_3 \frac{d(x+y)}{dt} - A \frac{dq_1}{dt} = 0.$$

$$-Gy + (C+D)x + L_2 \frac{dx}{dt} + L_4 \frac{dx}{dt} - D \frac{dq_2}{dt} = 0.$$

Solving for y , we have

$$\begin{aligned} \Delta y = (C+D) \left\{ A \frac{dq_1}{dt} - L_1 \frac{d(x+y)}{dt} - L_3 \frac{d(x+y)}{dt} \right\} \\ - (A+B) \left\{ D \frac{dq_2}{dt} - L_2 \frac{dx}{dt} - L_4 \frac{dx}{dt} \right\}. \end{aligned}$$

Integrating,

$$\begin{aligned}\Delta q &= (C+D) \left\{ A \int_{K_1 A x_0}^0 dq_1 - L_1 \int_{x_0}^0 d(x+y) - L_3 \int_{-\frac{A}{B}x_0}^0 d(x+y) \right\} \\ &\quad - (A+B) \left\{ D \int_{K_2 D x_0}^0 dq_2 - L_2 \int_{x_0}^0 dx - L_4 \int_{-\frac{A}{B}x_0}^0 dx \right\} \\ &= x_0 \frac{C+D}{C^2} (BCD^2 K_2 - B^2 D^2 K_1 + L_1 C^2 - L_3 CD - L_2 BC + L_4 BD), \\ &\quad \text{since } AC=BD.\end{aligned}$$

Also

$$x_0 = x_1 \frac{C}{D},$$

where x_1 is the permanent current in the branches B and C.

$$\therefore \Delta q = x_1 \cdot \frac{C+D}{CD} (BCD^2 K_2 - B^2 D^2 K_1 + L_1 C^2 - L_3 CD - L_2 BC + L_4 BD).$$

If $q=0$,

$$BD^2(K_2 C - K_1 B) + C(L_1 C - L_3 D) + B(L_4 D - L_2 C) = 0.$$

Hence

$$K_2 C = K_1 B, \quad L_1 C = L_3 D, \quad L_4 D = L_2 C;$$

or

$$\frac{K_1}{K_2} = \frac{C}{B} \quad \text{and} \quad \frac{L_1}{L_3} = \frac{L_2}{L_4} = \frac{D}{C}.$$

Again, since

$$\begin{aligned}\Delta &= \frac{C+D}{C} \{ B(C+D) + G(B+C) \}, \\ q &= x_1 \frac{BCDK_2 - B^2 DK_1 + L_1 \frac{C^2}{D} - L_2 \frac{BC}{D} - L_3 C + L_4 B}{B(C+D) + G(B+C)} \\ &= x_1 \frac{K_2 BC - K_1 B^2 + L_1 \frac{C^2}{D^2} - L_2 \frac{BC}{D^2} - L_3 \frac{C}{D} + L_4 \frac{B}{D}}{\frac{BC}{D} + B + \frac{G(B+C)}{D}}.\end{aligned}$$

If A and D are now made infinite,

$$q = x_1(K_2 C - K_1 B);$$

the same result that we obtain if there is no self-induction in the branches B and C.