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A RECTANGULAR-COMPONENT TWO-DIMENSIONAL ALTERNATING-CURRENT POTENTIOMETER.*

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It is here proposed to describe the construction, mode of operation, and use of a special form of alternating-current potentiometer, particularly adapted to telephonic-frequency measurements, and which gives its readings in two rectangular components of the voltage measured.

Brief History.—The name "Electric Potentiometer" was suggested in 1873, by Latimer Clark, for the instrument which he described, as serving to measure continuous-current potential differences, in a paper read in London, January 22, 1873, before the "Society of Telegraph Engineers," the forerunner of "The Institution of Electrical Engineers." This instrument has remained almost unchanged to the present time, and is one of the most valuable measuring instruments which electrical engineering possesses.

An early form of *a-c.* potentiometer was described by Franke in 1891. It employed a small *a-c.* generator with two armature windings mechanically adjustable with respect to a common rotating field; so that both the relative magnitudes and the relative

* Communicated by Dr. Kennelly.

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phase of the two induced *emfs.* at one and the same frequency, could be determined.¹

An improved *a-c.* potentiometer was described by Dr. C. V. Drysdale² in 1909. This instrument has come into fairly extensive use. It measures an *a-c.* potential difference in polar coördinates, over the complex plane; *i.e.*, in size and in slope or phase, with respect to a certain phase standard. Its readings are therefore presented in the general form $E \angle \beta^\circ$, such as $1.500 \angle 125.3^\circ$ volts. Its range, without any auxiliary multiplier, is from 0 to 1.8 volts in size, and 0 to 360° in slope. The range in frequency which it claims is from 25 to 1000 ω . The Drysdale *a-c.* potentiometer has filled a great need for a laboratory instrument capable of measuring planevector³ voltages. For many such purposes its use is invaluable. When, however, the source of testing current is an oscillator, and not an alternator, there is a difficulty in using this instrument, because the excitation of the necessary phase-shifting transformer requires more than 60 watts. When an oscillator is used as the source of alternating currents, it is necessary to reduce the power absorbed in the measuring instruments to the lowest available limits. The use of oscillators for such purposes is steadily increasing. It then becomes imperative to adopt such a form of *a-c.* potentiometer as will avoid the use of an electromagnetic phase-shifting device.

Prof. A. Larsen described in 1910,⁴ a form of two-dimensional potentiometer, the connections of which appear in Fig. 1. It consists of a non-inductive resistance *AB*, in series with the primary winding of an induction coil *BC*, an adjustable portion of the

¹ "Die elektrischen Vorgänge in den Fernsprechleitungen und-Apparaten," by Ad. Franke, Berlin, 1891. Thesis towards the Doctorate at Berlin University.

² "The Use of the Potentiometer on Alternating-Current Circuits." *Phil. Mag.*, Vol. 17, p. 402, Mar., 1909; also *Proc. Phys. Soc.*, London, Vol. 21, p. 561, 1909; also *The Electrician*, Vol. 63, p. 8, April 16, 1909; and Vol. 71, pp. 687-690, Aug. 1, 1913.

³ "A planevector" may be defined as a geometrically directed complex quantity in a plane of reference, and subject to the laws of complex arithmetic, as distinguished from a "vector" which is subject to the laws of vector arithmetic and is not necessarily confined to a plane. In this paper, the term "vector" is used as an abbreviation for "planevector."

⁴ A. Larsen: "Der Komplexe Kompensator, ein Apparat zur Messung von Wechselströmen durch Kompensation." *Elek. Zeitschrift*, 13th Oct., 1910, Vol. 31, pp. 1039-1041; also *The Electrical World*, Vol. 56, Nov. 3, 1910, pp. 1085-1088.

secondary winding being connected in series with an adjustable portion of the resistance AB , so that the vector sum of the resistance drop in AB between M_1 and K_1 , and the mutual reactance drop between M_2 and K_2 , shall be equal to the planevector $p.d.$ to be measured, between leads P_1P_2 , as determined by silence in the telephone.

The construction of a resistance-mutual-inductance potentiometer, on the Larsen principle, was taken up in 1916, in the electrical-engineering research laboratories of the Massachusetts Institute of Technology, by Mr. Alfred E. Hanson, in his thesis

FIG. 1.

Connections of Larsen potentiometer.

work towards a master's degree. In connection with this thesis,⁵ the new type of potentiometer here described was designed.

New Instrument.—The electrical connections of the new instrument are indicated in Fig. 2, and are in all essentials the same as in Fig. 1. The resistance OA contains 50 ohms, in short coils, of $Ia-Ia$ wire, wound both anti-inductively and anti-condensively, and also a short length of slide wire of 0.5 ohm. The mutual inductance coil has two equal copper windings, each having 8.8 ohms resistance at 20° C., and 3.9 millihenrys self-inductance.

⁵ "The Design and Construction of an Alternating-Current Potentiometer," by Alfred E. Hanson, September, 1916; a Thesis towards the degree of M.S. in Electrical Engineering at the Mass. Inst. of Technology.

The mutual inductance between the two windings is approximately 3.85 millihenrys. The *p.d.* to be measured is connected to the terminals *pp'*. A vibration galvanometer *V.G.* serves as the balance indicator.

FIG. 2.

Diagram of connections in new potentiometer.

Fig. 3 shows the general appearance of the Hanson form of potentiometer. The dial switches *A* and *B* control the resistance taps; while the slider *C*, moving over the resistance wire, serves for a fine adjustment of resistance. The dial switches *E*, *F*, and *G*, control secondary taps in the mutual inductor, *E* being for 200 turns per step, *F* 20 turns per step, and *G* 2 turns per step. A fraction of one turn may be added, by turning the handle *f*, as will be described later.

The interior of the mutual inductance box is shown in Fig. 4. The winding is toroidal in form, comprising 41 wooden sectors of suitable taper, as shown in Fig. 5b. The dimensions of one sector appear in Fig. 5a.

Each sector is first wound with a primary winding of 50 turns of No. 21 B. and S. gage double silk-covered copper wire (bare diam. 0.72 mm.) in 2 layers of 25 turns each. Over this primary winding is laid one layer of varnished cambric insulation. Over this insulation is a secondary winding, also 50 turns of the same size wire, in 2 layers of 25 turns each. In some of the bobbins, taps have to be brought out from the secondary winding. The mean diameter of the toroid is 36.8 cm. When all the sectors are assembled, and connected in series, the winding forms a

FIG. 3.

General view of potentiometer.

FIG. 4.

Interior of mutual inductor.

FIG. 5a.

Details of one sector of the mutual inductor. Dimensions in millimeters.

FIG. 5b.

Plan diagram of toroidal mutual inductor. Dimensions in millimeters..

closed circular solenoid with wooden core, devoid of screws or other metallic attachments. A wooden disk dd , Fig. 4, serves to support all the individual sectors in their proper places, when glued in position at DD . The advantages of the toroidal form of winding are, first, that the external magnetic field is then negligibly small, so that the primary current produces no appreciable stray field in the neighborhood of the apparatus; and second, that with accurate mechanical construction, the mutual inductance between the primary winding and the secondary coils should be proportional to their number of turns. One of the individual wooden

FIG. 6.

Range of impedance covered by the potentiometer, at $\omega = 10,000$ radians per second.

sectors is shown at e , held in a winding clamp. The handle f connects mechanically, by a spindle, with a single turn of secondary winding, situated in a hollow space at the centre of the core, between two adjacent coil sectors. The range of rotation of f , between the limits of -90° and $+90^\circ$, or half a turn, provides the same total change of mutual inductance as one step in the lowest dial.

Rectangular Range of Instrument.—Fig. 6 is a rectangular-coordinate diagram, indicating the range in impedance available with the apparatus for potentiometer measurement. The resistance $OR = 50.5$ ohms, and at $\omega = 10,000$ radians per second ($f = 1591 \text{ } \omega$), $j M \omega = j 38.5$ ohms. At any other frequency, the value of the mutual reactance will be varied proportionately.

Any impedance at $\omega = 10,000$, within the area $ORAX$ on the diagram, can be covered by the simple series connection of resistance and secondary winding shown in Fig. 2.

Fig. 7 is a similar rectangular coördinate diagram, indicating the range in planevector voltage available with the apparatus for potentiometer measurement. If the *r.m.s.* current I_P supplied to the instrument, is 0.1 ampère, or 100 milliamperes, taken at standard phase, and also at the frequency of reference ($\omega = 10,000$), the voltage that can be covered in the first quadrant is

Rectangular range of planevector voltage covered at $\omega = 10,000$ and $I_P = 0.1$ ampère.

comprised within the rectangle $ORAX$, and so the maximum *p.d.* within reach of the potentiometer is $5.05 + j\ 3.85$ volts. At any other current strength, the voltage developed in the apparatus will be varied proportionately. Any voltage within the field $ABCD$, Fig. 7, can similarly be covered at $\omega = 10,000$, and $I_P = 0.1$ ampère, by suitably reversing either the R or jX components, or both. These reversals are easily made, by means of switches j and k , Fig. 4.

Electrical Connections.—The full connections of the instrument are indicated in Fig. 8. The coil C is the secondary winding of an oscillator of adjustable frequency, which supplies testing current to the apparatus. The resistance R' and the condenser C' are used to adjust the potentiometer current in size and in slope.

The switch j serves to reverse both R and jX . The switch k reverses jX separately. B is a compensating rheostat, so arranged as to keep the total resistance in the circuit constant, while changing the resistance between the tapping points. The unknown $p.d.$ is connected to the terminals pp' . The vibration galvanometer, by means of which a balance is secured between the adjusted and unknown vector $p.d.s$, is indicated at $V.G.$

Method of Measurement.—The vibration galvanometer is at first heavily shunted, and is tuned to maximum response for the impressed frequency. The correct quadrant to employ is first found by using switches j and k . Successive adjustments are then made in R and jX , until the vibration galvanometer deflection is a

FIG. 8.

Detailed connections of the potentiometer.

minimum. The galvanometer shunt is now reduced, and the adjustments of R and jX repeated, until a sufficiently satisfactory zero balance has been obtained on the galvanometer.

Absolute and Relative Measurements of P.D.—Two general methods are available for use with potentiometers such as the one here described; namely,

- (1) The method of absolute potential differences, secured by passing a measured constant current through the potentiometer, which current is taken as of standard phase.
- (2) The method of relative potential differences, secured by passing a constant, but unmeasured current, through the potentiometer, whose readings are then uncalibrated, but which become capable of calibration, by being compared with the reading across a certain impedance which is maintained as a standard of reference.

The first method has the advantage that it enables the potenti-

ometer readings to be stated directly in rectangular-coördinate volts. It has, however, the disadvantage of requiring the measurement of the potentiometer current to an even greater degree of precision than that aimed at in the potentiometer work.

In most laboratory measurements of alternating *p.d.*, relative

Simplified diagram of connections, showing the rectangular potentiometer *P* arranged for exploration of the potential distribution over the working circuit *W*.

values only are required, and thus it is the ratio of two measured voltages which is of immediate importance. Such ratios can be determined by the relative method, without the necessity of measuring the potentiometer current. The absolute method has, however, to be resorted to, when, as in iron-cored instruments carrying alternating currents, the impedance is a function of the

current strength, and therefore departs from Ohm's law. In all measurements of *p.d.* in circuits which obey Ohm's law, the second or relative method is to be preferred. Both methods of using the new instrument have been tried.⁶ In the absolute method, the potentiometer current was measured by a differential dynamometer.⁷ This plan was found to work well, although a considerable amount of care was required for the measurement of the potentiometer current. In the progress of the experimental work, however, the relative method was found to be increasingly advantageous.

The essential connections of the second or relative method are presented in Fig. 9. The oscillator coil or source of alternating *emf.* is marked *O*. This coil is inductively connected with each of two circuits; namely (1) the potentiometer circuit *P*, and (2) the tested or working circuit *W*. The *P* circuit contains the tuning condenser *C'*, and the adjustable resistor *R'*, as well as the potentiometer *abc*. The *W* circuit may contain any set of apparatus in which potential differences are to be measured. As shown in the figure, the *p.d.* at the terminals *AD* is being led to the potentiometer through the tuned vibration galvanometer *VG*. In the second or relative method of testing, the drop of potential between terminals *B* and *C* on resistance *r* may be used as the standard *p.d.* Then if the device to be tested in the working circuit is the condenser *c*, the terminals *AB* can be connected immediately afterwards to the potentiometer. As will be shown later, it is important to maintain a ground connection at the point *A*. Consequently, the drop across any single element, such as *BC*, is obtainable as a vector difference between that across *AC* and that across *AB*.

Let *r* be the standard impedance of known slope (which in the case of a strictly non-inductive resistance would be zero) (ohms $\angle$)

Z be the unknown impedance of the condenser *c* to be measured (ohms $\angle$)

I_P be the *r.m.s.* current in the potentiometer circuit (ampères $\angle$)

I_W be the *r.m.s.* current in the working circuit (ampères $\angle$)

R₁ + jX₁ the reading of the potentiometer across terminals *AC*.

R₂ + jX₂ the reading of the potentiometer across terminals *AB*.

⁶ "Alternating-Current Potentiometry at Telephonic Frequencies," by Edy Velander; a thesis towards the A.M. degree at Harvard University, June, 1918.

⁷ "Precise Measurements of Alternating Currents," by C. O. Gibbon, *Electrical World*, May 11, 1918, pp. 979-981.

Then the drop across the standard resistance r will be

$$I_P(R_1 + jX_1) - I_P(R_2 + jX_2) = I_P(R_0 + jX_0) = I_W r \dots \text{volts} \angle (1)$$

In the AB connection, we have

$$I_P(R_2 + jX_2) = I_W Z \dots \text{volts} \angle (2)$$

Dividing (2) by (1)

$$\frac{Z}{r} = \frac{R_2 + jX_2}{R_0 + jX_0} \dots \text{numeric} \angle (3)$$

This procedure assumes that I_P remains the same in both tests, and also I_W . When, however, a high degree of accuracy is required, this assumption cannot be made. Suppose that in the first test I'_P is the vector potentiometer current, and I'_W the vector working current. Next suppose that when the second measurement is made, both the sizes and the slopes of these currents may have slightly changed, so that I''_P is the new potentiometer current, and I''_W the new working current. Then rewriting (1) and (2) accordingly, we have

$$\frac{Z + r}{r} \cdot \frac{I'_W}{I''_W} = \frac{R_1 + jX_1}{R_2 + jX_2} \cdot \frac{I'_P}{I''_P} \dots \text{numeric} \angle (4)$$

If, however, the potentiometer current and the working current vary together, so that the vector ratios remain equal:

$$\frac{I'_W}{I''_W} = \frac{I'_P}{I''_P} \dots \text{numeric} \angle (5)$$

Then equation (3) will still hold. If, then, the mutual inductances between the primary coil, and each of the two secondary coils K_P and K_W remain constant, also the mutual inductance between K_P and K_W ; likewise the impedance in each of these circuits; then any variation of current in the primary coil, due to unsteadiness in the oscillator, will not affect the vector ratios of equation (5), and will therefore not affect formula (3). If, however, the frequency impressed by the oscillator varies, then the above reasoning will not hold, and the method will be vitiated. It is therefore necessary that the frequency should be maintained constant within satisfactorily close limits. It should be noted that during $a-c$ potentiometer tests, no appreciable change should

be made in the load upon the oscillator; because any change in this load produces some corresponding change in the oscillator frequency.

Frequency Measurements.—It will be evident from the foregoing, that careful measurements of the impressed frequency are necessary in the use of the *a-c* potentiometer.

Probably the simplest device for the measurement of the impressed frequency is the Campbell condenser and mutual inductance branch circuit⁸ of Fig. 10. Here a condenser of C farads is shunted by the secondary coil of a mutual inductance of M henrys and a vibration galvanometer VG . One or both of these

FIG. 10.

Campbell frequency-measuring arrangement.

elements is varied, until the galvanometer shows no current. We then have, if I is the vector current in the circuit (ampères $\angle$)

$$jM\omega I + I \frac{1}{j\omega C} = 0 \dots\dots\dots \text{volts } \angle (6)$$

whence

$$\omega = \frac{1}{\sqrt{MC}} \dots\dots\dots \text{radians / sec. } (7)$$

Strictly speaking, this zero current in the vibration galvanometer can only be secured if (1) there is no effective resistance in the condenser, and (2) no effective capacitance in the secondary coil. Owing to the great difficulty of securing these conditions, to the necessary degree of precision, a sharp zero balance is hard to obtain. The modification of connections shown in Fig. 11 enables this difficulty to be overcome. A large resistance R , of say 2000 ohms, is connected through an adjustable small inductance l , as a shunt to both the condenser and mutual-inductance primary. The secondary is led through the vibration

⁸ A. Campbell. *Phil. Mag.*, May, 1908, Vol. 15, p. 166.

galvanometer to an adjustable tap on the resistance. If the condenser current is represented by I_c *r.m.s.* ampères $\angle$, and the current in the resistance by I_R *r.m.s.* ampères $\angle$, then with V_{AB} *r.m.s.* volts $\angle$ as the *p.d.* between *A* and *B*, at balance,

$$I_c = \frac{V_{AB}}{r + j \left(L\omega - \frac{1}{C\omega} \right)} \dots \dots \dots \text{ampères } \angle \quad (8)$$

and

$$I_R = \frac{V_{AB}}{R + j\omega} \dots \dots \dots \text{ampères } \angle \quad (9)$$

r being the effective resistance of the secondary winding in ohms. The *p.d.* in the galvanometer circuit will be

$$jM\omega I_c + kRI_R = 0 \dots \dots \dots \text{volts } \angle \quad (10)$$

where kR is the resistance included between *B* and the tapping point. After this zero balance has been obtained, first by assigning *k* and *C*, with a final adjustment in *M* and *l*, we have

$$\omega = \frac{1}{\sqrt{C \left(L + \frac{M}{k} \right)}} \dots \dots \dots \text{radians / sec.} \quad (11)$$

with the further condition that *l* has had to be adjusted to

$$l = CRr \left(1 + \frac{kL}{M} \right) \dots \dots \dots \text{henrys} \quad (12)$$

This method enables a sharp balance to be obtained at the expense of the additional adjustments. Moreover, since $1/k$ may conveniently be made a large number, *C* and *M* can both be kept reasonably small, even at low frequencies.

The same connections are presented in Fig. 12, under the form of a generalized Heaviside bridge. Formulas (8) to (12) apply equally well to this case.

In both Figs. 11 and 12, a ground connection is indicated in the frequency measuring set, at the point *B*. As will be mentioned in connection with sources of error in the use of the potentiometer, the selection of a proper ground point is vital to the accuracy of the measurements. At frequencies higher than, say, 100 cycles per second, parasitic alternating currents, due to distributed capacitance in the apparatus, play an increasingly prominent part. At very high frequencies, such parasitic currents may even swamp the working current. Either the point *A*, or the point *B* may be grounded in Figs. 11 and 12; but prefer-

FIG. 11.

Preferred modification of frequency measurer.

FIG. 12.

Bridge arrangement of connections in Fig. 11.

FIG. 13.

Pilotron connections; skeletron plan of connections.

ably B , owing to the preponderating distributed capacitance of the mutual inductor.

Oscillator.—A convenient set of vacuum-tube generator connections appears in Fig. 13. V is a 3-electrode vacuum-tube of a pliotron oscillator or similar type, with its plate, grid and filament. The main oscillation circuit consists of the adjustable condenser C and inductance L . The grid-coil secondary winding FG supplies the oscillating excitation to the grid. The secondary

FIG. 14.

Arrangement of the pliotron as a generator.

winding AB may be connected to the potentiometer circuit. Several such secondary coils may be used. The continuous *emf.* may be supplied by a small dynamo generator. If possible, this generator should be driven by a motor actuated by storage battery for steadiness of action. An auxiliary condenser of $2\mu f$ serves as a high-frequency bridge across the inductance of the generator.

A more detailed set of connections, showing an actual arrange-

ment employed, appears in Fig. 14. In this case, 240 volts of continuous plate *emf.* was derived from a storage battery, and 160 volts from a generator which was storage-battery-motor driven.

A Vreeland oscillator was also used in the earlier measurements. This apparatus is too well known to need special description. With the Vreeland oscillator employed, the available range of frequency, without external auxiliary apparatus, was from 400 $\sim$ to 2500 $\sim$. With the plotron oscillator, the available range of frequency was limited only by the capacitances and inductances employed in the oscillation circuit.

Vibration Galvanometer.—In nearly all of the measurements made with the apparatus, a Duddell bifilar vibration galvanometer, adapted for use up to a frequency of 2000 $\sim$, was employed as the balance detector. This served the purpose satisfactorily when properly tuned to the impressed frequency. When *p.d.* measurements are made upon apparatus of high internal impedance, a detector of greater sensitivity and corresponding impedance would be preferable. A pair of head telephones of suitable impedance were found to work well in such cases. A crystal detector with *d-c.* galvanometer was also used successfully.

Example of the Use of the Instrument in a Simple A-C. Circuit.—As an example of the measurements that may be made with the instrument, we may consider the simple series circuit *ABCDEF* in Fig. 15, consisting of a fixed mica condenser *AB* of 1.0 microfarad, in series with an anti-inductive metallic resistance *BC* of 200 ohms, followed by a non-ferric inductance *CD* of 100 millihenrys and 22.2 ohms, followed again by an anti-inductive resistance *DE* of 200 ohms similar to *BC*, and finally by a second mica condenser *EF* of 1.0 microfarad.

The electrical connections for exploring the potentials along the work series *AF*, Fig. 15, are indicated in Fig. 9. When using the absolute method, the differential dynamometer, *Dy*, measures the *r.m.s.* alternating current strength supplied in the potentiometer circuit, and thus furnishes the voltage scale $E = IR + jIX$, obtained for any reading of *R* and *jX*. Fig. 16 shows the results actually obtained in this way, at an impressed frequency of 500 $\sim$, with a current of 0.020 ampère in the potentiometer circuit. The grounded point *A* was kept fixed, and measurements were made by shifting the lead of the tuned vibration galvanometer along the points *B*, *C*, *D*, *E* and *F* in succession. These vector *p.d.s* are indicated in Fig. 16 by the vectors *AB*, *AC* . . .

AF , respectively. The straight lines connecting the adjoining pairs of points BC , CD , DE and EF on the diagram, then become the inferred vector potential drops in the successive elements of the circuit, all presented to the particular phase of the potentiometer current as standard.

It will be observed that the lines BC and DE are parallel. The direction of these lines shows the phase of the current in the working circuit, assuming that the anti-inductive resistances BC and DE may be regarded as pure resistances, at the test frequency

FIG. 15

Simple series circuit explored.

of 500ω . The lines AB and EF on the diagram are also parallel to each other, and are perpendicular to the lines BC and DE . This shows that the voltages at the terminals of the condensers lag substantially 90° behind the current in the working circuit, according to the regular theory. The line CD is the drop in the inductance coil, and may be analyzed into two mutually perpendicular components; namely, $D'D$ in phase with the current, due to the presence of resistance, and CD' in leading quadrature with the current, due to the inductive reactance of the coil.

If we replot the diagram to working-current standard phase,

as in Fig. 17, we rectify the diagram by a virtual rotation of approximately 65° in the clockwise direction. The various voltage drops are then read off, to rectangular coördinates, in the customary manner.

The diagram of Fig. 17 gives a correct presentation of the voltage drops in each or any series combination of the elements in the working circuit, provided that the potentiometer current I_P has been measured. In order to evaluate the impedance of the

FIG. 16.

Distribution of planevector voltage along series AF, Fig. 15, at 500 ω , to potentiometer current standard phase.

corresponding elements, it becomes also necessary to know either the magnitude of I_w , the working current; or, the vector impedance of one of the elements. Thus, if the resistance BC is a standard resistance of 200 ohms, the scale of the entire diagram can be interpreted in ohms. It should be observed that the measurement of I_P , the potentiometer current, with R and jX , is insufficient to determine the impedances of the elements in the working circuit, and that while it would be possible to measure both I_w and I_P , this double operation becomes burdensome and

unnecessary, so that the use of the instrument naturally leads to the relative method of measurement, in which no current strength has to be determined, but a standard impedance—preferably a standard pure resistance—is included in the working circuit. Measured potential drops on unknown impedances are then referred to the measured drop on this standard, as a working unit.

FIG. 17.

Distribution of planevector voltage in Fig. 16, referred to working current standard phase.

Sources of Error and Their Elimination.—It is impossible to work for any length of time with any *a-c.* potentiometer at a telephonic frequency, without encountering discrepancies in the measurements, which are attributable to the well-known disturbance from parasitic currents in the distributed capacitance of the tested apparatus. The first step in the elimination of these superposed parasitic currents is to ground a suitable point in the working circuit, so as to bring that point to zero potential, and prevent the distributed capacitance in the immediate neigh-

borhood of that point from producing a parasitic condenser current. In general, the ground should be established at one end of the range of the elements in the working circuit to be measured. As an example, Fig. 9 shows a ground established at the point *A*.

Suppose that the ground is established at the junction *B* of two impedances *AB* and *BC* to be compared, such as Z_1 and Z_2 in Fig. 18, and that I_W is the working current in the absence of distributed capacitance, and therefore equal in the two impe-

FIG. 18.

Diagram illustrating the effect of improper grounding.

dances. The current i'_c in Z_1 represents a small parasitic current of a certain magnitude and phase escaping to ground at *B*, while the current i''_c in Z_2 is a similar small parasitic current which, in general, will have a different magnitude and phase, also escaping to ground at *B*. The resultant working currents in the two impedances will then be the vector sums $(I_W + i'_c)$ and $(I_W - i''_c)$ amperes, respectively. The ratio of the potential drops in the two impedances will then be

$$\frac{(I_W + i'_c)Z_1}{(I_W - i''_c)Z_2}$$

which, in general, will differ from the required ratio, $\frac{Z_1}{Z_2}$ by an unknown amount.

In cases where the quantity to be determined is a potential difference distribution, rather than an impedance comparison, the importance of a proper grounding point in the working circuit is equally great. It is evident that, as a general rule, during a set of *p.d.* measurements on the same working circuit, the grounded point should not be shifted. This is for the reason that when the ground connection is shifted, the potential of all points along the circuit with respect to the ground is varied, thereby changing the entire distribution of parasitic currents.

An experimental example of the effect of shifting the ground connection in a working circuit appears in Fig. 19. Here a standard resistance box of 100,000 ohms, in 10 series coils of 10,000 ohms each, was inserted in the working circuit, as shown in Figs. 20a and 20b. In Fig. 20b, the vibration galvanometer connection to the potentiometer is carried permanently to the end *B* of the resistance box. Since it is manifestly inadmissible to establish a ground connection on this lead, as it would bring parasitic current directly through the galvanometer, the ground has to be established at *t*, the travelling lead to the potentiometer.

FIG. 19.

Distribution of planevector voltage along 100,000-ohm resistance box. Small circles indicate observed values, crosses indicate computed values, assuming that each section of the box subtends a hyperbolic angle of $0.08 \angle 45^\circ$ hyp. radian.

This involves, however, a change in the distribution of potential and capacitance current at each change in the position of *T*. The result is shown in the broken curve 1', 2', 3' of Fig. 19. The curve of *p.d.* between *A* and *B*, Fig. 20, should evidently be a uniform straight line *Oio'* in Fig. 19. The presence of the changing parasitic current distribution appears to have been responsible for the erratic curve shown. Assuming that the *p.d.* across the entire resistance box is correctly represented by the vector *Oio'*, Fig. 19, then all of the intermediate *p.d.s* should be

situated along this straight line; whereas the actually measured *p.d.s* from *O* to the successive points 1, 2, 3, etc., Fig. 20b, follow the bending line 1', 2', 3' . . . , etc.

The connections were then changed to those of Fig. 20a, in which the ground connection is kept fixed at *B*, and the travelling

Exploration by means of the potentiometer *P*, of the voltage drop along a high-resistance box *A B*, of 100,000 ohms. Note correct position of ground-connection at *B*.

lead contains the vibration galvanometer. The corresponding vector curve of *p.d.s* is shown at 1, 2, 3, 4, Fig. 19, and is a much closer approximation to a straight-line vector *OA*, representing the *p.d.* over the entire resistance. This curve resembles in form that presented by an artificial cable when an alternating *emf.*,

such as *OA* Fig. 19, is impressed on the sending end, while the distant end is grounded. The curve indicates that each section of the box subtends a small hyperbolic angle.

Applications of the New Form of A-C. Potentiometer.—A large field of experimental investigation in the laboratory lies

FIG. 20b.

Incorrect arrangement of ground-connection giving rise to the erroneous results
1', 2', 3' in Fig. 19.

open to the *a-c.* rectangular potentiometer. The power it consumes is a fraction of 1 watt, while its range of voltage is extensive. It enables the vector *p.d.* on different parts of an alternating-current instrument, or on different sections of an alternating-current line, to be measured conveniently. The results are also of considerable educational value to the student. It is pro-

posed to communicate in another paper a number of the results obtained experimentally at various telephonic frequencies.⁹

It should be pointed out that the degree of precision obtainable with the instrument, although very satisfactory from an alternating-current standpoint, is not to be compared with that readily obtainable from the *d-c.* potentiometer. This relatively low precision is attributable to parasitic currents from distributed capacitance, and also to stray alternating magnetic fields. If these two sources of error could be eliminated, the precision of the instrument might be of the same order as that of the *d-c.* potentiometer.

Suggestions for Improvements in Construction.—A weak point in the design of the new instrument is the relatively high internal capacitance between the primary and secondary windings of the mutual inductor, which, at high frequencies, tends to produce impurity in the vector mutual reactance jX . This defect could be reduced by increasing the insulating spacing between the concentric primary and secondary windings on the toroid. At 2000 ω , the discrepancy due to this mutual capacitance appears by measurement, in this instrument, to be $(0.25 + j1.2)$ per cent. in voltage, but rapidly diminishes as the frequency is reduced. There is also a weak point in the toroidal form of mutual inductor, that although there is negligibly small stray alternating field produced in the apparatus by an exciting primary current, yet the secondary winding is not exempt from the influence of stray magnetic fields from external sources at testing frequency. Precautions should therefore be taken to remove the apparatus from the vicinity of alternating magnetic fields.

SUMMARY.

- (1) The principle of the *a-c.* rectangular potentiometer here described is not new, but the form of the instrument appears to be new.
- (2) By the use of this instrument, alternating *p.d.s* can be measured, to rectangular coördinates, up to at least 2000 ω , with only a small expenditure of power.
- (3) The use of the instrument shows the marked influence of distributed capacitance in *a-c.* apparatus, and the importance of reducing this disturbing effect when measurements are made.

⁹ *Proc. American Philosophical Society*, 1919.

- (4) The importance of a proper ground connection in the working circuit is emphasized.
 - (5) The *a-c.* rectangular potentiometer escapes the necessity of measuring the strength of alternating current in either the potentiometer circuit, or the working circuit, provided that the relative method is used, and that the impressed frequency is maintained constant.
 - (6) Owing to the effects of distributed capacitance, the voltage distribution in a simple series resistance box, carrying alternating currents, fails to follow a vector straight-line law. The deviation tends to increase with the impressed frequency.
 - (7) The importance of reducing the mutual capacitance between the concentric primary and secondary windings of the toroidal induction coil, at high frequencies, is emphasized as the result of experimental tests.
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Analysis of Statically Indeterminate Structures by the Slope Deflection Method. M. W. WILSON, F. E. RICHART and CAMILLO WEISS. (*Bulletin No. 108, Engineering Experiment Station, University of Illinois, 1919.*)—In recent years rectangular steel frames with riveted joints have been used, and many types of monolithic reinforced concrete frames have been developed. The use of statically indeterminate stresses cannot, accordingly, always be avoided. Structures are frequently made of an indeterminate type to secure economy of material. It is felt that analyses of statically indeterminate structures are desirable since such information will do much to inspire confidence in the reliability and in the economy of such structures.

An investigation of statically indeterminate structures has been conducted by the Engineering Experiment Station of the University of Illinois to obtain a convenient method of analyzing the moments, stresses and deflections for a number of typical structures. The analyses were based upon the assumptions that the connections are perfectly rigid, that the length of a member of a rectangular frame is not changed by axial stress and that the shearing deformation is zero. The method has been explained in sufficient detail to enable the designing engineer to use it in the solution of his problems. It is believed that the fundamental principles presented may be readily coördinated with the ordinary principles of mechanics so that the more complex and even the simpler problems may be studied from a new viewpoint.