TUBES WITH SIDES AND WITHOUT, IN SHIP RESISTANCE—AN EXAMPLE FROM LORD KELVIN.

By MARSTON NILES.

1. LESS than thirty years ago the present or stream-line theory of ship resistance was adopted by naval architects, but it has long been held by mathematicians. It is shown in the six graceful curves of Fig. 1, taken from the treatises of Naval Constructor Taylor, U. S. N., and Prof. Durand of Cornell University, and in the straight lines into which the curves turn at a distance. By a single horizontal cut through her at the surface of the water, the vessel has lost all her upper portion. Water and ship are covered with an unbending sheet of frictionless ice, and beneath this and touching it she is steering north. A like sheet, parallel with the other, lies along her bottom. Both of these stretch to a very great distance on every side, and prevent all up and down movements. The curved lines seen about the vessel are the nearest members of an endless series which slice the whole expanse into so many independent streams. We might slice it into a hundred or a thousand times as many, all not so broad as deep, but none of them carrying more water than another; we might even have the streams no wider than a single file of particles. The lines or banks which divide them are immensely long strips of frictionless cast iron which in the figure are beheld edgewise through the ice. They are insensibly thin but unyielding. Each is fastened to the others, and all are fastened to the ship, and they and she are marching as a single body. Along their upper and lower edges the iron strips touch so evenly the ice-sheets between which they are traveling that there is no leak between ice and iron, yet the whole configuration slides smoothly northward. The skin of the ship, *skin*, and the mid-line, *n b*, taken together, form the inner bank of the inmost "port" stream, and in this stream the invisible innermost file of particles (which may or may not be reckoned a stream by itself) *hugs this bank and everywhere touches it*.

2. Ahead of the ship, a glance would show the strips drawing together a little at the first, but every inch of nothing brings them straighter and more nearly parallel. At some length ahead (as shown in the diagram) they become sensibly both straight and parallel. The bulge or broadening just off the bow at *i n*, made by the excess of pressure there, gives room for the congestion caused by the shock of the particles upon the bow and the consequent delay. Hard by the ship is where the difference of pressure between the streams is found the most striking. Any pressure difference between two neighboring streams is pointed out (and indeed is proved) by a corresponding difference between their breadths. Fig. 2 shows the system in miniature. Right ahead and off the bows is the greatest pressure, and there the streams are broadest, and each stream is broader and has a heavier pressure than its neighbor on the outboard hand, *i. e.*, than the stream next farther from the mid-line, *b n*. Along the mid-line the pressure is the heaviest of all. The weakest pressure is abeam, at *e, e', e''*, etc., and there the streams are narrowest, and each is narrower than its next outboard neighbor and has a weaker pressure. In the distant offing everywhere the streams are sensibly straight, of equal breadth, and parallel.

3. Of course each pair of strips, along with the top and bottom ice-sheets between which they and the ship are marching, forms a water-tight pipe, or "tube" as it is called, of right-angled cross-section and enormous length. Returning to Fig. 1, we see that at a point well north, *e. g.*, between *p* and *1*, the tube has become dead straight; and from there onward the breadth of its cross-section is found no narrower. Still further in the north each tube is fitted with a smoothly-slipping slug or piston, *p, p'*, etc. On its north side each piston has a very long handle, *h, h'*, etc.; and all handles fasten stiff to a single cross-piece lying due east and west. If we should push this cross-piece steadily southward, the pistons would be forced down the tubes toward the vessel through equal lengths and with equal swiftness. If we do not wish to get the moving force from pistons, we can do away with those and have the gush of sundry springs or "sources" welling out of the far north. The springs all have an equal strength of flow; they lie side by side, in row east and west, each spring answering to its own tube exactly; the tube goes well into the vein of the spring and the fit is cemented.

4. The motion is seen more easily from on board the ship, if we let her and the tubes stand fast and have the water flow past us. This is opposite to what we began with; yet all motion is but relative, and so the theory is wholly justified. The water now rushes down the tubes before the steady push of the pistons, or before the steady flow of the springs. Look at the inmost port tube, *p 1 x e*, in Fig. 1. It has to stay with the ship, however strong the flow. It will not stretch out longer. The shape or bend of it cannot be changed. The water is incompressible and the piston (or the spring-flow) is irresistible. Particles which have been idling between *i n* and *i' n'* in the broads will be sent shooting through the narrows at *e x*. Clearly the tube is a syringe, with the narrows for nozzle. The iron has been shaped exactly to a stream-line pattern, and the stream takes that shape exactly. This way of moulding the stream to a carefully-designed figure has been adopted by Prof. Hele-Shaw in those beautiful film-experiments which have generally been thought to prove the truth of the theory, and which for other purposes are certainly invaluable.

5. With equal readiness the theory takes up the situation in reverse, *viz.*, where the vessel is what travels and the water is the standing body. To bring this about, we first brace firm the cross-piece (see Fig. 1) into which the piston-handles are mortised, and then we push the tubes and ship northward as a single body. The pistons, however, cannot move with the tubes, since their rigid handles have a backing against the well-braced cross-piece. Thus the syringe is now a force-pump having a back-action through the narrows (between *s* and *s'*) abeam of the vessel; and the motion of the particles is displayed for Eulerian

investigation. On the other hand, when we kept fast the ship-and-tube system, and sent the pistons evenly down the tubing with a square southward movement of the cross-piece, we had the Lagrangian system, in which the water is taken as flowing past a vessel at her anchors.

6. The shape of each of these tube-streams is peculiar to the stream, and its peculiarity springs from two jointly-acting causes. The first cause is a purely ideal one which the theory bids us weigh by itself, all sundered from the metal stuff which has been moulded to that pattern. It is simply the form possessed by the particular tube, a form distinguishable from that of every other tube. Only through looking upon this form as a disembodied ideal or as a conceptual essence, can we really come to understand the theory. The second cause is a state of equilibrium between the following sets of forces: One set is found in the *vis viva* of the particles as (Fig. 1) they strike against the bow at *i n*. Before this shock the bow, however massive, cannot but give ground at first, although insensibly; but its reactive forces at once awaken and they heave back against the outer wall, at *i' n'*, etc., the whole onslaught. On the other hand, the fluid particles hurled over now against that wall must bend, though insensibly, its iron; and in so doing they awaken cohesive forces which have been sleeping among the particles of the metal. Instantly these forces resist and react; and between the action of the liquid particles against the wall, *i' n'*, and the reaction of the wall's solid particles against the liquid, there comes at once the equilibrium which is essential in the theory. For without this equilibrium, the wall, *i' n'*, might be bent sensibly, or might indeed be burst; in which case it would not be able to guide the fluid, or at least not guide it as the theory requires.

7. Yet it is only for a temporary purpose that the theory has framed this complicated mechanism. The use of the coarse device of pistons and a gross material tubing is merely to help us understand the theory's fine equilibrium and its peculiar movement. Grasping these last as ideals firmly we now fling away

and excepting also with heat, electricity, etc., in all of which phenomena there is that reciprocal, vibratory or back-and-forth motion whose equilibrium swing implies a nugatory march, an average equipoise and a general equipressure. The thing is not too difficult, but it may be too simple, for instant comprehension. In hydrostatics, or fluids at rest, an equality among the pressures from all quarters upon every particle is what keeps each particle motionless and maintains the hydrostatic plight. In hydrodynamics or moving fluids, on the other hand, a pressure from some quarter stronger than from others is what compels the motion and brings in the hydrodynamic rôle. For particles are not self-moving. They move only as they are pressed to move. And if particles are not pressed more strongly from one direction than from others, they will move no more in one direction than in another, or in other words they will not move at all. If we part with equipressure we part with statics. Unless we part with equipressure we cannot have dynamics.

10. In his paper "On Some Cases of Fluid Motion" (Trans. Cambr. Phil. Soc., Vol. VIII.) Sir George Stokes makes the following keen observation: "The equations of hydrostatics are founded on the principles that the mutual action of two adjacent elements of fluid is normal to the surface which separates them, and the pressure is equal in all directions. . . . In hydrostatics the accurate agreement of the results of our conclusions with experiments (those phenomena which depend on capillary attraction being excepted), fully justify our fundamental assumption. The same assumption is made in hydrodynamics, and from it are deduced the fundamental equations of fluid motion. But the verification of our fundamental assumption in the case of a fluid at rest does not at all prove it to be true in the case of a fluid in motion, except in the very limited case of a fluid moving as a solid."

11. This was written in 1843. Many years later, in these days of fast ships and experiments with models, the shrewd suspicion of this prince of science becomes

the stuffs or matters which had been sent into the form of tubes and pistons, and henceforth hold only to the abstract shape or pattern of the streams. A perfect equilibrium between the *vis viva* of the stream, *p 1 x e*, for example, and the *vis mortua* of its restraining wall, *i' n' b'*—an equilibrium we had earlier gotten only by stuffing the ideal wall shapes with actual materials—the theory now informs us is a native quality in every fluid, and that here the fluid particles in line will serve for a wall quite as effectively as if they had been frozen solid.

8. For the theory plants itself squarely on a hydrostatic doctrine which has been taken over into hydrodynamic science. This is that every fluid particle is pressed upon with equal force by all its neighbors in every direction. Take the line, *b' n' i' k' s'*, for example. Along this line, where earlier we had a wall of cast iron and now have nothing but a row of liquid corpuscles, the theory says there exists with every corpuscle an inborn and perfect equilibrium. It says that the pressure from shipward against the inboard side of that line, which tends at *i' n'* to drive it northwesterly or away from the vessel's bow, is exactly balanced by an equal and contrary pressure against the outboard side of it, tending to drive it southeasterly or shipward. All needless, then, the curbing metal. Equipoise alone will brace that mental or immaterial bank against the heavy bow-pressures. And along that line, says the theory, the equipoise is no less genuine and effective than that seen along the mid-line, *b n*—a line which, though equally non-materialized, we know will let no particle stray from starboard to port or from port to starboard.

9. The object of the present paper is to show, on the contrary, that the equipressure doctrine can by no means be brought into hydrodynamics, except in hydraulics or with the use of substantial pipes and tubes whose solid walls, placed *vis-à-vis* and reacting each against the other, both conserve within the stream limits the tube forces, and also bestow among them the equilibrium demanded by the equations;

fully justified by the absolute contradiction between the calculated results of the equations and the actual facts discovered by experiment. Mischief has been scented by other distinguished mathematicians. "Let us eschew the notion," said Poincaré, "that a science has been made when it has been reduced to mathematical formulas." Only two years ago, in his presidential address to the American Mathematical Society, Prof. Woodward observed as follows:

"Lagrange and most of his contemporaries in their eagerness to put mathematics on a sound analytical basis overlooked to a serious extent its more important physical basis. . . . It must not be inferred . . . that the simplicity of the phenomena of fluids in a state of relative rest extends to the phenomena of fluids in a state of relative motion; for the gap which separates hydrostatics from hydrokinetics is one which has not yet been fully bridged even by the aid of the most powerful resources of modern mathematics."

And referring to purely theoretical investigations, Prof. Osborne Reynolds has also said: "As regards the direct object in view, the revelation of the actual motion of fluids, the research has completely failed. And now that generations of mathematicians have passed away, . . . the simplest problems of fluid motion are yet unsolved." (Proc. Roy. Inst., February 2, 1877.)

12. I think it is upon the assumption of equipressure in fluid dynamics that we now may put the finger, declaring it the unsuspected but potent sire of the whole set of monstrous misconceptions and nugatory or abortive calculations. The proofs of this paternity are endless in number. Let me offer a simple specimen. In Fig. 3 we see on the right a great chamber having very strong walls. The top side, *s' s*, is a piston. On its left is a narrow cylinder whose mouth shuts tight with a cap screwed on at *s'' s'*. Both cap and mouth have the same cross-section as the cylinder. From *s'* to *k'* there is an unbending and unbreakable wall between the two ves-

sels; but from k' to i' , the only division is the thinnest possible curtain of India rubber. The opening, $i' n'$, has the same area as the cylinder's cross-section. Both vessels are full of liquid. The small circles seen on either side of the curtain may as yet be disregarded. A spiral spring, of strength equal to a ten-ton load, is placed upon the piston, begetting a fearsome pressure in the fluid of the chamber. This pressure, however, does not budge the curtain, $k' i'$, a hair's breadth. The curtain would not budge if a mountain lay upon the piston. Altering again and again the plan of the machine, we might shift the cylinder and the curtain so as to bring the curtain successively against each particle in either of these compartments. If the cap is only tight, the curtain will still be motionless.

13. The reason is that in hydrostatics, or unmoving fluids, the urgencies are not only felt all at once in every quarter, but also are instantly sent back thence, and therefore the pressures from all directions against each particle are ever equal. For along the curtain $k' i'$ (Fig. 3), on the cylinder side of it, pick out some particle, say a , that touches the curtain. If the pressure against the right side of this particle, proceeding from neighboring chamber-particles and transmitted through the yielding curtain, were greater than the pressure against the left side of it which proceeds from its companions in the cylinder, then the chamber-particles would push the cylinder-particle leftward. And in turn about, were the pressure against its left side, from its companions in the cylinder, greater than that against its right side, sent by the chamber-particles through the curtain, then the particle would be pushed over toward the right. In either case the curtain would be swayed with the pushing. In point of fact, however, the curtain does not budge. Now since both in cylinder and chamber each particle can be tried in like manner, yet never save with like result inevitable, and since with each a motion can be brought about only by more pressure in some direction than in others, and since non-motion can be maintained only through a non-excess of pressure in any direction, it follows that equipressure in all directions is the fundamental principle of fluid non-motion, i. e., of hydrostatics.

14. On the other hand, if we take off the cap and let the fluid shoot out of $s' s'$, on a sudden the curtain flies leftward. The next instant will find some part of the curtain pressed against the left wall of the cylinder, and the flow will be halted. If instead of a curtain of rubber, we have a plate of thin metal, $k' i'$, swinging on a hinge at k' , and held immovable at perpendicular until a current has been fairly established, and then let the plate go, it goes left as before. In either case the reason is, that the chamber-particles in touch with the curtain are now sending through it a heavier pressure against the side of any cylinder-particle in touch with the curtain, than its cylinder fellows are sending against its left side; therefore the cylinder-particles can only give ground and go toward the lesser pressure. It is plain now, that against any particle the pressures from different directions are not all equal. Non-equality among the pressures must in fact be reckoned the fundamental principle in the dynamics of a fluid; because non-equality, as we have seen, is what founds the *dynamis* or the Might that sends the motion.

15. We may pierce a good-sized opening, even wider than $i' n'$, in the bottom of the chamber anywhere, as from m' to m ; the spring still pressing against the piston. The curtain will go leftward as before, although less swiftly. For the motion of the chamber-particles (u, x, v , etc.) along the right side of the curtain is through a wider channel and therefore is slower than that of the cylinder-particles along its left side; and it has been established by experiment that of two neighboring streams having equal strength of flow, if other things are equal, the one whose motion is the slower has always the heavier pressure. It was by the general principle that "the average pressure is least where the average energy of motion is greatest," that Lord Kelvin explained so deftly the tremor experiments of Guyot, Prof. Guthrie and others—phenomena which some had thought would solve the mystery of gravitation (Clerk Maxwell on Attraction, Encyc. Brit., IX, ed.; Lord Kelvin in Phil. Mag., 1871).

16. A ready proof of Lord Kelvin's principle is found in the movement of the nostrils while a long swift breath is drawn in through them, the mouth being shut meanwhile. Suddenly they near each other. They do this because the streams of air-particles pressing upon them from the outside are broader and thicker, and hence flow more slowly, and therewith possess a stronger thrust sideways than those from the inside, although the latter have quite as great a strength of flow. This bringing together of the nostrils is by no means muscular, for it cannot be effected except while taking a breath.

17. Misleading, however, both in the foregoing examples and those in which Maxwell and Lord Kelvin have applied it, would seem the broad doctrine that "the average pressure is least where the average energy of motion is greatest." If this assertion may be said to account for these phenomena, does it answer the underlying question, *How?* By what actual steps? Is any assertion valid but that of some mechanical power? The unequal-velocity explanation seems like the præ-toricellian explanation of water's mysterious uprising in an ordinary pump, viz., "Nature abhorreth Vacuum." True, in a way, yet occult and not sufficiently materialized. The miles of upper air, down-thrusting on the reservoir with near a ton of push against each square foot of area, are the means whereby Nature lets Vacuum feel the weight of her displeasure. The misliking becomes impotent beyond three-and-thirty feet. But in admitting that the pressure can be less on one side of a surface in a fluid, and greater on the other side, would not Maxwell and Lord Kelvin themselves be abandoning the doctrine of equipressure which they reckon fundamental?

18. "A pressure," said Young, "is a counteracted force." Now a force cannot be counteracted but by some other and contrary-pushing force. Such counteraction may be only partial; but then also

there will be at least some resulting pressure or, in a compressible fluid, some compression. In Fig. 3 the "acting" force is the spring pressing on the piston. The "counteracting" (or *pro tanto* counteracting) force is the cohesive power a-slumber among the solid particles of the chamber-walls; which power the attack now awakens. The escape of the fluid through $i' n'$, the corner-opening, lasts for an instant, and from there to the outlet all the particles for that instant are in motion. The outlet $s' s'$, is as wide as the inlet or as any part of the cylinder. Therefore in the cylinder, the stream being once set a-going and the counteraction due to inertia being once overcome, there is no longer any counteraction and thus no longer any pressure. In the chamber, on the other hand, the particles encounter very serious counteraction; for the breadth of the relieving outlet, $i' n'$, is far narrower than the sweep of the oppressive marching piston. It is plain, then, that what sends the curtain leftward is not a slow motion in the chamber confronting swifter motion in the cylinder; but it is counteraction in the chamber confronted with lack of counteraction in the cylinder. Counteraction is but another name for pressure. In the foot of such a chamber, as in Fig. 4, pierce an opening at $m' m$; lengthen the curtain, $k' i'$, to n' ; knock out the whole bottom of the cylinder, making there at $n' n'$ the exit; and let the whole top, $s' s'$, be made a piston. Now push both pistons southward, taking care to push the piston of the cylinder more slowly than that of the chamber. If true is the doctrine that the swifter the motion in the chamber compared with that in the cylinder, the swifter leftward is the curtain's movement.

19. Our search for the cause of this phenomenon will not be over till we set eye upon the mechanism by which the curtain is driven. Returning to Fig. 3, take off the cylinder-cap, $s' s'$, and let a spring force the chamber-piston southward. The particles, u and v , for example, are now urged toward the opening, $i' n'$, yet are retarded by the counteraction sent back to them from everywhere but that comparatively narrow exit. The particle, x , itself urged onward, crowds in between them. The front half of its circumference, where it bears against their circumferences, makes practically a wedge or salient angle, and this wedge crowds into the practically re-entrant angle formed by the rearward semi-circumferences of the particles ahead, and it pries them asunder. Other particles lying along the whole breadth over to $k i$ are pried asunder in like manner, and all the stress is thrown back from $k i$ against the curtain.

20. In Fig. 1, if the bank at $s' k'$ should stand firm, as required by the theory, we should find the same phenomenon; and find it springing from the same cause. In that part of the most inboard stream 1 $x e$ which lies off the bow, the counteraction is greater than in its outboard neighbor, 2 $x' e'$, its exit being the narrower (see Sec. 2, at end); and therefore its particles, by means of that same wedge action, push northwestward or further outboard the division line, $b' n' i'$, between the streams; and thereby along this line as elsewhere they abolish the "steady motion" of the theory. A like greater narrowness, a like greater counteraction and pressure, a like wedging further outboard and a like abolishing of the "steady motion," must all be found ransacking the configuration, because every stream in it has an outboard neighbor of an equal strength of flow, whose exit ($e' x'$, $e' x''$, etc.) is less narrow (and therefore throws back northward less pressure) than its own. It is clear the only configuration that can be sensibly permanent is one in which every stream-line will everywhere find itself at balance between sensibly equal stream-breadths and thus equal pressures. But plainly the breadths cannot everywhere become sensibly equal until everywhere the streams lie sensibly parallel. If the ship's course lies in one straight line, $n b$, the only configuration in which it is geometrically possible to make all the streams sensibly parallel is one in which they are (relatively) flowing past an imperfect, very long, sensible parallelo-piped (or actual wedge) of water ahead of her bows, which is perfecting and lengthening itself to infinity and is coming to have at last no sensible motion relatively to her.

21. In the *Engineer* (London, May 25, August 10, 1900), and in the *Journal of the Society of American Engineers* (vol. xi., No. 3) may be found further demonstrations that the theory is untenable. The results of computation based upon the principles of the theory—results already known to be contradicted by the facts—were there contrasted with actual ship-waves taken by photograph. The contradictions, as I think, were traced back fairly, and with some minuteness, to the doctrine of equipressure. None of those proofs or arguments appear thus far to have been answered, nor do I know of any effort made at answer. It is improbable, however, that they have been found satisfactory in all quarters. That they have not been contested may well be owing to lack of clearness in stating them, and to the newness of the proofs as well as of my contention. I think a number of well-established "examples" in fluid motion should now be closely examined. The best authorities should be taken up in turn, and each example selected from them should be shown to be impossible; and every proof of impossibility should be made strictly ocular. Failing to make such proof, I might at any rate thereby succeed in finding an error of my own. The present paper has room only for an example from Lord Kelvin.

22. Fig. 5 shows a circular disk seen edgewise, lying east and west and perpendicular to the plane of the paper. It is at great depth in a fluid at rest, through which it is moving due north. The two filaments of (relatively) nearing particles which hug on either side the invisible or balance line of the configuration, part company at n , the middle of the disk; and (according to the theory), their corpuscles then pass to port and starboard along the disk's west-and-east front surface; which surface, says the theory, constitutes their "stream-line"; thus the stream-line of each of the pair is plotted to run here at right angles

to $b n$, the line of their relative approach just made from the northward. The port and starboard edges of the disk, originally right angles, have been rounded off with care, in order to be rid of the *infinite velocity* which the theory demands of a particle on its reaching any sharp turn in the bounding solid. "The occurrence of infinite values of the velocity may be avoided," says Prof. Lamb, "by supposing the edge to be slightly rounded" (Hydrodyn., Sec. 73); although, as he reminds us and as appears from the example now to be cited, the theory gives here still a very high velocity. The lay reader may profitably examine observations in Mr. Basset's *Hydrodynamics* (Secs. 125, 127) and in Helmholtz's "Discontinuirliche Flüssigkeitsbewegungen." It is in his paper "On the Doctrine of the Discontinuity of Fluid Motion" (Nature, vol. 50), that Lord Kelvin resorts to that device for avoiding the embarrassment of an infinite velocity. He takes "a stiff circular disk of 10 inches diameter and 1/10 of an inch thick at its middle, shaped truly to the figure of an oblate ellipsoid of revolution, . . . moved at . . . a velocity of 1 foot per second. . . . While the velocity of the fluid across the equator [i. e., across e or e_1] is 63.7 feet per second, the velocity across each of the two parallel circles [drawn at k and k_1] whose radii are 4.218 inches (the radius of the equator being 5 inches), is only 1 foot per second."

23. Yet on closer examination the east-and-west course assumed by Lord Kelvin may be seen to be impossible. In Fig. 6 it becomes clear that if at a given instant the disk is at $e k i n i, k, e_1$, and a given "port" particle at n , then when the particle shall have passed west as far as the "parallel circle" at k , the northward motion will have carried the disk on to $e' k' i' n' k', e'_1$, and the disk must have swept the particle north as far as k' . Thus the particle's path from n to the "parallel circle" will have been, not west, or in the direction $n i k$, as hitherto believed, but northwestward to k' . Moreover, the particles which later should arrive at n (to go thence westward like their fore-goers), will by no means get to that point at the time expected. For the first particles, being forced northward by the disk, will force to northward the neighbors on the north, and these last again their own. Such general northward urging, sent diagonally inboard from port and starboard (normal to $k' n, k' n_1$), will (through its due north element) affect those particles which ought (relatively) to approach along the mid-line, and it will hinder their arrival. With the disk's constant progress, this northward push (exerted also upon all the other filaments there) will constantly become intenser, and its general effect will become more near due north, and it will also spread still farther northward; making thus a more and more northerly set in all the fluid which lies due north of the whole breadth of the vessel, and wholly debarring the "steady motion" of the theory; so that a wedge-shaped mass of fluid will lengthen and sharpen itself in front of her and will also come more and more to lack motion relatively to her.

24. A like outcome is certain in the standard stream-line diagram Fig. 7. If the particle found at the cutwater, n , when the ship is at $s k i n$ could (as the theory proposes) hug the ship's side throughout, it would still be found, not at s , but at s' , on the instant the ship arrives at $s' k' i' n'$; and the path which the particle would describe in so moving, could by no means be the graceful stream-line curve $b n i k s$, as universally believed, but the jag $b n s'$, instead. In order, however, for it to follow even the jag, it would have to travel within an actual tube laid along the line $n i k s$, a tube whose then substantial northwestern wall would fling back shipward the northwestern pushes upon the particles bestowed by the ship's skin (which skin would then form the tube's opposite or southeastern wall); otherwise, as will be seen in sec. 29, the particle will be crowded still further on to the north.

25. If, however, there be given indeed the equilibrium which the theory takes for granted, the equations straightway tell a truthful story. In Fig. 8 let $E I S$ be a mass of colorless mathematical liquid frozen to a solid. It lies just north of Lord Kelvin's disk, thence parted by only the narrowest imaginable vein of colored liquid. The solid mass is braced so that it is immovable. Hence, though the disk is moving northward, the particles of the vein are yet in equilibrium between two equal opposing forces, viz., the disk's northward action and the southward reaction of the immovable solid in front. This vein being the narrowest imaginable, will be emptied by the disk's advance in the shortest imaginable time—a time not practically distinguishable from no time at all. Instantaneous in a sense, then, will have to be the journey of a particle from n , the middle point in front, to Lord Kelvin's parallel circle at k . Due west, practically, will also be its course, or indistinguishable from that predicted by the theory. Now this novel agreement with the theory will be owing solely to the newly-furnished equilibrium. For abolish in any way the equilibrium; for example in the following manner: Let the ice-mass be braced no longer, except by its own inertia, but let the motion of the disk be persistent; and let the ice-mass melt suddenly to a perfect fluid. Then before the thrust of the advancing disk, the melted particles will at once give ground in every direction, but especially toward the north. In the just-melted mass there being now no persistent southward reaction, but solely its own waning inertia, the particles of the colored vein will no longer find themselves in equilibrium, and evidently they will no longer travel due east and west as plotted by the theory, but will be borne northward, as well, along with the yielding melted particles north of them. And with both colored and colorless particles this new behavior will take place by virtue merely of *their fluid quality*—of their being by definition "a substance which if continuously acted upon by deforming force, yields continuously with continuously increasing deformation, so that the ratio of deformation to deforming force is not constant but increases with the lapse of time"—a definition, by the way, quite incompatible with the equipressure doctrine. Thus their motion, which the theory asserts is "steady," can be steady in

that novel sense alone that it is changing steadily in direction and that steadily it dwindles relatively to the advancing disk.

26. In Fig. 5 then, the theory's west course along the disk's front is (finitely) not possible, on account of the disk's meanwhile northward motion. In order to make good the course prescribed by the theory, viz., $b n i k$, the particles, being unable to pass through the advancing disk, must have, along the disk's front, that same infinite velocity which, after discovering it at the corners, the masters have hitherto escaped in that neighborhood by the device of rounding them. But with an infinite velocity, as well observed by Mr. Basset (Hydrodyn., sec. 125), there comes of need an infinite minus pressure, indicating that a hollow has now been formed.

"It would be necessary," he says, "that at every point of the liquid hollow the pressure should be constant, and therefore the liquid boundary would have to be a line of constant pressure as well as a stream-line; but it is not difficult to show from the formulæ that it is not possible for a line of constant pressure to coincide with a stream-line, and hence the formulæ fail."

He is writing, it is true, of sharp edges or square corners, and not of this straight course conceived as lying along the front of the disk; for till now, I think, no infinite velocity has ever been pointed out as here inevitable; but his observation is quite just in respect to all infinite velocity, whatever the site. Rounded corners, then, cannot save the formulæ.

27. The same insuperable objection intervenes with the ship-shaped solid which Constructor Taylor's brilliant expedient (Trans. Inst. Nav. Arch., 1894) has enabled the theory to treat mathematically as it had treated the old circular and elliptical forms. But no expedient can reconcile fact with fact-denial. In Fig. 9, although the scale is too small to show the "vein" along the vessel's waist, we yet can see the same proof which is afforded by the disk of Lord Kelvin. On the north of the vessel lies the frozen fluid $E I S$, whose south side, $s' k' i' n' i', k', s'$, has been carved out to the same ship-shaped form. As in Fig. 8, being immovable, it gives the particles within the "vein" a counteraction just equal to the action of the advancing vessel, and thus supplies the equilibrium on which are based the equations. Here, as in Fig. 8, the fluid in traveling through so narrow a vein cannot but take practically the stream-line course, $n i k s$, the line of the ship's skin. But to get absolute identity with that stream-line, the vein must, precisely as in the case of Lord Kelvin's disk (Fig. 8), be infinitely narrow, and its particles must consequently receive an infinite velocity. An infinitely narrow vein, however, allows only an infinitely slight advance to the vessel; and such an advance, being practically indistinguishable from rest, ousts practically the hydrodynamic condition and brings in a quasi-hydrostatic rôle. It is doubtful if "the gap which separates hydrostatics from hydrokinetics," and the still from the moving, and the simultaneous from the successive, and equal pressures from pressures not equal, can ever be spanned save by some such bridge as this; yet this one would seem at best but a draw, and we now are caught improvidently upon it with our infinitesimal, and the bridge half swung. For a really infinite velocity, as Mr. Basset reminds us, forbids any stream-line whatever. But let me waste the reader's time no longer in hopeless efforts to break these nightmares to the saddle. In point of fact, it could be only when the ice-mass should be prolonged to an infinite length northward that its inertia would suffice to bestow on the fluid particles along the ship's skin that equilibrium-producing reaction which would enable them to pursue the stream-line course provided by the skin; but even then this course could be followed for only an infinitesimal and nugatory period. If this mass be melted, as indeed is the case, each particle therein will each instant be crowded more and more swiftly toward the north by the advancing ship's accelerating force, and in consequence less and less swiftly to eastward and westward; so that no particle in the mass will preserve "steady" its course of motion, but its course will be changed each instant, becoming ever more and more northerly, and in the end becoming sensibly north.

28. Like the præ-copernican astronomies, the theory is founded upon an optical illusion. Depicted on the eye of a landsman stationed aboard the moving vessel, the particle n (Fig. 7) appears to pass from n , the cutwater, to s , along the course $n i k s$, averaging say south-by-west, half-west, and thus to move in accordance with the theory. This apparent course is the resultant of two motions; only one of which, however, viz., that from n to s' (or equal to from p to s) westerly, belongs really to the particle; the other, from n to p southerly, is only an appearance. In the painting effected upon the retina, this last movement does not come properly from a stroke of the brush (so to speak), but from an unobserved shifting of the canvas. While the bow is crowding the particle a distance $n s'$, westward, the observer himself is being carried a distance $p n$, northward; and with so much distance made by him in that direction he unwittingly credits the particle as having been made by it to the southward, and thus he mistakes the curve $n i k s$, one of whose components is his own journey, $p n$, for the genuine path of the particle.

29. But this is not the whole of the illusion. Water looks alike everywhere and presents no point d'appui or identifiable place of reference by which the eye may correlate the motions. The ship as a solid object, on the other hand, with its well-marked outline, supplies the lacking means of reference; and from her the untrained observer, even if he is at a fixed station outside the vessel, will instinctively and always estimate the movement. Not until he posits and identifies a number of convenient spots on the watery waste—which easily he may do by scattering there some chips of wood or other floating material—is the glamour broken. When the bow, however sharp, strikes one of these chips, the chip is borne actually somewhat forward, so as at once to change either its distance or its bearing from some near chip not stricken; and then for the first time does he under-

stand that in Fig. 7 the particle which starts from n , will (instead of making a southwesterly course, and of being found later at s , as the theory depicts), be found not even at s' , or due west of the cutwater, but at a point north of that, viz., s'' , having been sent seemingly farther forward than the ship's side which furthered it, although there has been with it no proper rebounding. To make it come to s' , we must turn the line $n i k s$ into an actual tube accompanying the vessel, as suggested in Sec. 24, and inside the tube must go the particle. The tube, moreover, must be provided with a piston, braced as in Sec. 5, so that the piston will (relatively) travel southward down the tube while the vessel is moving northward. If the piston is lacking, then as soon as the vis inertia of the water lying about the tube's mouth further northward is overcome, the tube-water

WIND MACHINE.

will be carried bodily on with the vessel. It is plain that the same cantrip sleight haunts also the example given by Lord Kelvin. Between Figs. 6 and 7 the only difference in result is such as springs from the difference between Lord Kelvin's bow-angle (the half of 180°) in Fig. 6 and the bow-angle, $s k i n p$, in Fig. 7, which latter angle may average the half of twenty-five or thirty degrees. In both figures, and in all cases, the northward transportation of the particle will vary as the sine of half the bow-angle. Certain movements which in a viscous fluid become vortical and have therefore been mistaken for effects of friction and viscosity, are pure geometrical and mechanical factors in this hitherto unsuspected headward motion, and they would be found as well in a perfect fluid.

If my theoretical conclusions as above stated may seem unbelievable because contrary to the universal belief of physicists and mathematicians, less believable yet are the results of certain analogous experiments in reducing the wave-resistance, though it was altogether self-standing and apart from any theory whatever that I carried them on. I would thank the reader for a hint of any mistake or oversight into which I may have been betrayed. In a fundamental study of

hydromechanics, perhaps as fully as in that of any other science, a man's best efforts to clear his work of errors can be successful in no other sense or manner than that though constantly he will be falling into them, he may constantly be pulled out of some portion of those into which he has fallen. Only unremitting re-examination and many-times repeated experiments can give the student much help in this lifelong process of alternate sinking and emerging. But often the criticism and experiments of others may help him more than can be done by self-criticism and his own observations. For such help (seldom, alas! to be gotten) he should be very grateful. On this account, and also because my meaning in one or more places may not be clear to every reader, I have cut these remarks into sections numbered for easy reference, and add an address.

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BEHIND THE WINGS IN THE HOF THEATER IN DRESDEN.

THE times in which we live make serious demands upon us. There is a spirit of realism abroad. Everything must be real. Things merely indicated will not suffice. This is especially true in reference to the stage. The most refined accessories of mechanics are utilized to assure an appearance of reality. The sinking sun must slowly gild the castle up yonder with a red glow before he disappears behind the mountains. The little village fountain must tinkle its actual waters into the trough below, so that the spectator (induced by the heat of the crowded house, no doubt) feels an intense longing to dip into the trough and quaff a vivifying drink. The sere and withered leaves of autumn fall from the trees with melancholy rustling; through the dark night, the storm howls so that we who sit safely down below in the parquet circle feel pleasantly horrified and fearsome; masses of white flakes slowly heap themselves in corners of windows and cover the roadways and spread over the wanderer who has lost his way, so horrible a winding sheet, that we feel delightfully creepy.

"The vapors rise
To lurid skies,
And the mad flames roar
About window and door,
With flare and flash,
Falling walls crash."

Our hearts beat anxiously. We sit motionless with bated breath.

And here comes the indiscreet scene painter. He turns an amiable and a trifle cynical smile to the audience and says, "Have no fear, gentlemen, the lion is stuffed."

It is not pleasant to expose the bare wilderness of wires that lies back of and within the beautiful world of appearances. But I have one comfort. In this respect, we are like children—eager to see what is inside of the toys; but, in this case, there is no need of the toys being broken in order to further such examination.

We can enter fearlessly through the little iron door, in spite of its stern admonition, "No admittance," and investigate what there is back of the scenes. We need not fear that anything will happen to us, in spite of the chaos of laths and ropes and linen and ladders and lamps. Confused as it looks, everything is in good order. Each one of the many intricate things is in control of one hand that masters it. The danger to be

LOHENGRIN'S FAREWELL TO THE SWAN.