50. On the Cumulative Vote as Exercised in School Board Elections

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50. On the cumulative vote as exercised in School Board elections.

The number of ways in which an elector can distribute r votes among n candidates is the coefficient of x^r in $(1-x)^{-n}$. Hence the total number of ways in which an elector can vote when k seats are to be filled

$$= \text{sum of coefficients of } x, x^2, x^3, \dots x^k \text{ in } (1-x)^{-n}$$

$$= \text{coefficient of } x^k \text{ in } (1-x)^{-n-1} - 1$$

$$= \frac{(n+1)(n+2) \dots (n+k)}{k} - 1$$

$$= (\text{number of selections of } k \text{ things out of } n+k) - 1.$$

The following investigation of the *a priori* grounds for the form of the result may be of interest :

Let (k, r) denote the number of ways in which an elector, having selected r candidates, may distribute his k votes among them, giving each at least one. Taking the case of any particular candidate of these r , he must receive 1, 2, 3, or $k-(r-1)$ votes ; and the number of ways the remaining $k-1, k-2, k-3, \dots$ or $r-1$ votes can be given is $(k-1, r-1), (k-2, r-1), (k-3, r-1), \dots (r-1, r-1)$ respectively.

$$\text{Hence } (k, r) = (k-1, r-1) + (k-2, r-1) + (k-3, r-1) \dots + (r-1, r-1).$$

But this equation would hold if (k, r) denoted the number of selections that could be made out of k things a, b, c , etc., r at a time, for this must be made up of (all that we could make containing a) + (all that we could make containing b and not a) + (all that we could make containing c but not a and b), and so on.

Also $(k, 1) = k = \text{number of ways of selecting one thing out of } k$.

Hence $(k, r) = \text{number of selections out of } k \text{ things } r \text{ at a time}$.

Now, if n be the whole number of candidates for k seats, each voter can select r candidates in (n, r) ways, and may vote for them in (k, r) ways after he has selected them.

Hence whole possible number of ways of voting

$$= (n, 1) \cdot (k, 1) + (n, 2) \cdot (k, 2) + (n, 3) \cdot (k, 3) + \dots + (n, k) \cdot (k, k)$$

$$= (n, 1) \cdot (k, k-1) + (n, 2) \cdot (k, k-2) + (n, 3) \cdot (k, k-3) + \dots + (n, k) \cdot 1.$$

Now, any term $(n, r) \cdot (k, k-r)$ represents the number of ways of selecting r candidates out of a group n and $k-r$ candidates out of a group k , and therefore the whole expression gives the number of possible ways of selecting k candidates out of $n+k$, except that we are to take at least one from group n [the term $1 \cdot (k, k)$ not appearing].

Hence whole possible number of ways $= (n+k, k) - 1$. E. M. LANGLEY.

51. The mid-points of the three diagonals of a complete quadrilateral are collinear.

Through D, E draw DQP, ERS parallel to AB , and through F, B draw FPS, BQR parallel to AD . Then it is known that the diagonals BE, DF, QS co-intersect, i.e. that S, Q, C are collinear.

Hence the mid-points L, M, N of AC, AQ, AS are on a line parallel to CS . But M, N are also the mid-points of BD, EF . E. M. LANGLEY.

Another Proof.—Join XY, XZ , and YZ .

Then, since Z is the mid-point of EF ,

$$2BYZ = EBY - FBY$$

$$= \frac{1}{2}EBD - \frac{1}{2}FBD. \dots \dots \dots (1)$$