ART. XI.—*On the Measurement of Extinction Angles in the Thin Section*; by FRED. EUGENE WRIGHT.

1. The Measurement of Extinction Angles of Minerals in the Thin Section.
2. The Adjustment of the Petrographic Microscope with Special Reference to the Measurement of Extinction Angles.
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1. *The Measurement of Extinction Angles of Minerals in the Thin Section.*

THE petrographic microscope as an instrument has undergone many changes and modifications since its introduction nearly forty years ago, but from the very first, each improvement has tended to increase its efficiency in such a way that the optical features of minerals in the thin section can be ascertained more readily and more accurately. In the hands of geologists, the microscope is merely the means to an end—an apparatus to aid in recognizing the minerals composing a given rock (mineral composition) and the relations of such rock components to each other (rock texture); and for such purposes the modern petrographic microscope is admirably adapted, especially since, as a rule, approximate results only are required. But with the increased knowledge of rocks thus attained, the demand for data which are precise and quantitative in character rather than qualitative has become more imperative, with the result that, at the present time, one expects to find in a thorough petrographic investigation accurately determined optical constants of each of the rock-forming minerals examined, and in critical points, the probable error of each determination given. This passage from qualitative to quantitative work implies consequences of profound importance; an additional burden is placed on the working geologist, and the time and energy required for the investigation of a given problem is much greater under the present régime than formerly; at the same time, this transition indicates that, in one phase of geology, at least, the step from the lower first plane of preliminary reconnaissance work to the higher level of precise and detailed work is being taken.

The optical properties which are made use of in the practical determination of minerals under the microscope are, briefly: refractive index, birefringence, optical axial angle, optical character, extinction angles, color and pleochroism. By means of these properties alone it is possible to ascertain the crystal system to which a given mineral belongs, and by a short process of elimination to determine definitely the mineral in ques-

tion. This process has been carried to such refinement in certain instances, as in the isomorphous series of plagioclase feldspars, that it is now possible, from extinction angles alone, to determine very closely the actual chemical composition of the particular plagioclase in hand.

For a given mineral plate in the thin section, the term extinction angle signifies the angle between a known crystallographic direction (cleavage line, or trace of a crystal face on that plate) and one of its optic ellipsoidal axes or directions along which it extinguishes when these directions are parallel with the principal planes of the crossed nicols. In order to ascertain this angle satisfactorily one must be able not only to measure plane angles accurately, but also to locate correctly the position of the optic ellipsoidal axes of the particular crystal plate. The first condition is easily accomplished and demands no special comment, while the second requirement is extremely difficult to meet with any degree of satisfaction without great expenditure of time.

Many methods have been suggested by which the position of the optic ellipsoidal axes of a given crystal section can be located more or less exactly, and all are based on the fact that when the optic ellipsoidal axes are parallel with the principal planes of the crossed nicols the plane polarized light normally incident from the lower nicol passes through the crystal plate without changing its plane of vibration. In case the optic ellipsoidal axes of the plate do not coincide with the planes of the nicols, interference in general takes place and some light passes through the upper nicol. The different methods proposed have for their common object the rendering apparent the extremely small percentage of light which thus emerges from the analyzer when the angle of revolution of the crystal plate from its position of absolute darkness is very small.

Before considering in detail the different methods for accomplishing this result and their relative merits and defects, it will be well to treat the subject mathematically and to derive the formulas for the intensity of light with special reference to the subject of extinction angles. This treatment is given in some detail in the following paragraphs, since the deductions given later are all drawn from these fundamental equations.

Theoretical.

Mathematical.—The phenomena of light are considered to be produced by periodic changes or disturbances in the ether, transverse to the direction of propagation. Different hypotheses have been proposed which assign different properties to this medium, but no one of the hypotheses yet suggested is

satisfactory in all its details.* For the purposes of this paper, however, these disturbances may be considered vibrations of ether particles about positions of rest and in a plane normal to the line of wave propagation. Adopting this view chiefly as a matter of easy expression, we may assert that in plane polarized light the disturbances or vibrations are confined to a plane, each particle vibrating then with simple periodic motion, to and fro, pendulum like, along a straight line. An equation which satisfies such a periodic vibration and which has been found to represent satisfactorily the ether disturbances, is the following:

$$y = a \sin \frac{2\pi}{T} (t - t_1) \quad (1)$$

in which a represents the amplitude, T the periodic time, $\frac{2\pi t_1}{T}$ the initial phase, t the time which has elapsed at any given instant, and y the distance of the ether particle from its position of rest. The velocity of the ether particle at any instant is given by

$$v = \frac{dy}{dt} = a \frac{2\pi}{T} \cos \frac{2\pi}{T} (t - t_1)$$

and if m be its mass, its kinetic energy is $1/2 mv^2$.

The above equation shows that v varies from 0 to $\frac{2\pi a}{T}$; accordingly the average kinetic energy during a complete vibration will be

$$\begin{aligned} \frac{1}{T} \int_0^T \frac{1}{2} mv^2 dt &= \frac{ma^2\pi^2}{T^3} \int_0^T 2\cos^2 \frac{2\pi}{T} (t - t_1) dt \\ &= \frac{ma^2\pi^2}{T^3} \int_0^T \left(1 + \cos \frac{4\pi}{T} (t - t_1)\right) dt = \frac{ma^2\pi^2}{T^3} \left[t + \frac{t}{4\pi} \sin \frac{4\pi}{T} (t - t_1) \right]_0^T \\ &= \frac{ma^2\pi^2}{T^2} \end{aligned}$$

Now, a vibrating particle possesses at any instant a definite amount of kinetic energy and also a definite amount of potential energy, and the sum of these two amounts of energy remains constant throughout the vibration, so that as the kinetic energy in the particle increases its potential energy

* These theories, as well as the mathematical treatment of the same, are given in the standard text-books on light (Preston's *Theory of Light*; P. Drude, *Lehrbuch d. Optik*, and many others; also Rosenbusch-Wülfig, *Mikrosk. Physiographie* i, 1; and Duparc and Pearce, *Traité de Technique Minéralogique et Pétrographique*).

decreases, and vice versa. At the moment the kinetic energy of the particle becomes zero, the total energy is potential, and similarly, when the kinetic energy attains its maximum, the potential energy is zero. In other words, the average potential energy is equal to the average kinetic energy, and the whole energy is twice the average kinetic energy, given in the above expression.

A measure for the intensity of light is the energy per unit volume of the vibrating ether, and if in the above expression for the kinetic energy m stands for the mass per unit volume, the whole energy or intensity will be measured by the expression,

$$I = \frac{2m\pi^2 a^2}{T^2}$$

In practice, only relative intensities are encountered. The relative intensities of two light vibrations of equal period (T) at a given point will be in the ratio of the square of their amplitudes:

$$\frac{I_1}{I_2} = \frac{2m\pi^2 a_1^2}{T^2} \div \frac{2m\pi^2 a_2^2}{T^2} = \frac{a_1^2}{a_2^2} \quad (2)$$

In other words, the intensity of light of a given period of vibration (color) varies as the square of its amplitude (a) of vibration.

This relation will now be made use in determining the relative intensities of the plane polarized light waves which emerge from the upper nicol of the microscope, after having passed through the lower nicol and an intervening crystal plate in different positions.

Disturbances in the ether which produce light phenomena are ascribed to the action of forces on the ether mass, and if two or more separate disturbances are simultaneously impressed on the same element, the resultant disturbance can be calculated according to the principle of the resolution of forces on the assumption of direct superposition of the forces. If, in the case of plane polarized light, two separate vibrations be imposed simultaneously on an element, the resultant vibration will also be in that plane, and its amplitude, on the principle of superposition, is the algebraic sum of the amplitudes of the components. The mathematical expression for the resultant vibration of a particle simultaneously impressed by two periodic disturbances of the same period but differing in phase and amplitude, can be deduced from the equations of the separate vibrations.

$$y_1 = a_1 \sin \frac{2\pi (t - t_1)}{T} \quad \text{and} \quad y_2 = a_2 \sin \frac{2\pi (t - t_2)}{T}$$

The resultant displacement at any time t is

$$\begin{aligned} y &= y_1 + y_2 = a_1 \sin \frac{2\pi}{T} (t - t_1) + a_2 \sin \frac{2\pi}{T} (t - t_2) \\ &= \sin \frac{2\pi}{T} t (a_1 \cos \frac{2\pi}{T} t_1 + a_2 \cos \frac{2\pi}{T} t_2) - \cos \frac{2\pi}{T} t (a_1 \sin \frac{2\pi}{T} t_1 + a_2 \sin \frac{2\pi}{T} t_2) \\ &= A \sin \frac{2\pi}{T} (t - t_3) \end{aligned}$$

if

$$A \cos t_3 = a_1 \cos \frac{2\pi}{T} t_1 + a_2 \cos \frac{2\pi}{T} t_2$$

and

$$A \sin t_3 = a_1 \sin \frac{2\pi}{T} t_1 + a_2 \sin \frac{2\pi}{T} t_2$$

By squaring and adding the last two expressions, we obtain

$$A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \frac{2\pi}{T} (t_2 - t_1) \quad (3)$$

In this expression $\frac{2\pi}{T} (t_2 - t_1)$ denotes the difference in phase of the two component periodic displacements and A the amplitude of the resultant vibration.

In considering the effects which different crystals exert on transmitted light waves, it has been found, both in practice and theory, that these influences can be predicted accurately and satisfactorily by reference to a triaxial ellipsoid, the optical ellipsoid, the position and relative axial lengths of which vary in general with different minerals, and with the wave length of light employed. Thus the directions of vibration of light waves emerging normally from a mineral plate are parallel with the major and minor axes of the ellipse which a central diametral plane parallel to the given plate cuts out of the optical ellipsoid for the particular mineral and wave length used. The determination of the actual position of these directions in the plate is accomplished in polarized light by observing the relative intensity of the transmitted light for different positions of the plate parallel to the principal planes of the nicols.

Light waves emerging from the lower nicol are plane polarized and their vibration is given by the equation

$$u = a \sin \frac{2\pi t}{T}$$

On entering the crystal plate, this vibration is resolved into two vibrations in planes normal to each other. If θ (fig. 1) be the angle included between the major optic ellipsoidal axis

of the plate and the plane of the incident vibrations, the equations for the resultant waves are

$$x = u \cos \theta = a \cos \theta \sin \frac{2\pi t}{T} \text{ and } y = u \sin \theta = a \sin \theta \sin \frac{2\pi t}{T}$$

Each of these vibrations traverses the plate with a different velocity and the time required by the fast wave to traverse the plate of thickness d will be $t_1 = d/a'$, while the time required

FIG. 1.

by the slow wave is $t_2 = d/\gamma'$ where a' and γ' are respectively the refractive indices of the two waves. On emergence, therefore, the equations for the periodic displacements will be

$$x' = a \cos \theta \sin \frac{2\pi}{T} (t - da') \text{ and } y' = a \sin \theta \sin \frac{2\pi}{T} (t - d\gamma')$$

On reaching the upper nicol each of these vibrations is again resolved further into two component vibrations again normal to each other, one of which, however, is annulled by total reflection. If ϕ be the angle between the principal planes of the nicols, then the component vibrations emerging from the upper nicol are

$$\xi = x' \cos (\theta - \phi) = a \cos (\theta - \phi) \cos \theta \sin \frac{2\pi}{T} (t - da') = A_1 \sin \frac{2\pi}{T} (t - da')$$

$$\eta = y' \sin (\theta - \phi) = a \sin (\theta - \phi) \sin \theta \sin \frac{2\pi}{T} (t - d\gamma') = A_2 \sin \frac{2\pi}{T} (t - d\gamma')$$

and the resultant amplitude

$$A = \xi + \eta = A_1 \sin \frac{2\pi}{T} (t - da') + A_2 \sin \frac{2\pi}{T} (t - d\gamma')$$

The intensity I' of the emergent wave is then proportional to the square of the amplitude

$$\frac{I'}{I} = \frac{A^2}{a^2}$$

The equations (3) above, moreover, show that

$$A^2 = A_1^2 + A_2^2 + 2 A_1 A_2 \cos \frac{2\pi}{T} d (\gamma' - a')$$

On substituting the values of A_1 and A_2 in this equation, and noting that

$$\cos \frac{2\pi}{T} d (\gamma' - a') = 1 - 2 \sin^2 \frac{\pi}{T} d (\gamma' - a')$$

we obtain $A^2 = a^2 [\cos^2 \phi - \sin 2(\theta - \phi) \sin 2\theta \sin^2 \frac{\pi}{T} d (\gamma' - a')] \quad (4)$

and finally, $\frac{I'}{I} = \frac{A^2}{a^2} = \cos^2 \phi - \sin 2(\theta - \phi) \sin 2\theta \sin^2 \pi d (\gamma' - a') \quad (5)$

But the velocity of light V , period of vibration T , and wave length λ , are so related that $VT = \lambda$ and if we consider the velocity unity, then we may replace T by λ and the equation (5) reads:

$$\frac{I'}{I} = I_1 = \cos^2 \phi - \sin 2(\theta - \phi) \sin 2\theta \sin^2 \frac{\pi}{\lambda} d (\gamma' - a')$$

This is the usual expression for the relative intensity of the emergent waves; it may, however, be brought into more convenient form for practical purposes. To save space, let $\sin^2 \frac{\pi d}{\lambda} (\gamma' - a') = K$, where K may have any value from 0 to +1;

then

$$I_1 = \frac{1 + \cos 2\phi}{2} - K \sin 2\theta \sin 2(\theta - \phi)$$

$$2I_1 = 1 + \cos 2\phi - 2K \sin 2\theta \sin 2(\theta - \phi) \quad (a)$$

$$= 1 + \cos 2\phi - K (\cos 2\phi (1 - \cos 4\theta) - \sin 2\phi \sin 4\theta) \quad (b)$$

$$= 1 + (1 - K) \cos 2\phi + K \cos 2(\phi - 2\theta) \quad (6)$$

For a given angle θ to find the condition that the intensity will be zero, the equation (a) of the foregoing can be changed to

$$\begin{aligned} 2I_1 &= 1 + (1 - 2K \sin^2 2\theta) \cos 2\phi + 2K \sin 2\theta \cos 2\theta \sin 2\phi \\ &= 1 + K_1 \cos 2\phi + K_2 \sin 2\phi \end{aligned} \quad (7)$$

in which

$$K_1 = 1 - 2K \sin^2 2\theta \text{ and } K_2 = 2K \sin 2\theta \cos 2\theta.$$

If $2I_1 = 0$, then

$$\begin{aligned} & 1 + K_1 \cos 2\phi + K_2 \sin 2\phi = 0 \\ & \text{or } 1 + K_1 \cos 2\phi = -K_2 \sin 2\phi \\ & \text{squaring } 1 + 2K_1 \cos 2\phi + K_1^2 \cos^2 2\phi = K_2^2 - K_2^2 \cos^2 2\phi \\ & \cos 2\phi = \frac{-K_1}{K_1^2 + K_2^2} \pm \sqrt{\left(\frac{K_1}{K_1^2 + K_2^2}\right)^2 + \frac{K_2^2 - 1}{K_1^2 + K_2^2}} = \\ & \frac{-K_1}{K_1^2 + K_2^2} \pm \frac{K_2^2}{K_1^2 + K_2^2} \sqrt{K_1^2 + K_2^2 - 1} \end{aligned} \quad (8)$$

In order that $\cos 2\phi$ have a real value, the expression $K_1^2 + K_2^2 - 1$ must be zero or positive. But,

$$\begin{aligned} K_1^2 &= 1 - 4K \sin^2 2\theta + 4K^2 \sin^4 2\theta \\ K_2^2 &= 4K^2 \sin^2 2\theta - 4K^2 \sin^4 2\theta. \end{aligned}$$

Accordingly,

$$K_1^2 + K_2^2 - 1 = -4K \sin^2 2\theta (1 - K) \quad (9)$$

The right hand of this equation is a negative quantity, and $\cos 2\phi$ can have a real value only when $K_1^2 + K_2^2 - 1 = 0$, and this condition is fulfilled only when

$$(1) K=0; \text{ or } \sin^2 \frac{2\pi}{T} d (\gamma' - \alpha') = 0, \text{ i. e. } \frac{\pi}{T} d (\gamma' - \alpha') = n\pi$$

$$(2) K=1; \text{ or } \sin^2 \frac{2\pi}{T} d (\gamma' - \alpha') = 1, \text{ i. e. } \frac{\pi}{T} d (\gamma' - \alpha') = (n+1) \frac{\pi}{2}$$

$$(3) \sin^2 2\theta = 0; \text{ i. e. } \theta = n\pi.$$

The value for $\cos 2\phi$ then reduces to

$$\cos 2\phi = \frac{-K_1}{K_1^2 + K_2^2} = -K_1 = -(1 - 2K \sin^2 2\theta)$$

For the three different cases the value of $\cos 2\phi$ becomes

$$(1) \cos 2\phi = -1 \quad \text{i. e. } \phi = (2n+1) \frac{\pi}{2}$$

$$(2) \cos 2\phi = -(1 - 2 \sin^2 2\theta) = -\cos 4\theta, \text{ i. e. } \phi = \pi - 2\theta$$

$$(3) \cos 2\phi = -1 \quad \text{i. e., } \phi = (2n+1) \frac{\pi}{2}$$

If the nicols be not crossed, therefore, it is not possible to obtain absolute darkness for a given section unless

$$2 \frac{\pi d (\gamma' - \alpha')}{T} = \frac{(2n+1) \pi}{2} \text{ i. e., unless monochromatic light be}$$

used of such a wave length that the one wave is an odd number of half wave lengths ahead of the second, and in this case, $\phi = \pi - 2\theta$. If white light be employed, abnormal interference colors will appear because of the abnormal conditions, and at no point will darkness ensue.

The condition that the entire light be transmitted is

$$I_1 = \frac{I'}{I} = 1$$

or $(1-K) \cos 2\phi + K \cos 2(\phi - 2\theta) = 1$
which is satisfied if both $\phi = 0$ and $\theta = 0$.

In case either ϕ or θ be given, the problem of finding the particular disposition of upper nicol or crystal plate for which the intensity of the transmitted light reduces to a minimum or maximum, involves the first partial differential quotients of the function $2I_1$ (equation 6) after either ϕ or θ . If ϕ be given, the point in question is determined by

$$\frac{\delta(2I_1)}{\delta \theta} = 4K \sin 2(\phi - 2\theta) = 0$$

$$\therefore \phi = 2\theta \text{ or } \phi = \pi - 2\theta$$

The second partial differential quotient shows that for the first value of θ the intensity is a minimum, while for $\phi = \pi - 2\theta$ the intensity is a maximum. This relation is of importance in certain of the methods described below.

If θ be given and ϕ is the variable,

$$\frac{\delta(2I_1)}{\delta \phi} = -2(1-K) \sin 2\phi - 2K \sin 2(\phi - 2\theta) = 0$$

From this equation we find

$$\sin^2 2\phi = \frac{K^2 \sin^2 4\theta}{(1-K)^2 + 2K(1-K) \cos 4\theta + K^2}$$

a complicated expression which for $K=1$ simplifies to

$$\sin^2 2\phi = \sin^2 4\theta \quad \text{and this equation}$$

is satisfied for

$$\phi = 2\theta$$

$$\phi = \pi - 2\theta$$

It is of interest to plot the values given by the equation for different values of ϕ and θ .

$$2I_1 = 1 + (1-K) \cos 2\phi + K \cos 2(\phi - 2\theta). \quad (6)$$

In fig. 2, curve V, the rate of increase in intensity of light is given for the special case of $\theta = 0$, where simply the upper nicol is turned and the crystal plate has no effect in the polarization of the waves from the lower nicol. In this case

$$2I_1 = 1 + \cos 2\phi \quad (10)$$

From the curve it is evident that the rate of increase is very slow at first, but rises rapidly and reaches a flexion point at 45° , after which the intensity increases with decreasing rapidity to its maximum value at 90° .

FIG. 2.

FIG. 2.—Curves showing relative intensity of light emerging from upper nicol after transmission through polarizer, crystal plate and analyzer, the positions of the crystal plate and also the analyzer ranging from 0° – 90° . The abscissa values refer to angular distances of the major ellipsoidal axis of the crystal plate and also of the plane of the analyzer. For curves I–IV, the nicols are considered crossed ($\phi = \frac{\pi}{2}$) and the crystal plate alone to be

revolved from 0° to 90° . In curve I, $\sin^2 \frac{2\pi}{T} d (\gamma^1 - a^1) = K = 1$; in curve II, $K = 1/2$; in curve III, $K = 1/4$; in curve IV, $K = 0$. Curve V shows the relative intensity of the emerging light for different positions of the analyzer alone ($\theta = 0$, ϕ ranging from 0° to 90°). Curves calculated from the general formula

$$I_1 = 1/2 (1 + (1 - K) \cos 2\phi + K \cos 2(\phi - 2\theta)).$$

In case the nicols are crossed ($\phi = \frac{\pi}{2}$), the rates of increase for different values of K are given by the reduced equation

$$2I_1 = K(1 + \cos 4\theta) = 2K \sin^2 2\theta \quad (11)$$

which defines a curve similar in aspect to the foregoing except that ϕ is replaced by 2θ and the factor K tends to reduce all values proportionately. The curves I–IV of fig. 2 represent the relative intensities for values of $K=1, 1/2, 1/4$ and 0 respectively. The greatest possible intensity is thus attained when $K=1$, i. e., when the waves, after emerging from the crystal plate, are an odd number of half wave lengths apart (in opposite phase); the intensity is zero for all positions of the plate when $K=0$, i. e., when the distance between the two emergent waves is a whole number of wave lengths.

In figs. 3–6, intensity curves are drawn showing the relative intensity of the emergent light for different positions of the crystal plate (θ usually $0^\circ, 15', 30', 45'$, and 1°) with the principal plane of the lower nicol, and for different positions of the upper nicol (ϕ ranging from 88° to 92°). The heavy curve in each figure is the relative intensity curve of the crystal alone (nicols crossed, $\phi = 90^\circ$) and θ (ranging from -2° to $+2^\circ$). The narrow range of intensities only is considered, since in general it represents about the order of magnitude of the probable error of a single determination made in the usual manner.

In each of the figures the unit ordinate division represents .025 per cent of the total intensity and the unit abscissa division $10'$ of arc.

In fig. 3, K is considered $=1$ or $\sin^2 2 \frac{\pi}{\lambda} d(\gamma' - a') = 1$, which obtains when the one wave is any odd number of half wave lengths ahead of the second on emergence from the plate; in figs. 4, 5, 6 and 7, the relative intensity curves are drawn for $K=3/4, 1/2, 1/4$ and 0 respectively.

It is not a difficult matter to grasp the meaning of these curves, as the following example will show: let it be required to find the percentage of light which emerges from the nicol in the case of a mineral plate of such thickness and birefringence that for yellow light the faster waves after emerging from the plate will be precisely one half wave length ahead of the slow waves ($K = 1$, fig. 3), the direction of extinction of the plate to make an angle of $30'$ ($\theta = 30'$) with the principal plane of the lower nicol, and the principal plane of the upper nicol to include an angle of $89^\circ 10'$ with the lower nicol ($\phi = 89^\circ 10'$). On the $30'$ curve of fig. 3 the ordinate for $89^\circ 10'$ is .104 and the relative intensity is therefore .104 of 1 per cent of the total intensity.

FIG. 3.

FIG. 3.—Intensity curves for crystal plates making angles 0', 5', 10', 15', 20', 25', 30', 40', 45', 50' and 1° with plane of polarizer. Analyzer revolved about axis from 88° to 92° with lower nicol plane.

$$\sin^2 \frac{2\pi t}{T} (\gamma' - a') = K = 1$$

Curves calculated from the formula

$$I = \frac{1}{2} (1 + \cos 2(\phi - 2\theta)).$$

The

FIG. 4.

FIG. 4.—Intensity curves for crystal plates at angular distances of 0', 15', 30', 45', and 1° from plane of polarizer, the analyzer being revolved about axis from 88° to 92° with plane of polarizer. $\sin^2 \frac{2\pi t}{T} (\gamma' - \alpha') = K = 3/4$. Curves calculated from the formula

$$I_1 = 1/8 [4 + \cos 2\phi + 3 \cos 2 (\phi - 2\theta)].$$

As in fig. 3, the heavy curve indicates the relative intensity of emergent light for different positions of the crystal plate (θ ranging from 88° to 92°, or -2° to $+2^\circ$) under crossed nicols ($\phi = \frac{\pi}{2}$) and $K = 3/4$. Calculated from the formula

$$I_1 = 3/8 (1 + \cos 4\theta)$$

heavy curve indicates the relative light intensity under crossed nicols ($\phi = \frac{\pi}{2}$) for different positions of the crystal plate (θ ranging from 88° to 92° or -2° to $+2^\circ$) and $K = 1$. Calculated from the formula $2 I_1 = 1 + \cos 4\theta$.

These figures 2-6 are well adapted to show graphically certain facts which are evident from a mathematical consideration of the intensity formula. (1) If $K = 0$, which occurs when the one wave is any number of whole wave lengths ahead of the second, the crystal plate is dark and remains dark for all positions of revolution, as indicated by the heavy abscissa line

FIG. 5.

FIG. 5.—This figure differs from the two preceding figures only in the value of K , which is $1/2$. The curves were calculated from the formula

$$I_1 = 1/4 (2 + \cos 2\phi + \cos 2(\phi - 2\theta)).$$

The heavy curve from the formula

$$I_1 = 1/4 (1 + \cos 4\theta).$$

of fig. 7. (2) In case $K = 1/4$, fig. 6, the intensity curve for crystal plate, coincides very closely with that for the revolving nicol. The extinction curves, moreover, for the crystal plate at different angles ($\theta = 15', 30', 45'$, and 1°) with the principal plane of the lower nicol and for different positions of the upper nicol (ϕ ranging from 88° to 92°) are similar and lie close together, so that, in this particular case, methods involving the

revolution of the upper nicol for the location of zero intensity directions are not greatly different in their degree of accuracy from those in which the nicols remain crossed and the crystal plate is revolved. Nevertheless, even in this instance the former are the more sensitive methods and results attained by their use are correspondingly more accurate. For $K=1/2$, fig. 5, the extinction curve for the crystal plate alone (nicols crossed and plate only revolved) no longer coincides with that for the upper nicol alone, but similar conclusions can be drawn as to the relative sensitiveness of the two methods, the one involving the revolution of the crystal plate (while the nicols remain crossed), and the second, the revolution of the upper nicol while the crystal plate remains stationary. The amount of light

FIG 6.

FIG. 6.—Differs from fig. 5 only in K , which is $1/4$. The curves are expressed by the formula

$$I_1 = 1/8 [4 + 3 \cos 2\phi + \cos 2(\phi - 2\theta)]$$

and the heavy curve by

$$I_1 = 1/8 (1 + \cos 4\theta)$$

which is required to produce the sensation of light in the human eye is different for different persons. But for a given eye the limit of the actual sensation of monochromatic light is fixed for any particular instant and may be represented by one of the horizontal percentage lines of the figures. Let us assume that for a source of monochromatic light of definite intensity I , the limit for the sensation of light is .050 per cent of the

total intensity and represented by second horizontal line above the base line of fig. 5.

Then the curve for the crystal alone shows that for all points below that line, i. e., between $89^{\circ} 04'$ and $90^{\circ} 56'$, the crystal will appear absolutely dark and on a single determination an error of nearly $\pm 1^{\circ}$ may be made. If, however, the crystal plate remain stationary, and the upper nicol be revolved through small angles from its normal, crossed position ($\phi = 88^{\circ}$ to 92°), it is evident from the figure that if, for example, the

FIG. 7.

FIG. 7.—In this particular case K is considered $= 0$ and the general formula reduces to

$$I = \frac{1}{2}(1 + \cos 2\phi)$$

which is independent of θ . In other words, if the thickness of the plate be such that $\sin^2 \frac{2\pi t}{T} (\gamma' - a') = 0$, or the emerging waves are any number of whole wave lengths apart, total interference takes place and the plate is dark under crossed nicols for every angle of revolution about its normal axis.

crystal plate is $30'$ distant from its position of true extinction and still dark under crossed nicols so far as the eye of the observer can detect, the differences in intensity between the field and crystal plate for different angles of revolution of the upper nicol (measured by the ordinate intercepts between the curve $0'$ and $30'$ of figure), are of such a character that at the point where the illumination of the field can just be observed ($88^{\circ} 43'$) the intensity of illumination of the crystal is more than twice as great ($.106$ per cent instead of $.05$ per cent), whereas

on the other side, where first indications of illumination on the central plate can be detected at $91^{\circ} 41'$, the field is lighted up by .085 per cent instead of .050 per cent of the total intensity. These differences of intensity are of such a character that they can readily be observed, and the sensitiveness of any method involving the revolution of the upper nicol while the crystal remains stationary is in this case at least twice as great as that for which the nicols remain crossed and the crystal plate alone is revolved.

Similar theoretical conclusions can be drawn from figs. 3 and 4.

If white light be used and the upper nicol be revolved abnormal interference colors result. The rapid and pronounced change in interference colors near the position of crossed nicols, on a plate which is not precisely in the position of true extinction, is well adapted for use in the location of its ellipsoidal axes.

With a given color of monochromatic light extinction angles should be determined on plates of such a thickness that K is about $+1$ (the two emergent waves are a whole number of half wave lengths apart). Thus if sodium light be used the plates should show in white light an interference color of about straw-yellow of the first order but not sensitive violet, since for this particular thickness the two waves are $589\mu\mu$ apart and the yellow waves are totally destroyed, with the result that the plate appears dark in all positions. It follows, furthermore, that a plate which is well adapted for determinations in one kind of monochromatic light may be useless for another color.

It has been found that the insertion between the crossed nicols of specially cut plates and wedges of birefracting substances, as quartz and selenite, is often well adapted to increase the accuracy of the measurement of the extinction angle on a given plate. The principle there involved is that of the superposition of birefracting plates, the action of which is to produce a resultant which differs from that of either component. It is possible to select a wedge or plate of such a character that the interference phenomena produced by it alone are extremely sensitive to the slight changes produced by a second crystal plate when it is not precisely in the position of true extinction.

From the mathematical standpoint, the insertion of a second plate involves a new set of conditions for the vibrating ether elements and the equations for the resultant are correspondingly more complex. Their derivation, however, is exactly counterpart to that for the intensity of a single crystal plate and the final result only need be given here. If the nicols be crossed and θ_1 be the angle which γ_1 of the crystal plate of

thickness d_1 includes with the principal plane of the polarizer and θ_2 , the angle between γ_2' of the inserted plate or wedge of thickness d_2 and the polarizer plane, the relative intensity is given by the formula, and $d_1(\gamma_1' - \alpha_1') = T_1$ and $d_2(\gamma_2' - \alpha_2') = T_2$, then

$$\begin{aligned} I_1 = & \sin 2(\theta_2 - \theta_1) \sin 2\theta_1 \cos 2\theta_2 \sin \frac{2\pi}{T_1} T_1 \\ & + \sin 2(\theta_2 - \theta_1) \cos 2\theta_2 \sin 2\theta_2 \sin \frac{2\pi}{T_1} T_2 \\ & + \cos^2 (\theta_2 - \theta_1) \sin 2\theta_1 \sin 2\theta_2 \sin \frac{2\pi}{T_1} (T_1 - T_2) \\ & - \sin^2 (\theta_2 - \theta_1) \sin 2\theta_1 \sin 2\theta_2 \sin \frac{2\pi}{T_1} (T_1 - T_2) \end{aligned}$$

From this formula the relative intensity can be calculated for any given values of (θ_1, θ_2) and T_1 and T_2 .

In case the crystal plate is of such a thickness that $\sin \frac{2\pi}{T_1} T_1 = 1$ and at the same time the inserted plate is also

of a thickness that $\sin \frac{2\pi}{T_2} T_2 = 1$, this equation reduces to

$$I_1 = \sin^2 2(\theta_2 - \theta_1)$$

an expression for a curve similar in every respect to those of fig. 2, but which is zero for $\theta_2 = \theta_1$ and also for $\theta_2 = \frac{\pi}{2} + \theta_1$ and

reaches its maximum of 1 at $\theta_2 = \frac{\pi}{2} + \theta_1$. It can also be shown

that for a given increment of θ_2 as $d\theta_2$, the ratio of the value of the function for $(\theta_2 + d\theta_2)$ to its value for θ_2 is greatest when θ_2 is equal to θ_1 . If, therefore, the angle θ_2 be so chosen that the field is just illuminated, the change resulting from small angles of θ_1 will be greater than for any other position of the inserted plate.

In the Calderon method described below, the calcite plates are purposely so thick that they show the white interference colors of higher orders in white light, in which case the thickness is so great that for a number of different colors throughout the spectrum the path difference of the emergent waves is a whole number of wave lengths, in other words, in the Calderon method it is permissible for practical purposes to consider the plate of such a thickness that for white light the expression $\sin^2 \frac{\pi}{\lambda} T_2$ is unity, and that therefore the angle θ_2 should be small in order to secure the best results, so small in fact that the illumination of the field is just visible.

In several of the other methods cited below for the exact location of the ellipsoidal axes of a given plate, use is made of quartz plates or wedges, cut normally to the principal axis, which rotate the planes of vibration of normally incident, plane polarized light. For the purposes of this paper it is not necessary to enter into the mathematical discussion and theory of the rotatory power of quartz, but simply to apply the known laws of rotatory polarization as they were first proved experimentally by Arago and Biot on this mineral. A quartz plate perpendicular to the principal axis rotates the plane of normally incident, plane polarized waves, through an angle which is proportional to the thickness of the quartz plate and also approximately proportional to the inverse square of the wave length used. The rotation effected by two superimposed plates is moreover the algebraic sum of the rotations produced by each separately.

By using, therefore, a properly constructed quartz wedge, it is possible to counteract exactly the effect, in plane polarized monochromatic light, of any crystal plate in any given position with respect to the nicols, by rotating the new planes of vibration, determined by the crystal plate back to the original plane of the nicols.

In the intensity formula (5),

$$I_1 = \cos^2 \phi - \sin 2\theta \sin 2(\theta - \phi) \sin^2 \frac{\pi}{\lambda} d (\gamma' - \alpha')$$

this rotation affects the angle θ only, and if the nicols be crossed, then

$$I_1 = -\sin^2 2\theta \sin^2 \frac{\pi}{\lambda} d (\gamma' - \alpha') \quad \text{Equation (11, page 359)}$$

In all measurements of extinction angles, however, θ is a small quantity and in place of the sine we may use the angle itself without sensible error; accordingly,

$$I_1 = 4K\theta^2. \quad (13)$$

This formula, which for small angles θ states that the light intensity is proportioned to the square of the angle θ , will be employed later in the description of a new combination quartz wedge for use in determining extinction angles.

In certain other methods, convergent polarized light is employed and the disturbing effects of an intervening crystal plate observed whose optic ellipsoidal axes are not precisely parallel with the planes of the nicols. The intensity formulae applying to such conditions are similar to those for plane polarized and the general deductions from the latter may be considered to apply to the phenomena in convergent polarized light. The methods involving convergent polarized light, however, have several

important defects which render their general application cumbersome and unpractical.

In the foregoing pages, the intensity formulae for light transmitted by crystalline plates under different conditions have been developed and the attempt has been made to treat the subject in such a way that the results attained shall be directly applicable to the practical methods for determining extinction angles under the microscope. In the following sections, the different methods for accomplishing this end will be described, with special reference to their general applicability and relative accuracy, and the conclusions reached in this mathematical part will be used constantly as criteria of fundamental importance.

Methods.

Extinction angles can be measured either in plane polarized light or in convergent polarized light; and in plane polarized light the exact location of the positions of zero extinction is fixed, either by observing relative intensities of monochromatic light under special conditions or by means of the interference colors resulting from the use of white light. In all measurements of extinction angles it is imperative that careful attention be given to the source of light, especially if monochromatic light be used. The source should be as intense and steady and uniform as possible in order that the variation in the source of light itself be not mistaken for actual differences in the microscopic field. The rays of light incident on the preparation should, moreover, be as nearly parallel as it is possible to obtain them. To meet these requirements satisfactorily requires both time and patience, but in order to attain the best results they cannot be overlooked.

The microscope, moreover, should be in perfect adjustment, the optical system should be accurately centered and the cross hairs in the ocular should be precisely parallel with the principal planes of the crossed nicols. The adjustment of the microscope is not a difficult task to accomplish if suitable apparatus is at hand, and will be discussed briefly in part 2 of this paper.

Assuming the microscope to be in perfect adjustment and the source of light satisfactory, we may use any one of the following direct methods for measuring the extinction angle of a particular crystal plate:

Parallel Polarized Light.

(1) The ordinary method, which consists in turning the crystal plate under crossed nicols until the position of maximum

darkness is attained. This method is in general use and is equally well adapted for white light and for monochromatic light. With it any degree of accuracy can be attained provided a sufficient number of measurements be taken to reduce the probable error. In applying this method it is customary to note not only the positions of maximum darkness attained by the crystal when revolved clockwise from a position of bright illumination, but also when revolved counter clockwise from such position. This was the method used by Max Schuster* in his classic measurements of the extinction angles of plagioclase feldspars. He determined for each cleavage flake the position of zero extinction eighty times for clockwise revolutions of the plate and eighty times for counter clockwise revolutions, and averaged the two readings. His work in this line remains unsurpassed, even to the present time.

To increase the accuracy of each determination on a crystal plate under crossed nicols, different schemes have been devised, all of which depend on the disturbing influence of the plate on inserted plates or wedges of birefracting substances. Each of the inserted plates or wedges is constructed in such a way that the interference phenomena which it presents are markedly influenced by the slight disturbing effects from the crystal plate when it is not precisely in its position of zero extinction.

Sensitive Tint Plate.—Plates showing this interference color (violet of the second order) are usually made of selenite or quartz and are under certain conditions very sensitive to the slight changes which the crystal plate produces when it is not precisely in its position of total extinction. As a general rule, the eye is more sensitive to slight differences in color than in intensity, and in certain cases the sensitive tint plate can be used to advantage to increase the accuracy of the ordinary method. Its use is most effective on and practically limited to colorless plates showing low interference colors of the first order. Its efficiency is seriously impaired in the case of deeply colored minerals which veil the true interference color and also in thick plates of strongly birefracting minerals showing high interference colors, even red of the first order. It can, moreover, only be used with white light and accordingly cannot take cognizance of the dispersion of the bisectrices in the monoclinic and triclinic systems. This method is therefore not of general application and can be employed to advantage only under specially favorable conditions.

Bravais†-Stöber‡ plate.—This plate is also cut to show the sensitive violet interference color, and consists of two such

* Tschermak's Min. Petr. Mitth. v. 189, 1882.

† Comptes Rendus. xxxii. 113, 1851; also Pogg. Ann. xevi, 397. 1855.

‡ Zeitschr. Kryst. xxix, 22-24. 1898.

plates in combination instead of a single one. A single sensitive tint plate of mica or quartz is taken and cut along a line at 45° with the directions of extinction; the one half is then turned through angle of 180° and the two halves recemented as indicated in the figure. By this combination plate, which is placed in the focal plane of the ocular, the interference color is made to fall in the one half and to rise an equal amount in the second, thus doubling the sensitiveness of the single plate.

FIG. 8.

This plate is intended for use only in white light, but under certain conditions it may serve to good advantage in monochromatic light.

*The Combination Wedge.**—On the principle of the Bravais-Stöber plate, the writer has had a combination wedge prepared in which the interference colors range from total darkness to green of the second order. This wedge was made by taking

*This wedge was prepared with great care by Voigt & Hochgesang, Göttingen, Germany (cost, 48 mks.), and the writer desires to express his appreciation of the interest taken by the firm in the same. The compensation on different ends of the wedge, however, proved to be of unequal value, with the result that although the dark zero interference bands were precisely adjacent, the interference colors near the ends of the wedge did not coincide exactly. This defect could be eliminated by combining two quartz plates (45^{mm} long by 5^{mm} wide and of such a thickness as to show interference color green-yellow second order, the ellipsoidal axis c of the one to be parallel to the long direction, while in the second a is parallel), with two wedges of the same pitch (45^{mm} long by 5^{mm} wide and ranging in interference colors pale gray of the first order to violet gray at the top of the third order, and likewise the ellipsoidal axis c parallel to the direction of elongation in one and a to the same direction in the second); the wedge of long direction c to be combined with the plate of long direction a . In this manner the plate and wedge compensate in the center of the wedge and the interference colors rise to about blue green of the second order at both ends.

an ordinary combination wedge* showing the zero interference band exactly in the center and green of the second order on each end, cutting the same longitudinally in half parallel to the ellipsoidal axes; the edges were then polished and the halves again recemented, the one half, however, having been rotated first through 180° , so that in the resultant combination by wedge the phase difference of the adjacent half at any point of insertion is always equal and opposite. By this method the principle of the Bravais-Stöber plate is extended to cover interference colors from total darkness to blue green of the second order, and to allow the observer to select an interference color which, in combination with that of the mineral plate examined, is most sensitive. The low gray tints of this wedge (particularly the dark band region on both sides of which the interference colors rise and thus divide the field into four quadrants and produce an effect similar to that of the Bertrand ocular) have been found specially useful with minerals showing interference colors from red first order to blue second order. This wedge is held in a brass carriage, which in turn slides in the wedge holder shown in fig. 10, and is viewed by the Ramsden ocular.

Calderon† Calcite plate.—This plate is also placed in the focal plane of the ocular and consists of two calcite plates placed side by side and so cut that the direction of extinction in each plate makes an angle of about $3\frac{1}{2}^\circ$ on opposite sides of the common line of junction. The plate is so thick that the interference color is white of the higher orders and when used alone without intervening crystal plate, lights the entire field under crossed nicols with a dull gray tone. If a crystal plate whose lines of extinction do not coincide with the principal nicol planes be then observed, the field appears divided into two unequally illuminated halves and only when the extinction directions coincide with the nicol planes is the intensity of illumination in both halves equal. Calderon claims an accuracy of $\pm 2'$ with this ocular, but for a single determination and for general preparations the probable error is considerably larger.

* Compare F. E. Wright, Tschermak's Min. Petr. Mitth. xx, 233-306, 1901; also Jour. Geol., 33-35, 1902.

† Zeitschr. Kryst., ii, 70, 1878.—The calcite twin plates of a Calderon ocular from R. Fuess in Steglitz were tested by the writer and found to be inaccurately ground. The plate was 3.18 mm thick and cut at an angle of about 45° with optic axis. The extinction angle in each half of the plate was measured in convergent polarized light by means of the dark bar in the center of the field and found to be $+4.4^\circ$ on the one half and 3.2° on the other. Extinction angles measured with this ocular, using the junction line of the plate as the line of reference, would therefore be out 0.6° from this source alone. The field of the ocular is, moreover, small and unfavorably lighted because of the thickness of the plate and of the wide dark junction line across the center of the field, which in turn disturbs the exact matching of the halves of the field.

(10'–15'). The principle on which this method is based is evident from the intensity formula, for in case the ellipsoidal axes of the plate do not coincide precisely with the principal nicol planes, they make unequal angles with the optic ellipsoidal axes of the calcite (in the one half, this angle is $3\frac{1}{2}^\circ + \theta$ and for the second $3\frac{1}{2}^\circ - \theta$) and this produces at once a marked difference in intensity of illumination.

Quarter-undulation plate of H. Traube.†—This plate consists of two adjacent quarter-undulation mica plates so cut that

FIG. 9.

the optic axial plane of each includes an angle $3\frac{1}{2}^\circ$ with the common line of junction and for slight deviations of a crystal plate from its true position of extinction, the two halves appear unequally lighted, and only when the crystal is precisely in its position of zero extinction do the halves show the same intensity of illumination.

Twinned Selenite plate.—The use of a twinned selenite plate has been recommended recently by E. Sommerfeldt* for the accurate adjustment of the ocular crosshairs to the planes of the nicols. But the same twins can be made to serve admirably in the measurement of extinction angles. The extinction angle which the ellipsoidal axis makes in each plate with the twinning plane is $37\frac{1}{2}^\circ$, and if the twinning line on such a plate be turned to the diagonal position with the crossed nicols, the extinction angle on each side of the nicol measures $45^\circ - 37\frac{1}{2}^\circ = 7\frac{1}{2}^\circ$, but in the opposite halves different optic ellipsoidal axes are adjacent the principal nicol plane. The net result of this arrangement is a change in intensity dependent not only on

* Neues Jahrbuch, 1898, i, 251.

† Zeitschr. f. wissenschaft. Mikroskopie, xxiv, 24–25, 1907.

FIG. 10.

FIG. 10.—Mikroskop-polymeter of Voigt and Hochgesang carrying wedge holder H and also device described below for holding small crystals. The wedge holder in turn carries a cap nicol, K, and a Ramsden ocular in the focal plane of which the wedge, W, is introduced. The crystal-holding device consists of the following parts: S, the stand; C, graduated vertical circle with 1° divisions; A, centering plate; B, ball and socket adjusting device; D, crystal holder; L, plano-concave lens for holding drop of liquid of same refractive index as crystal and held by stand, E.

The mikroskop-polymeter is a useful instrument in many ways but for accurate work the mechanical workmanship on the same leaves much to be desired. The fine adjustment screw is practically worthless for even approximate readings, the accurate adjustment of the different circles to the common optic axis is not possible, and even if once accomplished does not remain in adjustment: the divisions on the different verniers are inaccurate; many other defects have been felt by the writer and several of them remedied in the workshop of the Geophysical Laboratory.—A stop diaphragm M has been introduced into the upper tube, also an optic axial angle reflector, R (compare F. E. Wright, this Jour., x:ii, 19-20, 1906); a holder for the combination wedge is shown at T, which fits on the objective supporting arm and is revolvable about the axis, an arrangement which has proved advantageous in place of revolving the microscope stage when the interference plates are inserted.—The fine adjustment screw P has been lengthened so that it can be reached from either side of the microscope. The base of the stand at N has been milled out to allow space for the reflector when the condenser lens is lowered.

the angle but also on the different compensations of the path differences in the two plates, and if white light be used this results in a rapid change in interference color in the two halves if the crystal be only a small angular distance from its position of true extinction.

The writer has had such a plate cut showing the sensitive tint and also a wedge, so that on insertion different interference colors, or intensities in monochromatic light, can be used for which the eye under certain conditions is most sensitive. These plates, as well as the preceding, are inserted in the focal plane of the objective and the junction line serves for the vertical crosshair. For such plates the Ramsden positive ocular has been found by experience to be best suited and a specially constructed holder convenient.

Artificially twinned Quartz plate.—Still another advantageous arrangement can be had by cutting on a polished quartz plate parallel to the principal axis a vertical edge making an angle of about 3° – 6° with the principal axis. The quartz plate is then divided transversely to the polished edge and the polished edges cemented together, thus producing an artificial twin whose two halves extinguish at equal and opposite angles from the common line of junction. Such plates may then be ground to a thin plate showing either the sensitive tint or dull gray of the first order or to wedge form, thus increasing the range and usefulness of the device.

All of the preceding plates, the Bravais-Stöber, the Calderon and the Traube, the selenite twin plate and the quartz combination plate wedges of the last paragraph, can be made somewhat more sensitive by dividing the field into quadrants instead of halves, after the example of Bertrand in his rotatory polarizing quartz plates described below.

Bi-nicol ocular.—In the practical application of these different types of plates the angle θ has been small (2° – 4°) and found to furnish good results, but in each case there is a particular angle θ which is best adapted for the observations; the limit of sensitiveness of different eyes introduces, moreover, a variable element of such wide range that the angle θ cannot be calculated and fixed once for all. In order therefore to have control over all angles θ and thus in each instance to be in position to procure the best possible conditions, the writer has had constructed the following ocular attachment.

The principle of construction of the apparatus is apparent from the figure, and need not be expressed at length at this point. The light after passing through the lower nicol and the crystal plate reaches the lower reflecting prism pair of this ocular and passes thence through appropriate nicol prisms (Thompson prisms) or birefracting plates of exactly the same

character, and after total reflection in the upper prism pair is again brought back to the common field of vision and viewed by the Ramsden ocular and upper nicol. The nicol prisms or birefracting plates fit in collars and can simultaneously be revolved about the axis and in opposite directions so that the angle θ can be made to vary from $+90^\circ$ to -90° in each plate and at any instant θ of the first plate is equal and opposite to θ

FIG. 11.

FIG. 11.—Bi-nicol ocular: consists of the following parts: Two pairs of reflecting prisms, P_1 and P_2 , (ground specially for the purpose by Steeg & Reuter of Homburg v. d. Höhe, Germany): two tapering revolving brass holders, M , into which either two Thompson prisms, N , or birefracting plates are introduced. These conical brass carriages are revolved in opposite directions and through equal angles by means of the worm thread S and grooved wheels T , the angle of revolution being read off directly on the head H . The bi-nicol ocular fits in the microscope as an ordinary ocular and into it in turn a Ramsden ocular is introduced and above this the cap nicol.

of the second plate. In the mechanical construction of this apparatus, special care has been taken to make the angular movements of both plates exactly equal and opposite.

With this revolving bi-nicol ocular, it is thus possible to allow any proportion of the light incident on the crystal plate

to pass through the upper nicols by simply revolving the same and at every instant to state what percentage is passing through. The particular angle of revolution θ for which the intensity of light transmitted is best suited for the maximum sensitiveness of the eye of the observer can be readily ascertained, and the actual position of extinction for any given mineral plate be determined by its use. By means of the bi-nicol ocular, the adjustment of two crossed nicol prisms can also be tested accurately and easily.

This ocular, although serviceable, suffers from one defect which it is difficult to overcome satisfactorily, namely, the depolarizing of the total reflecting prism pairs on light waves transmitted when the planes of the revolving nicols are not parallel with the planes of the polarizer and analyzer. As a result a certain amount of false light is introduced into the field and tends to veil the sharp contrast of the two halves and thus to decrease the sensitiveness of the instrument.

Bertrand plate—In place of birefracting plates, which introduce an entirely new set of conditions in the path of light waves and which complicate the expression for the relative intensity correspondingly, Klein* and Bertrand† have used the rotary power of quartz plates, cut normal to the principal axis, on the plane of polarization of normally incident, plane polarized waves. As shown above, the total effect of such a quartz plate in monochromatic light is merely to increase the angular distance θ in the intensity formula (6). This power of rotation of quartz varies with different wave lengths and with the thickness of the plate. If white light be used, interference colors result.

A quartz plate 7.50^{mm} thick shows the sensitive tint under crossed nicols and can be used to good advantage in measuring extinction angles, since for slight deviations of the crystal plate from the position of true extinction its interference color rises or falls. To increase its sensitiveness, Bertrand combined two plates (2^{mm} in thickness) of right-handed with two plates of left-handed quartz, so that each right-handed plate is adjacent to a left-handed plate. This plate is inserted in the focal plane of the ocular and the sharp junction lines serve as crosshairs. The Bertrand plate can be used in monochromatic light, provided for the particular wave length used, its angle of rotation is not a multiple of π , in which case darkness ensues and the observed effect is nil. By revolving the upper nicol it is possible in white light to bring out the sensitive interference tint over the entire field covered by the Bertrand plate, and in such a position a very slight turn of an intervening crystal from its position of true extinction is sufficient to disturb this equality

* Neues Jahrbuch, 1874, p. 9.

† Zeitschr. Kryst., i, 69, 1877.

of interference color and to divide the field into four quadrants, the opposite sections of which are similarly colored, while adjacent sections are differently colored.

The Bertrand plate is best adapted for use in white light, although it is possible to use it in monochromatic light provided its thickness be correct for the particular wave length employed.

The quartz half shade plate of S. Nakamura.—In a recent paper,* S. Nakamura discussed the problem of the sensitiveness of the half shade system and arrived at practically the same conclusions as those noted above. He suggests the use of a double quartz plate of $.4^{\text{mm}}$ thickness instead of 3.5^{mm} or 7^{mm} thick as in the Bertrand ocular, and by actual tests finds the theoretical deductions valid and the plate useful. The thickness of $.4^{\text{mm}}$ is equivalent to an angle $(90-\phi)$ of about 8.67° on each side of the junction line; under certain conditions of illumination this angle is undoubtedly the best, and with the plate the accuracy of the measurements thereby attained equal to that of any of the other measuring devices.

Bi-quartz wedge-plate.—It is possible, however, to construct a combination wedge of quartz plates of such a character that any angle of rotation from 0° to any other value, positive or negative, can be had on insertion of the wedge, thus adapting to wedge form the advantage of the revolving bi-nicol ocular. This has been accomplished by combining two plates of quartz cut normal to an axis and of specified thickness, the one of right-handed, the other of left-handed quartz, each with a wedge of quartz of the opposite sign of rotary polarization, as indicated in figure 12.†

The effect of this combination is to produce zero rotation in each half wedge where plate and wedge have the same thickness and as the wedge is inserted or drawn out from this point of zero rotation the angle of rotation increases proportionately and in a positive sense on one side of the junction line of the combination and in a negative sense on the opposite half. This combination wedge, which is introduced at the focal plane of the ocular, divides the field under crossed nicols into two halves, the intensity of color of which at any instant is equal, provided no intervening crystal plate is present or is rendered inactive

* Centralblatt f. Mineralogie. 1905. 267–279. Compare also J. Macé de Lépinay, Jour. de Phys. (2), iv, 267. 1885; (3), ix, 585, 1900. Unfortunately this paper did not come to notice until after the manuscript of the present article had been sent to the press and the mathematical discussion by S. Nakamura, which considers the problem from a somewhat different standpoint, could not well receive the analysis and recognition which otherwise might have been given it in the general theoretical part.

† Made for the writer by Steeg & Reuter of Homburg v. d. Höhe, Germany. Cost, 100 marks. The accuracy of this wedge was tested by the writer and the grinding found to be exceptionally perfect.

by the parallelism of its ellipsoidal axis with the principal planes of the nicols. So soon as the crystal is turned even a very small angle out of this position, the intensity of illumination of the two fields is no longer equal. By inserting or withdrawing the combination wedge, the most advantageous angle of rotation in the two fields can be procured so that the difference in intensity between the two halves is most apparent. In effect this wedge is identical with that of the bi-nicol ocular described above, is much simpler in con-

FIG. 12.

FIG. 12. Bi-quartz wedge-plate. In the plate-wedge ground for the writer the two quartz plates are 35^{mm} long, 6^{mm} wide, and $.35^{\text{mm}}$ thick. The wedges are $.3^{\text{mm}}$ thick at the one end and $.5^{\text{mm}}$ at the thick edge. Cementing material is Canada balsam whose refractive index is 1.54, while ω for quartz is 1.544, a difference so slight as to render inappreciable the exceedingly slight deviation of the waves caused by the slight wedge surface of the wedge. This inclined surface is mounted next the Canada balsam and care is taken by inserting a thin glass strip at the thin end to make the upper and under surfaces of the completed wedge parallel. The thickness of the wedge is $.9^{\text{mm}}$. At the one end the rotation is $\pm 1.1^\circ$; at the thick end, $\pm 3.2^\circ$ for sodium light, while at $.85^{\text{mm}}$ from the thin edge the rotation is zero in both halves. For a wedge-plate of an angle of rotation 0° to 10° the following specifications are suitable: length 50^{mm} , width of each half 6^{mm} , total width of wedge 12^{mm} ; thickness of plate, $.4^{\text{mm}}$; thickness of wedge at thin end $.35^{\text{mm}}$; at thick end, $.85^{\text{mm}}$. In such a wedge the point of zero rotation is 5^{mm} from the thin end. At the thin end the rotation is $\pm 1.1^\circ$; at the thick end, $\pm 9.9^\circ$. In the article following this, specifications for a wedge with rotation of 15° at the thick end are given. In preparing the wedge it is necessary that the edges be ground and polished in order that the central division line (fig. 11) be as sharp as possible. The two halves are eventually cemented side by side with Canada balsam and any disturbing influence thus eliminated which might arise from total reflexion on the sides.

struction, and requires no adjustment; the one condition which must be fulfilled for satisfactory results is that the wedge be not tilted on insertion but that the optic axis remain always parallel with the optic axis of the microscope, otherwise disturbing birefringence phenomena appear. The wedge carriage should, therefore, slide in an accurately fitting holder such as

shown in fig. 10 above, which was constructed in the workshop of the Geophysical Laboratory.

Methods involving revolution of upper nicol.—In all of the preceding methods the nicols have been considered crossed and the crystal plate revolved. The intensity formula shows, however, that the relative intensity is dependent not only on the angle θ of the crystal plate but also on ϕ , the angle between the principal planes of the nicols. It was shown in the general mathematical treatment that this method is in general at least twice as sensitive as the method based on the revolution of the crystal plate under crossed nicols. The mode of application of this method to any particular crystal plate is obvious and consists simply in placing the crystal under crossed nicols in its position of apparent true extinction and then observing, either in white or monochromatic light, the changes which occur on revolving the upper or lower nicol through small angles with its normal position. In case the crystal is actually in its position of true extinction, the crystal and field attain their position of maximum darkness simultaneously and show the same increase in its intensity of illumination; if, however, the crystal be not in its position of true extinction, but a small $+$ angle, as $30'$ distant, then for a position of the nicol $+2^\circ$ from its normal position, the crystal plate will appear lighter than the field; and vice versa for the nicol -2° from its normal position the crystal plate will appear darker than the field. This method is extremely simple in manipulation and does not require special apparatus, but seems not to have been applied before to the measurement of extinction angles. Weinschenk,* in describing the adjustment of the nicols in the microscope, uses the interference phenomena which occur under these conditions, but does not appear to have applied conversely the principle to the practical determination of the optic ellipsoidal axis in a given crystal plate.

To double the sensitiveness of this method of revolving the upper nicol, the bi-nicol ocular attachment of fig. 11 can be used. By this device alone, without the upper nicol used in the above methods, the two halves of the field in the ocular preserve the same intensity of illumination at every instant, provided no disturbing crystal plate intervenes. If the position of the latter does not coincide precisely with its true position of zero extinction, the two halves of the field appear unequally illuminated and by revolving the nicols that position of the nicols can be found for which the effect is most pronounced for a given angle θ .

In its effect the bi-quartz wedge plate is identical with the revolving bi-nicol scheme, and has the advantage of requiring

* Zeitschr. Krystall. xxiv, 581-583, 1895.

no adjustment and of not suffering from the false light of depolarization noted above.

Convergent polarized light.—Two methods have been proposed which require convergent polarized light and are based on the change in aspect of symmetrical interference figures caused by the intervening crystal plate when it is not precisely in the position of zero extinction. The idea underlying the methods is that the eye can detect more readily slight changes in the shape of a symmetrical interference figure than proportionate changes in intensity or color. Theoretically, this principle is excellent, but its practical application to mineral sections is less satisfactory. The first method of this type was proposed by Kobell in 1851, who used a plate of calcite normal to the optic axis as his test plate. The microscope was arranged for convergent polarized light and the crystal plate with the calcite test plate above it placed on the microscope stage and turned until the interference figure appeared perfectly normal and undistorted. Practically, the following objections apply to this method. The optical system of the microscope requires changing each time to meet the new conditions; during the observations the crystal itself is lost sight of, and in the case of minute crystals or crystals with undulatory extinction this is a serious drawback. Moreover it is tacitly assumed that in the crystal plate itself for directions other than the normal to its surface of the crystal plate the planes of polarization remain parallel, which in general is only approximately true even for small fields which include only a small angle with the normal.

In the Brezina† method a more complicated interference figure is produced by two calcite plates cut at a small angle with the optic axis and cemented together one above the other in such a way that the optic axes of the two are in the same plane and at equal angles with the normal. The interference figure from such a combination is noteworthy because of a dark vertical bar through the center of the field. A slight revolution of an intervening crystal plate displaces this bar noticeably, but the same objections noted in the Kobell method apply with equal force to this method, with the result that neither method is made use of at the present time by working petrologists. In fact, both these methods were suggested before the petrographic microscope had been introduced.

The relative sensitiveness of the different methods.—The term position of extinction means practically that position of a birefracting plate for which light waves are transmitted without changing their plane of polarization and for which no light

* Pogg. Ann., xcv, 320, 1855.

† Described in Schrauf's Lehrb. d. Phys. Min. ii, 219-220, 1868.

passes the upper nicol, i. e., the field is just as dark as though no crystal plate were there. A revolution of the plate through a very small angle from its position of true extinction allows an equally small percentage of the total amount of incident light through the upper nicol and the field is very dimly illuminated. For a given angle of revolution, the actual amount of transmitted light can be increased only by increasing the original source of light. Since, however, it is not possible to increase the intensity of such a source indefinitely, and the human eye is sensitive only to a certain limit, the position of actual extinction can only be determined within a definite degree of exactness. By means of the above devices, however, certain phenomena are introduced which increase the accuracy of such a determination, even though the field of original illumination remains the same. That method or device is obviously the best for which the probable error of a single determination under the same conditions is the least.

In comparing the relative accuracy of the methods described above it will facilitate the presentation to assume definite conditions and then by means of the theoretical intensity curves (figs. 2-7) to test the results attainable by the different methods under the most favorable conditions.

Let it be assumed that under the conditions of experiment the eye of the observer is of such sensitiveness that he is able to detect .05 of one per cent of the total light intensity; in other words, he can just detect the difference between the dark field of the microscope under crossed nicols and a crystal section turned at such an angle as to allow .05 of one per cent of the total intensity through the upper nicol. For all positions of the crystal, then, for which the intensity of the emergent light is less than .05 per cent, the crystal will appear absolutely dark. The heavy curves in figs. 3-7 indicate the relative intensity of illumination of a crystal under crossed nicols for all positions of its major ellipsoidal axis from 88° to 92° or -2° to $+2^\circ$ with the plane of the polarizer; in fig. 3 there is an interval of $38'$ at least on each side of the true extinction position for which the eye is unable to detect any interference illumination. The possible error on a single determination under the most favorable conditions is in this case at least $\pm 38'$ while for fig. 4 it is $\pm 44'$; for fig. 5 ($K=\frac{1}{2}$) $\pm 55'$; for fig. 6 $\pm 1^\circ 17'$; while for the $K=0$ the crystal is dark for all positions. In any crystal, therefore, the conditions are most favorable when the plate is of such thickness that $K=1$ or the emergent waves are half a wave length apart (in opposite phase). Conversely, having given a crystal plate, not all wave lengths are best adapted for extinction-angle measurements. If yellow

sodium light be used, a plate showing the sensitive violet interference tint is worthless since for that tint the path difference is $572\ \mu\mu$, nearly a whole wave length of Na light ($589\ \mu\mu$), and for this difference $K = 0$ and fig. 7 applies. If sodium light be used, then plates should be chosen for which the phase difference of the two emerging waves is $\frac{(n+1)\lambda}{2}$, bright yellow of the first order or pure yellow of the second order or green yellow of the third order, etc. This is an important consideration and applies to all methods involving the intensity equations.

The visible spectrum includes wave lengths ranging from about $400\ \mu\mu$, and $700\ \mu\mu$, the interference color in white light ranges from about red of the first order to blue of the second order; in short, the sensitive interference tint region of the Newton color scale as determined by G. Quincke.* For this interval the distance between the emergent waves is not far from a whole wave length for the major part of the visible spectrum; in other words, the phase difference is such that K is a small fraction not greatly different from zero and the intensity curves for practically all wave lengths will be covered by 5-7. These are, however, the least favorable for showing difference in intensity and such plates are, therefore, the least suitable for the measurement of extinction angles by methods based on intensity differences. On the other hand, plates showing interference colors gray to yellow of the first order are best suited for such measurements. If the methods involving interference tints be used, however, these objections do not hold with equal force. Experience has shown that in case the mineral plate does show red or blue interference tints of the first and second orders the best determinations can be made either by the method of revolving the upper nicol or by the bi-quartz wedge plate, and the true position fixed by noting the absence of abnormal interference colors on revolving the nicol very slightly or inserting the wedge.

After this digression on the most suitable sections for the measurement of extinction angles, fig. 3 may again be considered and the relative accuracy of the different methods under the same conditions of experiment deduced.

The heavy curve indicates that for the assumed limit of sensitiveness .05 per cent of the total intensity, an error of at least $\pm 38'$ on a single determination is possible by revolving the crystal plate alone under crossed nicols. On the other hand, if the crystal plate remains stationary and the upper nicol alone is revolved, the other intensity curves of fig. 3 are valid, each curve indicating the intensity of illumination of

* Pogg. Ann., cxxix, 177, 1866.

the crystal plate for a specified angular distance from its position of true extinction during the revolution of the upper nicol from 88° to 92° . These curves indicate that the probable error with this method is less than half as great as in the preceding method, for if the crystal be only $\pm 15'$ distant from its position of true extinction, differences in intensity can even then be detected on revolving the upper nicol.

The changes in intensity of illumination of the microscopic field on revolution of the analyzer are indicated by the 0' curve, while for the crystal plate the 15' curve is applicable. At $88^{\circ} 43'$ (fig. 3) the field is just beginning to show detectable illumination (0.05 per cent of total intensity), while for the same angle the crystal is illuminated with 0.97 per cent of the total intensity, nearly twice as great and easily noticeable. In this position the crystal plate appears, therefore, decidedly lighter than the field. On the other side of 90° the crystal plate passes its limit of light sensibility under the assumed conditions at $91^{\circ} 42'$, while for the same angle the microscopic field is illuminated by 0.97 per cent of the total intensity; in this case the field is appreciably brighter than the crystal and the difference can be readily detected by the eye.

If white light be used, these differences are accentuated by the abnormal interference colors which appear in the crystal plate when it is not precisely in the position of true extinction. This method of revolving the upper nicol has the advantage, furthermore, of not being dependent on the accuracy with which the nicols are crossed, since all data are referred at once to the plane of the analyzer. It is not, however, so advantageous in very weakly birefracting or deeply colored mineral plates.

The sensitiveness of the latter method can, moreover, be doubled by devices which allow the phenomena on both sides of the 90° position to be observed simultaneously. This is the end striven for in the ocular plate, the Bravais-Stöber plate, the Calderon plate, the Tranbe plate, and accomplished most effectively by the new circularly polarizing bi-quartz wedge and also by the bi-nicol ocular, though less satisfactorily. In each of these last two devices the plane of polarization of the incident waves is turned through equal angles on both sides of the junction line of the two parts, so that the field appears equally lighted throughout, while if the crystal plate be not in its position of true extinction, it will appear lighter than the field in the one half and darker in the second. Since, however, there is an angle best suited under the given conditions to show these differences most clearly, it follows that the best results can be had with a plate or apparatus in which the

angle ϕ can be varied at will. This is true of both the circularly polarizing wedge plate and the bi-nicol ocular, and by their use the probable error of the extinction position of any crystal plate is at least one-fourth that of a determination after the usual method by revolving the crystal plate under crossed nicols. Experience has shown that with favorable sections, extinction angles can be determined by the use of the bi-quartz wedge with a probable error of less than $\pm 10'$ on a single trial.

Still another method for obtaining the most favorable conditions of experiment with a given plate is that suggested on page 374 with the artificially twinned quartz wedge. The two halves of this wedge extinguish at a small angle (as 3°) on opposite sides of the line of junction, and by inserting the wedge that particular interference color, or phase difference if monochromatic light be employed, can be produced for which the given angle of revolution (3°) is the best. This wedge, however, is less favorable than the circularly polarizing bi-quartz wedge, since its twinning line must be inserted precisely parallel with the plane of the polarizer, while with the circularly polarizing bi-quartz wedge the rotation of the planes of polarization of transmitted waves is entirely independent of the line of junction of the adjacent halves.

In the preceding pages, special emphasis has been placed on those methods for measuring extinction angles which are of general application and which are based on intensity differences. The other methods, which are of limited application, and can be used only in white light on favorable sections, depend on differences in interference colors produced by slight deviations of the crystal plate from its position of true extinction. Although these methods are serviceable in many instances, their application and the results obtained thereby are so dependent on the conditions of experiment that they are difficult to treat satisfactorily in a general way. Experience has shown that they are not more sensitive than the other methods and usually much less so. This is true both of the selenite sensitive tint plate and of all combinations of the same.

Experimental Tests.—To test the different methods under different conditions, different mineral plates were chosen and the position of true extinction on each determined by the different methods under precisely the same conditions of illumination with white light.—On an anhydrite plate showing white interference tints of the higher orders the possible error of a single determination by revolving the crystal plate under crossed nicols was found to be about 1.1° ; by revolving the upper nicol alone, $.4^\circ$; by inserting the quartz wedge, about 0.1° ; by using the Calderon ocular, about $.5^\circ$; by means of

the Bertrand ocular, about 0.1° ; with such a plate the sensitive tint plate is of no value since the interference color of the anhydrite plate itself is so high that the violet of the inserted plate has no effect and the differences in intensity which occur, do so in a strongly lighted field and are not easily discernible.—Similar measurements were made on an apatite plate parallel to 10 $\bar{1}0$ and showing the interference tint, red of the first order. The possible error of a single determination of the position of true extinction on turning the crystal plate alone under crossed nicols was found to be $\pm .9^\circ$; on revolving upper nicol about $.2^\circ$, accurate because of abnormal interference colors which appear when the plate is distant only a slight distance from its correct extinction position; on inserting the bi-quartz wedge plate 0.2° to 0.3° ; with the Calderon ocular, about 0.3° ; with the Bertrand ocular, about 0.3° ; the sensitive tint plate is again of no value since the interference color changes comparatively slowly as crystal is revolved.—A section of nephelite parallel to 10 $\bar{1}0$ and showing the interference color, yellow first order, gave the following results: On revolving the crystal plate alone, possible error $.4^\circ$; on rotating upper nicol, less than 0.1° ; with bi-quartz wedge less than 0.1° ; Calderon ocular about 0.2° ; Bertrand ocular less than 0.1° ; sensitive violet plate still of very little value as a method; only slight changes in color for large angles of revolution of plate. On a plate of colorless gehlenite of very low interference color, dull gray, first order, the sensitive tint plate proved as satisfactory as any other and more so than the method of turning the crystal plate under crossed nicols or of revolving the upper nicol or the Calderon ocular. The Bertrand ocular and the bi-quartz wedge plate proved about as favorable, the probable error being slightly less than 0.5° .—A plate of strongly pleochroic tourmaline was also used and the following results obtained: Probable error of determination on revolving crystal plate alone, about 1.6° ; the method of revolving upper nicol is of little value because of deep natural color of mineral and consequent inability to match fields; with the bi-quartz wedge plate 0.3° ; Calderon ocular, about 0.4° ; Bertrand ocular, about 0.5° . The sensitive tint plate is useless because of strong natural color of mineral which veils the true interference colors.

The results of these tests show that the theoretical deductions from the general equations are in general valid, but that in certain instances other factors, as natural color and very low birefringence, become dominant and tend to render some of the methods less sensitive and to favor the use of other, in general less suitable methods. The bi-quartz wedge plate, however, seems to apply in all cases with equally favorable

results and to equal in sensitiveness any of the methods, whether of local or of general application.

2. *The Adjustment of the Petrographic Microscope with Special Reference to the Measurement of Extinction Angles.*

A petrographic microscope in perfect adjustment should satisfy the following requirements: (1) Its optical system should be accurately centered; (2) the axes of revolution of all revolvable parts, whether stage or ocular, should coincide with the optic axis of the microscope; (3) the principal planes of the nicols when crossed should be precisely 90° apart; (4) the crosshairs of the ocular should be parallel with the principal planes of the nicols. Of the four conditions, the first two can be accomplished without difficulty, and with the adjustment screws fitted on every petrographic microscope. The last two, however, require special appliances for accurate adjustment, and without these can be effected only with difficulty.

The test usually applied in ascertaining the correct position for crossed nicols is that of the Bertrand ocular. A cap nicol is used over the ocular and turned until the field of the Bertrand ocular shows uniform intensity of illumination throughout. This can be accomplished readily and with an error of less than $\pm 15'$ if strong illumination be used. The cap nicol is then revolved through an angle of 90° , the lower nicol removed in its carriage, the upper nicol inserted and tested by the cap nicol in its new position and adjusted until it is actually crossed. The Bertrand ocular, however, furnishes only one angle of rotation for the emerging waves, and allows of no variability in this angle to meet different conditions in the best way possible. This can be accomplished, however, by use of the circularly polarizing bi-quartz wedge plate described above or the bi-nicol ocular. With the bi-quartz wedge plate the cap nicol is unnecessary and crossing of the microscope nicols can be tested directly. For this purpose, parallel light should be used and the entire lens system, both condenser lenses, objective and ocular, removed from the microscope; parallel incident rays are then allowed to fall on the reflector of the microscope (either sun rays or strong white light or the rays from a Nernst filament or arc light emerging at the focal point of a large lens). The parallelism of the incident rays is necessary and an important factor, since with the thick quartz plates a slight deviation from normally incident and parallel light produces disturbing interference phenomena. The bi-quartz wedge-plate in its metal casing may then be placed on the microscope stage and with it the accuracy of the crossing of the nicols tested directly, just as the position of zero extinction of a mineral plate is tested. The error of such a determination should be considerably less than $10'$ of arc.

The final step in the adjustment is the alignment of the crosshairs of the ocular with the principal planes of the nicols. Many methods have been suggested for this purpose which may be used to advantage. A mineral showing good cleavage or lines of growth and parallel extinction (flakes of anhydrite or crystallites of quartz) serve well for the purpose. These are first placed in the position of zero extinction (determined accurately by means of bi-quartz wedge plate), and the crosshairs of the ocular brought to parallelism with the crystal edge or cleavage line. The chief difficulty in this method lies in the fact that it is exceedingly difficult to obtain suitable material.

E. Sommerfeldt* has recently suggested the use of a twinned plate of selenite of sensitive tint. The plate is turned until under crossed nicols the interference colors in the two halves are of precisely the same tint, in which position the twinning line parallels the principal planes of the nicols. This method is sensitive and satisfactory, especially if, instead of a plate, a wedge of the material be used, with which the interference color can be changed until that particular tint, for which the observer's eye is most sensitive, covers the field. In this connection care should be taken to select a selenite plate in which the twinning line is perfectly straight. In the selenite plate, however, the angle which the adjacent ellipsoidal axis in each half makes with the twinning line is $37\frac{1}{2}^\circ$, a very large angle and not so well adapted to show slight deviations from the true position as a twin of smaller symmetrical angle. In this connection experiments with plagioclase lamellæ were tried but abandoned, since it was found by experience to be exceedingly difficult to procure suitable material for the purpose; a simpler method can, however, be used, which accomplishes the same purpose more readily. On a thin plate of quartz, cut parallel to the principal axis, an edge surface, making an angle of 4° – 6° with the principal axis, is first ground and polished. The plate is then cut in half at right angles with the polished edge surface and the two halves placed with their polished edges side by side and cemented with canada balsam, thus producing an artificial twin of any angle which may be selected as most suitable and best adapted for fixing the crosshair in the ocular. The twin is finally ground thin and polished either to the sensitive tint or a pale gray of the first order or in wedge form.† Like the selenite twin plate, it is placed on the stage of the microscope and revolved under

* *Zeitschr. für wissenschaftliche Mikroskopie*, xxiv, 24–25, 1907.

† Such a plate and also wedge were prepared for the writer by Voigt & Hochgesang of Göttingen, and have proved satisfactory in every respect. The twinned selenite plate of sensitive tint cost 5 marks: the wedge (I–III order interference colors), 21 marks.

crossed nicols until its halves show equal intensity of illumination, in which position the line of junction fixes the direction for one of the crosshairs of the ocular.

By use of the bi-quartz wedge plate and the artificial quartz twin plate or wedge, the adjustment of the nicols and also of the crosshairs in the ocular is a matter of only a few moments, and the method followed is theoretically and practically more accurate than the other methods for adjustment which have been suggested.

3. *A Device for Holding Small Crystals for the Purpose of Measuring Extinction Angles in Zones ; also for Measuring the Optic Axial Angle of such Fragments directly.*

In measuring the extinction angles of certain minute artificial pyroxene and other crystals in the prism zone, the writer has had occasion to use the following holding device or finger, which has proved both convenient and practical.* (Fig. 10 on stage of microscope.)

As indicated in the figure, it consists of two parts, a holding or clamp device for the crystal itself, and a universal ball and socket joint and centering plate for adjusting and centering the crystal, and a vertical circle for reading any specified angle of revolution of the crystal. A small crystal (1–2^{mm} in length) thus held and adjusted is immersed in a drop of liquid of the same refractive index and thus the disturbing phenomena of refraction and total reflection eliminated. The liquid drop is contained in the concave side of a planoconcave lens of 6^{mm} diameter (ground for the purpose by the Scientific Shop of Chicago), which in turn is held by an adjustable and supporting arm.

In work with artificial preparations particularly it is often desirable to measure extinction angles on certain minute faces, or optical axial angles, of small fragments too minute for the optic axial angle apparatus and not suitable for measurement by one of the microscopic methods,† and this device has been made to fill that want. After adjustment and immersion in a liquid of refractive index β the optic axial angle can be read off directly on the vertical circle of this apparatus.

Summary.

The measurement of extinction angles of minerals in the thin section is one of the most common methods of petrographic microscopic practice, and at the same time one of

* This apparatus was made in the workshop of the Geophysical Laboratory at the instigation of the writer, and can be constructed by any good mechanic.

† Compare F. E. Wright, Measurement of the Optic Axial Angle of Minerals in the Thin Section, this Journal (4), xxiv, 317–369, 1907.

the least satisfactory when accurate results are desired. It is an exceedingly easy matter to measure, with one trial only and on favorable sections, extinction angles with a probable error of $\pm 1^\circ$ to 2° , but to do so within $\pm 1^\circ$ is a very different matter. In the foregoing pages the problem is first discussed theoretically, and the mathematical equations covering the different methods applied to extinction angle measurements derived and discussed briefly. The methods for this purpose may be grouped into two classes,—those of general application and those of limited applicability. The first class may again be subdivided into two classes; either (1) the crystal is revolved between crossed nicols about its position of true extinction, or (2) the crystal remains stationary, and the accuracy of its position of true extinction tested by revolving the upper nicol or by inserting one of several different optical devices to increase the sensitiveness of the test under the prescribed conditions of illumination. These devices include the Calderon ocular, the Bertrand ocular, the Bravais-Stöber plate, the Traube plate, also twinned plates and wedges of selenite, artificially twinned plates and wedges of quartz, the circularly polarizing bi-quartz wedge plate and the bi-nicol ocular, all of which are described briefly above. Of these devices the last two are the most universal and can be so used under any given conditions of illumination that the phenomena observed are the most sensitive possible to attain by devices of this type.

On comparing the relative sensitiveness of the different methods under the same conditions, it is found that the method of testing the position of true extinction for the crystal by revolution of the upper nicol is, on colorless mineral-plates, at least twice as sensitive as that of simply turning the crystal to its position of apparent maximum darkness under crossed nicols. Under the same conditions the methods requiring the use of one of the several plates or wedges mentioned above are at least four times as sensitive as the ordinary method. With the exception of the last two devices, however, these different plates do not furnish equally sensitive results for the different conditions of illumination which may arise. In accurate work adjustable sensibility is of prime importance, particularly if a given device is to be of general application. These requirements are best filled by the bi-quartz wedge plate, by means of which the angle of rotation can be varied from 0° to any desired angle. The two halves of this wedge rotate in opposite directions, and on insertion that angle of rotation can be secured for which the contrast in the intensity of the halves of the field is most striking for a slight deviation of the crystal from its true position of extinction.

An application of the bi-quartz wedge plate, together with an artificially twinned quartz-plate or wedge, to the accurate adjustment of the petrographic microscope is considered in outline in the second part above, and a method of procedure for accomplishing the same indicated.

In part 3 a simple device for holding and rotating small crystals for the purpose of determining extinction angles or for measuring optic angles directly, is described briefly, such an apparatus having been found peculiarly useful in work with artificial crystals.

Geophysical Laboratory,
Carnegie Institution of Washington,
Washington, D. C., June, 1908.