science generally from the standpoint of descriptive natural history, in which it is just the subject-predicate proposition that is most prominent. It was in this domination of Aristotelian thought by the subject-predicate proposition with "existent" subject which I had in mind in writing the words referred to by Mr. Ross, and I do not see that their force, whatever it may be, is lessened by appealing to the predominance of mathematical illustrations in the *Posterior Analytics*. Of course, in any case, examples of rigidly demonstrated results are bound to be taken from the mathematical sciences; my point was that Aristotle's conception of the nature of the proposition and of the scope of deduction is defective in just those respects which might be expected of a thinker who came to the study of logic with an unconscious bias derived from an early training in biology and natural history.

Incidentally, may I suggest that at 987 and 10 *μετρίωτερον* perhaps derives some confirmation from a rather similar use of the word in Sextus *Empiricus adversus grammaticon*, 288. *ὁ μὲν σκεπτικός ἀναγορευθεὶς φιλόσοφος ἔτι μετρίωτερος φαίνεται, λέγων μὴ εἰδέναι ὃ προσεύχεται*, which appears to mean "the poet who got the title of the 'philosopher of the stage' will be found even more unsatisfactory, since he confesses that he does not know who it is to whom he prays". (Sextus had just said that the opinions of the poets about theology are *worse* than those of illiterate men.) Possibly, however, *μετρίωτερος* is here ironical, "even more modest in his pretensions". And may this not, after all, be the sense of the phrase of Aristotle; the philosophers enumerated "treated the subject less pretentiously" than their successors; the implication being that their "unpretentious treatment was also inadequate". Of course I should assume that if *μετρίωτερον* is the right word in one passage, the other readings represent *deliberate* conjectural emendations of copyists who were dissatisfied with the text before them.

A. E. TAYLOR.

'IF' AND 'IMPLY,' A REPLY TO MR. MACCOLL.

In the January number of *MIND*, Mr. MacColl accuses me of holding premisses from which it would follow that either every red-haired Englishman is a doctor, or every English doctor is red-haired. The reason why Mr. MacColl supposes me to hold these premisses (which I do not hold) is that he has failed to grasp what seems to me a vital distinction, namely that which I call the distinction between *propositions* and *propositional functions*. The nomenclature is unimportant; but what is important is to remember that what Mr. MacColl calls a proposition is usually something which I should not call a proposition, so that when my statements about propositions are interpreted as applying to what Mr. MacColl calls propositions, the result, naturally, is apt to be curious.

Let us illustrate the distinction between propositions and propositional functions by the case which Mr. MacColl chooses. He says (p. 152): "Take the two statements 'He is a doctor' and 'He is red-haired,' each of which, observe, is a *variable*, because it may be true or false. Is it really the fact that one of these statements implies the other?" It will be observed that what I affirm (as quoted by Mr. MacColl on the previous page) is, that "of any two *propositions* there must be one which implies the other". It by no means follows that of any two *statements* one must imply the other. Mr. MacColl assumes it undecided who 'he' is to be, for as soon as it is decided, the statement ceases to be 'variable'. Now I should not call 'he is a doctor' the expression of a proposition unless, from the context, it is known who 'he' refers to, and then it is a mere abbreviation for (say) "Mr. Smith is a doctor". This is a pro-

position, but it is not variable: Mr. Smith either is or is not a doctor. Of the propositions 'Mr. Smith is a doctor' and 'Mr. Smith is red-haired,' it is easy to see that one must imply the other, using the word 'imply' in the sense in which I use it. (That this is not the usual sense, may be admitted; all that I affirm is that it is the sense which I most often have to speak of, and therefore for me the most convenient sense.) I say that p implies q if either p is false or q is true. This is not to be regarded as a proposition, but as a definition. Now consider 'Mr. Smith is a doctor' and 'Mr. Smith is red-haired'. Four cases are possible, namely: (1) both are true, (2) both are false, (3) the first is true and the second false, (4) the first is false and the second true. In cases (1) and (2) each implies the other according to the above definition. In case (3) the second implies the first; and in case (4) the first implies the second. These facts are all immediate consequences of the above definition of 'implies,' together with the fact that a disjunction is true when either or both of its alternatives is true. Hence in all four cases, at least one of our two propositions implies the other.

It will be seen, however, that we do not always have "'He is a doctor' implies 'he is red-haired,'" nor do we always have "'He is red-haired' implies 'he is a doctor'". We have always one or other (or both), but not always one, nor yet always the other. Hence, when 'he' is not specified, we cannot affirm "'He is a doctor' implies 'he is red-haired,'" nor yet "'He is red-haired' implies 'he is a doctor'". But when 'he' is unspecified, 'he is a doctor' and 'he is red-haired' have no definite meaning: they are forms awaiting determination. They are what I call 'propositional functions,' not what I call 'propositions'. It is customary to suppose that 'propositions' are whatever is either true or false, but 'he is a doctor' is neither true nor false until 'he' is specified; for this reason, it seems undesirable to call it a proposition. I should not, however, object to Mr. MacColl's calling it so, if he would realise that it is something different from the things that are true or false, and from the things that I call propositions.

The formal statement of the above is as follows. Let ϕx , ψx be two propositional functions, i.e. expressions which, so soon as we give a definite value to x (such as 'Mr. Smith'), become propositions. Then for every possible value of x , either ϕx implies ψx or ψx implies ϕx . But it is not in general true that either ϕx implies ψx for every possible value of x , or ψx implies ϕx for every possible value of x . Hence, if x is not assigned, we cannot assert ' ϕx implies ψx ,' and we cannot assert ' ψx implies ϕx ,' because sometimes the one is true and sometimes the other, according to the value assigned to x , and we can only assert an ambiguous expression containing an undetermined x when, however, we may determine x , the resulting proposition is true—as, for example, when we say ' x is x '.¹ We thus arrive at Mr. MacColl's division of "statements" into the three classes of *certain*, *doubtful* and *impossible*, but "statements" must be understood as meaning "propositional functions," not "propositions". A propositional function ϕx is *certain* when all its values are true, *impossible* when all its values are false, and *doubtful* when some of its values are true and some false. But the values themselves, which are "propositions," are subject, not to this threefold division, but merely to the twofold division into *true* and *false*.

B. RUSSELL.

¹ For further explanations on the above question, see my *Principles of Mathematics*, pp. 12, 13, 22.