Missing-Figure Problems

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MISSING-FIGURE PROBLEMS.

BY W. E. H. BERWICK.

IN a recent number of the *Mathematical Gazette* (March 1920) I proposed the following problem for solution :

THE FOUR FOURS.

$$\begin{array}{r}
 \times \times \times) \times \times \times \times \times 4 (\times 4 \times \times \\
 \underline{\times \times \times} \\
 \times \times 4 \times \\
 \underline{\times \times \times \times} \\
 \times \times \times \times \\
 \underline{\times 4 \times} \\
 \times \times \times \times \\
 \underline{\times \times \times \times}
 \end{array}
 \begin{array}{l}
 (1) \\
 (2) \\
 (3) \\
 (4) \\
 (5) \\
 (6) \\
 (7) \\
 (8)
 \end{array}$$

Some of the figures in the long division sum have been replaced by crosses, and it is required to restore them. Four solutions are possible—of course in scale ten. It is understood that the sum is to be worked as in elementary arithmetic and that there is nothing fictitious. There must be no line of zeros, neither must the left-hand figure of any line be zero. In the divisor the digits figure ought not to be zero (for such a zero would ordinarily be cancelled before working the sum), but it is not enough to assume this: proof must be possible from the data. In the FOUR FOURS the proof is instantaneous, because there is no remainder and the right-hand figure of the dividend is 4.

The reasoning which gives rise to the solution of the FOUR FOURS is as follows :

(i) Let the divisor be $D \equiv abc$ and the quotient $Q \equiv def$, the notation being that of positive integers in scale ten. Since $e.D$ and $4.D$ are respectively three-figure and four-figure numbers [from lines (6) and (4)], e cannot exceed 3. Further $(e+1).D \geq 10^3$ otherwise, from line (5), the third figure in the quotient would be $e+1$ at least.

(ii) Trying $e=3$, $3.D = \times 4 \times$ and $4.D \geq 1000$. The sixth line is $84 \times$ or $94 \times$ and $f \geq 4$. Since $cf \equiv 4 \pmod{10}$ from the 4 in the dividend we have as possibilities for D and Q :

$$\begin{array}{rcl}
 D & - & 280, \quad 281, \quad 282, \quad 283, \quad 314, \quad 315, \quad 316, \quad 316, \\
 Q & - & \times 434, \quad \times 437, \quad \times 438, \quad \times 436, \quad \times 434, \quad \times 439.
 \end{array}$$

All these have to be rejected through their failure to satisfy one (or both) of the considerations :

- (α) $D.ef$ is a five-figure number from line (5),
 (β) the third figure in $D.4ef$ is 4 from line (3).

Accordingly $e \neq 3$.

(iii) Next trying $e=2$, reasoning similar to that in (ii) gives the following possibilities for D and Q :

$$\begin{array}{rcl}
 D & - & \begin{cases} 370, & 371, & 372, & 373, & 374, \\ 420, & 421, & 422, & 423, & 424, \\ 470, & 471, & 472, & 473, & 474, \end{cases} \\
 Q & - & \times 424, \quad \times 427, \quad \times 428, \quad \times 426.
 \end{array}$$

All fail to satisfy one or both of (α), (β) in (ii). Hence $e \neq 2$, and the only remaining choice is $e=1$.

(iv) When $e=1$, $a > 4$ from line (5), $b=4$, $d=1$ and $Q < 1500$. If $a=5$ or 6, $D=54 \times$ or $64 \times$ and $D.Q < 650.1500$, a six-figure number. These

the FIVE] FIVES most nearly approaches the ideal of what a missing-figure problem should be.

THE FIVE FIVES.

$$\begin{array}{r}
 \times \times \times \times \times \times 5 \ 5 \times \times 5 \times (\times 5 \times \\
 \times \times 5 \times \times \\
 \hline
 \times \times \times \times \times \\
 \times \times \times \times \times \\
 \hline
 \times \times \times \times \\
 \times \times \times \times
 \end{array}$$

The number of figures in the data can be reduced to a lower level in a division sum with no remainder than when a remainder has to be found as part of the solution. A division sum without remainder can be inverted into a multiplication, but a missing-figure problem equivalent to the latter usually has more solutions than the inverted problem in division. In the THREE

THE THREE THREES.

$$\begin{array}{r|l}
 \times \times \times \times \times \times 3 \times \times (\times \times & \times \times \times \\
 \times 3 \times & \times \times \\
 \hline
 \times \times \times \times & \times \times 3 \times \\
 \times \times 3 \times & \times 3 \times \\
 \hline
 & \times \times 3 \times \times
 \end{array}$$

THREES the division on the left and the multiplication on the right have each one solution. On the other hand, in the FOUR SEVENS the multiplications on the left have solutions not admitted by the divisions on the right, the reason

THE FOUR SEVENS (I).

$$\begin{array}{r|l}
 \times \times \times \\
 7 \times \times \\
 \times 7 \times \times \\
 \times \times \times \\
 \times \times 7 \times \\
 \hline
 \times \times \times \times \times 7 & \times \times \times \times \times \times 7 (7 \times \times \\
 & \times 7 \times \times \\
 & \times \times \times \times \\
 & \times \times \times \\
 & \times \times \times \times \\
 & \times \times 7 \times
 \end{array}$$

being that the third line of each division gives a piece of information which is wanting on its left. I hope that one of these positions will satisfy Prof. H. H. Turner, who, after seeing the earlier SEVEN SEVENS and FOUR FOURS, suggested (*v. The Observatory*, July 1920) that I should propound a problem

THE FOUR SEVENS (II).

$$\begin{array}{r|l}
 \times \times \times \times \\
 \times \times \times \\
 \times \times 7 \times \\
 \times 7 \times \times \\
 \times \times 7 \times \times \\
 \hline
 \times \times \times 7 \times \times \times & \times \times \times \times \times \times \times 7 \times \times \times (\times \times \times \\
 & \times \times 7 \times \\
 & \times \times \times \times \times \\
 & \times 7 \times \times \\
 & \times \times \times \times \times \\
 & \times \times 7 \times \times
 \end{array}$$

involving either four sevens or seven fours. Professor Turner will see that the unique solution to the problem on the right and the four to that on the left give the FOUR SEVENS (I) an additional reminiscent flavour of the other two. Possibly a SEVEN FOURS will be forthcoming later.

Introduction of a remainder usually requires one figure in the data for each figure in the remainder, though the figures used to determine the remainder should serve some other purpose as well. Thus, in the SEVEN SIXES the sixes in the third, fifth and seventh lines fix the remainder and also render