148. A Graphical Solution of the Typical Quadratic Equation $ax^2 \pm bx \pm c = 0$

Author(s): W. O. Hemming

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From the equation of the orbits given me it was apparent that all conics having the centre of force and given line for pole and polar were not obtained unless we took

- (1) all orbits described under an attractive force of above magnitude ;
- (2) all orbits described under a repulsive force of above magnitude.

But I deduced from that equation that the circles under consideration came under case (1).

Hence the theorem that the circles can be simultaneously described under an attractive central force is true.

H. L. TRACHTENBERG.

Trinity Coll., Cambridge.

147. [v. 1. a.]. *On decimalisation of money.* (Cf. Note 140, p. 383, Vol. II.)

Shillings and sixpences can be expressed accurately as decimals of a pound.

$$\frac{1}{4}\text{d.} = \text{£}001 + \cdot 01\text{d. accurately.}$$

To multiply £3. 9s. $7\frac{1}{2}\text{d.}$ by 365.

$$\begin{aligned}\text{£}3. 9\text{s. } 7\frac{1}{2}\text{d.} &= \text{£}3. 9\text{s. } 6\text{d.} + 6 \text{ f.} = \text{£}3\cdot 475 + \text{£}006 + \cdot 06\text{d.} \\ &= \text{£}3\cdot 481 + \cdot 06\text{d.}\end{aligned}$$

£	d.
3·481 +	·06
365	
1044·3	21·90
208·86	
17·405	

$$\text{£}1270\cdot 565 + 21\cdot 90\text{d.}$$

$$= (\text{£}1270. 11\text{s.} + \text{£}015) + (1\text{s. } 9\cdot 75\text{d.} + \cdot 15\text{d.})$$

$$= \text{£}1270. 11\text{s. } 3\frac{3}{4}\text{d.} + 1\text{s. } 9\frac{3}{4}\text{d.}$$

$$= \text{£}1270. 13\text{s. } 1\frac{1}{2}\text{d.}$$

This work can be abbreviated. I have found it valuable as a help to accuracy (a) to look for a £015 in the first product (after sixpences have been removed), (b) to then look for a ·15d. in the second part.

E. E. CHAMBERS.

148. [x. 4. b. a.] *A graphical solution of the typical quadratic equation*
 $ax^2 \pm bx \pm c = 0.$

(i) The fact that the rectangles contained by the segments of intersecting chords of a circle are equal leads to a very simple graphical solution of the equations of the form $ax^2 \pm bx + c = 0$ where a, b, c are all positive quantities.

(a) When $b^2 - 4ac$ is positive or the roots are real.

Take any two numbers p, q whose product is $\frac{c}{a}$ and sum greater than $\frac{b}{a}$.

Draw ACB such that $AC = p$ and $CB = q$.

On AB as diameter describe a semicircle and through C by the ordinary geometrical method draw a chord DCE of length $\frac{b}{a}$. Then DC and CE represent the roots of $ax^2 - bx + c = 0$ and $-DC$ and $-CE$ the roots of

$$ax^2 + bx + c = 0.$$

For $DC \cdot CE = AC \cdot CB = p \cdot q = \frac{c}{a}$ and $DC + CE = \frac{b}{a}$.

(β) When $b^2 - 4ac$ is negative or the roots are imaginary.

As before, take any two numbers p, q whose product is $\frac{c}{a}$ and sum greater than $\frac{b}{a}$. Draw ACB such that $AC=p$ and $CB=q$. On AB as diameter describe a semicircle, and it will now be found that all chords drawn through C are greater than $\frac{b}{a}$. Hence we conclude the roots are imaginary and proceed as follows.

Through C draw the chord DCF perpendicular to AB , and on DC as diameter describe a circle. From D draw the chord DE equal to $\frac{b}{2a}$ and join EC .

The roots of $ax^2 - bx + c = 0$ will be $DE + \iota \cdot EC$ and $DE - \iota \cdot EC$ where 2 represents $\sqrt{-1}$.

Now $ED = \frac{b}{2a}$ and $DC^2 = AC \cdot CB = p \cdot q = \frac{c}{a}$.

Hence $EC^2 = DC^2 - ED^2 = \frac{c}{a} - \frac{b^2}{4a^2} = \frac{4ac - b^2}{4a^2}$;

$$\therefore -EC^2 = \frac{b^2 - 4ac}{4a^2};$$

$$\therefore \iota \cdot EC = \frac{\sqrt{b^2 - 4ac}}{2a};$$

$$\therefore DE \pm \iota \cdot EC = \frac{b \pm \sqrt{b^2 - 4ac}}{2a},$$

that is, $DE \pm \iota \cdot EC$ represent the roots of $ax^2 - bx + c = 0$.

(ii) The corresponding proposition for chords intersecting outside the circle enables us to solve $ax^2 \pm bx - c = 0$.

Take any two numbers p, q whose product is $\frac{c}{a}$, provided their difference is greater than $\frac{b}{a}$.

Draw ABC so that $AC=p$ and $AB=q$.

On BC as diameter describe a circle and from A draw a secant ADE such that DE is equal to $\frac{b}{a}$.

Then AE and $-AD$ are the roots of $ax^2 - bx - c = 0$ and $-AE$ and AD are the roots of $ax^2 + bx - c = 0$.

For $-AE \cdot AD = -AB \cdot AC = -p \cdot q = \frac{c}{a}$ and $AE - AD = DE = \frac{b}{a}$.

W. O. HEMMING.

REVIEWS.

A Treatise on the Line Complex. By C. M. JESSOP. Pp. xvi, 364. 10s. 1903. (Cam. Univ. Press.)

This is a welcome addition to the number of books in the English Language dealing with modern developments of pure mathematics. It is in fact the only