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“Limiting Vacuum for Steam Turbines, with Tests of Large Turbo-Generators and Air-Compressors on the Rand.”

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THE supply of power to the gold mines of the Witwatersrand area in South Africa is in the hands of two companies, namely, the Victoria Falls and Transvaal Power Company, Limited, founded in 1906, and the Rand Mines Power Supply Company, Limited, founded about 2 years later; but as the capital of the latter company is provided by the former, and as the two companies are worked by the same management and staff, they may for the purpose of this Paper be included under the parent title.

The operations of the Power Company extend over a T shaped area, of which the upper bar represents nearly 50 miles of gold-bearing reef, extending east and west, and the vertical part is the transmission line from Vereeniging, 40 miles due south. Three other stations are situated on the Reef, Brakpan near the eastern end, Simmer Pan about 12 miles westward from Brakpan; and Rosherville about 3 miles west again of Simmer Pan, thus bringing it nearly into the geographical centre of the distributing-area. The situation of the power-stations was largely decided by the proximity of “pans,” or reservoirs, for the supply of sufficient water for condensing purposes. Vereeniging is on the Vaal river and has a local coal-supply; but for the Reef stations coal is drawn from the colliery districts in the Eastern Transvaal, which centre about Witbank, and are within 80 miles of the gold-bearing area.

Supply is given to the mines at 2,100 volts and 525 volts, but generation takes place at 5,000 volts, stepping up to 80,000 volts in the case of the Vereeniging transmission, and to 10,000, 20,000, and 40,000 volts for the Reef sections of the distributing network.

There are in all twenty-one three-phase steam-driven turbo-generators of the Curtis-Rateau type, variously rated from 3,750 kilovolt-amperes, to 18,000 kilovolt-amperes, with a total of 219,000 kilovolt-amperes. The total output for the year 1915 was close upon 750 million kilowatt-hours, so that there are only two other Companies in the world, the Commonwealth Edison Company (1,200 millions), and the Niagara Falls Power Company (900 millions), which have a substantially greater annual output. The total coal burned for such an output is roughly 1 million tons a year.

Whilst the output of these American power-companies is entirely electrical, that given for the Rand includes both electrical and air units. A supply of compressed air is distributed to the mines of the Rand Mines Group, situated in the central area, through a pipeline network, and to some outlying mines through localized plants. The principal system extends 10 miles east and west, and is supplied by steam-driven turbo-compressors at Rosherville, and from electrically-driven compressors at Robinson, a substation situated 5 miles west of Rosherville. The Rosherville plant consists of six sets rated at 3,200 kilowatts, and three sets of 7,000 kilowatts (these correspond with free air ratings of 21,000 and 45,000 cubic feet per minute), whilst at Robinson there are six sets of 3,200 kilowatts, making a total of 59,000 kilowatts. The pressure of delivery to consumers ranges from 100 lbs. to 110 lbs. per square inch by gauge, and the intake pressure at the altitude of the Rand is 12 lbs. per square inch. The weight of air compressed is nearly 2 million tons per annum, whilst the maximum rate of supply is 500 tons per hour. The total length of air-mains is 30 miles. These range in size from $27\frac{1}{2}$ inches to 9 inches in diameter, and the isothermal efficiency of distribution is now 95 per cent.

Extensive and specially accurate tests were carried out on behalf of the makers and of the users, and the following is a short account of some of these tests and a discussion of the results obtained.

As a preliminary, however, it is desired to submit a method of determining the "limiting vacuum"—that is, the vacuum which will give the maximum available energy to a given turbine supplied with steam of a known pressure and temperature. Incidentally, suggestions will be made as to the units for measuring vacuum and steam energy, and a simple formula will be established by means of which a check over the ordinary station operations can easily be kept. That this limiting vacuum exists can be seen from the consideration that as the vacuum improves the specific volume of the steam increases, and therefore the velocity of the steam through the

spaces between the blades increases also. The steam energy thus rejected as kinetic energy into the condenser varies as the square of the leaving velocity, and the vacuum at which this rejected energy or "leaving loss" is equal to the gain due to improved vacuum is the "limiting vacuum" sought.

In 1912 an improved Mollier diagram was published, on double the scale of the former diagrams. This diagram in addition to the work and entropy lines gives pressure and temperature lines, but the volume lines are omitted. The ordinates are in calories per kilogram of steam, and are calculated from 0° C. Whilst this diagram is excellent for reference, the amount of information that it contains is far in excess of the requirements of those concerned with a particular type of machine. For example, in any particular engine-room the range of working steam-pressures and temperatures is not great, and thus only a small portion of the diagram has to be utilized. The diagram was therefore re-drawn, as reproduced to a small scale in *Fig. 1*, selecting only such portions as fit in with the working conditions of the Victoria Falls and Transvaal Power Company, Limited. Moreover, as the calorie is not the energy unit with which engineers who deal with power-stations are most familiar, the scale for the ordinates of the diagram, i.e., the work or energy scale, has been re-drawn in terms of kilowatt-hours per pound of steam.¹ The scale of the modified diagram is such that 0.1 kilowatt-hour is about 15 inches, so that the steam energy can be read off easily to 1 per cent. and very nearly to 1 in 1,000.

As it has been found more accurate to observe the temperature of the exhaust steam, instead of the more usual pressure observations, the diagram has been drawn showing the temperatures of the exhaust in the saturated-steam region. These temperature lines meet the saturated-steam line which divides the saturated from the superheated-steam regions of the diagram at the same points as the nearly-horizontal temperature lines in the superheated region cut the saturated-steam line, so that if a temperature line is required in the upper region it can be drawn horizontally from the point of intersection of the temperature lines in the lower region with the saturated-steam line. None of these horizontal lines have been drawn in the diagram, as the steam always appears to be saturated, and to have a dryness-fraction of about 98 per cent.—that is, about 2 per cent. of the steam has been already condensed into water.

¹ One kilowatt-hour = 3,412 B.Th.U.
= 1,896 pound-calories.

Fig. 1.

These dryness-fractions have been drawn from 0.95 to 1.00 (the saturation line).

If some initial conditions of pressure and superheat are assumed, corresponding to the design of the turbine, then the diagram gives the total energy of the steam in kilowatt-hours per pound, calculated from 0° C. For example, if the pressure is fourteen atmospheres¹ (absolute) and the temperature of the steam is 330° C., then the diagram shows that the total energy in each pound of steam is 0.3917 kilowatt-hour. Following vertically downward along the entropy line corresponding with these initial steam conditions to the temperature of the exhaust gives the total energy that the steam would have at the exhaust temperature if the turbine had 100 per cent. efficiency ratio. For example, at 40° C. it would be 0.2784 kilowatt-hour per pound. The difference between the two (0.3917—0.2784), or 0.1133 kilowatt-hour per pound, is the gross available energy at the disposal of the turbine. This gross value of the available energy must be diminished by the kinetic energy due to the velocity of the outflowing steam through the last row of blades in order to obtain the net available energy.

The velocity of the outflowing steam is found by multiplying the weight of steam passing in pounds per second by the specific volume in cubic feet per pound at the exhaust temperature, and dividing by the area in square feet of the space between the blades. The square of this velocity divided by $2g$ will give the foot-pounds of kinetic energy per pound of steam; and if this is divided by the number of foot-pounds in a kilowatt-hour (2,654,000), the kilowatt-hours of kinetic energy per pound of steam is obtained. If this is subtracted from the gross available energy of the Mollier diagram the net available energy is obtained, and the maximum value of this gives the "limiting vacuum." Moreover, the rate at which the value of the net available energy changes will show the loss that may be expected by an improvement or the reverse in the vacuum.

Thus, for example, the Rosherville and Vereeniging turbo-generators have an area of 12.5 square feet through the last row of blades, and may be loaded so that 45 lbs. of steam are passing per second. If the initial conditions of the steam are fourteen atmospheres pressure and a temperature of 330° C., for

¹ Continental atmospheres, kilograms per square centimetre, which are equivalent to 14.223 lbs. per square inch and not English atmospheres, which are equivalent to 14.696 lbs. per square inch,

which the energy value is 0·3917 kilowatt-hour per pound, then Table I shows the calculation of the net available energy.

TABLE I.—NET AVAILABLE ENERGY.

(Steam passing = 45 lbs. per second.)

Vacuum T _v ° C.	Specific Volume, cubic feet per lb.	Velocity, feet per second.	Kinetic Energy, kilo- watt-hours per lb. ($g = 32 \cdot 17$).	Available Energy.	
				Gross kilowatt- hours per lb.	Net kilowatt- hours per lb.
20	928·0	3,340	0·0653	0·1304	0·0651
25	698·0	2,513	0·0370	0·1260	0·0890
30	529·5	1,906	0·0213	0·1217	0·1004
35	405·0	1,458	0·0124	0·1175	0·1050
40	313·3	1,128	0·0075	0·1134	0·1060
45	234·7	881	0·0045	0·1093	0·1048
50	192·8	694	0·0028	0·1054	0·1026
55	153·2	551	0·0018	0·1013	0·0995
60	122·8	442	0·0011	0·0975	0·0964

It will be seen that the maximum value of the net available energy is at a vacuum of 40° C. and has the value of 0·1060 kilowatt-hour per pound of steam. At the much-improved vacuum of 20° C. the gross available energy is 0·1304 kilowatt-hour per pound, but almost exactly one-half of this is wasted as kinetic energy, and the net available energy is reduced to 0·0651 kilowatt-hour per pound.

In order to calculate the kinetic energy at other loads the figures in the fourth column of Table I must be multiplied by the square of the new load divided by 45 squared, since that column was calculated for a load of 45 lbs. per second. Subtracting these values of the kinetic energy from the gross available energy of the Mollier diagram gives the net available energy as before. As in power-station work the steam-consumption is generally known in pounds per hour instead of in pounds per second, the former unit will be used instead of the latter.

Table II then gives the net available energy for all loads on the turbine from 50,000 to 200,000 lbs. of steam per hour, and for all vacua between 20° C. and 60° C.

TABLE II.—NET AVAILABLE ENERGY—ALL LOADS.

Load.	Vacuum Temperature.								
	20° C.	25° C.	30° C.	35° C.	40° C.	45° C.	50° C.	55° C.	60° C.
Lbs. per Hour.									
50,000	0.1242	0.1225	0.1197	0.1163	0.1127	0.1089	0.1051	0.1011	0.0974
60,000	0.1214	0.1209	0.1188	0.1158	0.1124	0.1087	0.1050	0.1011	0.0973
70,000	0.1182	0.1191	0.1177	0.1152	0.1120	0.1085	0.1049	0.1010	0.0973
80,000	0.1145	0.1170	0.1165	0.1145	0.1116	0.1082	0.1047	0.1009	0.0972
90,000	0.1102	0.1146	0.1151	0.1137	0.1111	0.1079	0.1045	0.1008	0.0971
100,000	0.1055	0.1119	0.1136	0.1128	0.1106	0.1076	0.1043	0.1006	0.0971
110,000	0.1003	0.1089	0.1119	0.1118	0.1100	0.1072	0.1041	0.1005	0.0970
120,000	0.0946	0.1057	0.1100	0.1107	0.1093	0.1068	0.1039	0.1003	0.0969
130,000	0.0884	0.1022	0.1080	0.1095	0.1086	0.1064	0.1036	0.1002	0.0968
140,000	0.0816	0.0984	0.1058	0.1082	0.1078	0.1059	0.1033	0.1000	0.0966
150,000	0.0744	0.0943	0.1035	0.1068	0.1070	0.1054	0.1030	0.0998	0.0965
160,000	0.0667	0.0899	0.1009	0.1054	0.1061	0.1049	0.1026	0.0996	0.0964
170,000	0.0585	0.0853	0.0983	0.1038	0.1052	0.1043	0.1023	0.0993	0.0962
180,000	0.0498	0.0803	0.0954	0.1021	0.1042	0.1037	0.1019	0.0991	0.0961
190,000	0.0406	0.0701	0.0924	0.1004	0.1032	0.1031	0.1015	0.0989	0.0959
200,000	0.0309	0.0696	0.0893	0.0985	0.1021	0.1024	0.1011	0.0986	0.0958

The maximum value of the net available energy in each line of the above Table is in *italics* and gives the approximate limiting vacuum at that load, but as the vacua in Table II have only been tabulated for every 5° C., a closer examination of the values in the neighbourhood of these maxima is required. As it is difficult, owing to uncertainties in reading the last figure of the available energy from the diagram, to place the maximum with great certainty, it will be assumed that the value of the kinetic energy at the maximum is 0·0080 kilowatt-hour per pound, which corresponds to a velocity of 1,170 feet per second in the steam.

Table III then gives the maximum value of the net available energy for any load and the vacuum temperature at which it is obtained.

TABLE III.—LIMITING VACUUM AND MAXIMUM ENERGY.

(Rosherville and Vereeniging Turbo-Generators.)

Load, lbs. per hour.	Limiting Vacuum Temperature.	Maximum Net Available Energy.
50,000	under 20° C.	0·1245
60,000	21·1	0·1214
70,000	23·8	0·1190
80,000	26·1	0·1171
90,000	28·2	0·1153
100,000	30·1	0·1137
110,000	31·9	0·1122
120,000	33·5	0·1108
130,000	35·0	0·1095
140,000	36·4	0·1083
150,000	37·8	0·1072
160,000	39·1	0·1061
170,000	40·3	0·1051
180,000	41·4	0·1041
190,000	42·5	0·1031
200,000	43·5	0·1023

Fig. 2 shows a portion of the family of curves of Table II, giving the net available energy at all loads, and *Fig. 3* shows the temperature of the limiting vacuum of each load as given in Table III.

The foregoing calculations have been based on the assumption of an initial condition of the steam of fourteen atmospheres pressure and 330° C. temperature—that is, an entropy of 1·696. If the pressure and temperature both change, so as to maintain constant entropy, then the total energy of the steam will be altered, and the gross available energy and net available energy will be altered by

the same amount, but the temperature of the limiting vacuum for the load will not be affected.

If, however, an alteration takes place in the pressure and/or

Fig. 2.

temperature in such a way as to alter the entropy, then the effect on the temperature of the limiting vacuum must be examined. Assuming as before that the steam-consumption of the machine is 45 lbs. per second, two cases will be considered, one in which the

Fig. 3.

entropy is 1.62, corresponding with steam conditions of 14.4 atmospheres and 250° C., and the other in which entropy is 1.74, corresponding to steam conditions of 12 atmospheres and 350° C. This range of entropy covers the margin of initial steam-conditions

that should exist where the "nominal" conditions are 14 atmospheres and 330° C.

The gross energy per pound is 0·3700 and 0·3975 kilowatt-hour respectively.

Table IV corresponds with Table I for the smaller value 1·62 of the entropy, and Table V for the higher value 1·74. The first four columns of these three Tables are identical, but for the fifth column the gross available energy is different.

TABLE IV.—NET AVAILABLE ENERGY.

Steam passing=45 lbs. per second.

Entropy 1·62.

Vacuum T_v ° C.	Specific Volume, cubic feet per lb.	Velocity, feet per second.	Kinetic Energy, kilo- watt-hours per lb.	Available Energy.	
				Gross kilowatt- hours per lb.	Net kilowatt- hours per lb.
20	928·0	3,340	0·0653	0·1200	0·0547
25	698·0	2,513	0·0370	0·1160	0·0790
30	529·5	1,906	0·0213	0·1120	0·0907
35	405·0	1,458	0·0124	0·1080	0·0956
40	313·3	1,128	0·0075	0·1040	0·0965
45	244·7	881	0·0045	0·1000	0·0955
50	192·8	694	0·0028	0·0960	0·0932
55	153·2	551	0·0018	0·0925	0·0907
60	122·8	442	0·0011	0·0890	0·0879

In all three Tables (I, IV and V) the maximum is well marked near to 40° C., so that it may be taken that the temperature of the limiting vacuum is not materially affected by the initial conditions of pressure and temperature of the steam.

The exhaust temperatures on the Mollier diagram can be marked with the load for which that temperature is the limiting vacuum for a given area through the last row of blades. The area of 12·5 square feet has been used corresponding with the dimensions of the Rosherville and Vereeniging sets; but it might be more convenient to use, say, 10 square feet or even 10·764 square feet, which is 1 square metre, as a standard area through the last row of blades.

TABLE V.—NET AVAILABLE ENERGY.

Steam passing = 45 lbs. per second.

Entropy 1.74.

Vacuum T _v ° C.	Specific Volume, cubic feet per lb.	Velocity, feet per second.	Kinetic Energy, kilo- watt-hours per lb.	Available Energy.	
				Gross kilowatt- hours per lb.	Net kilowatt- hours per lb.
20	928.0	3,340	0.0653	0.1290	0.0637
25	698.0	2,513	0.0370	0.1245	0.0875
30	529.5	1,906	0.0213	0.1205	0.0992
35	405.0	1,458	0.0124	0.1160	0.1036
40	313.3	1,128	0.0075	0.1120	0.1045
45	244.7	881	0.0045	0.1075	0.1030
50	192.8	694	0.0028	0.1035	0.1007
55	153.2	551	0.0018	0.0995	0.0977
60	122.8	442	0.0011	0.0955	0.0944

For other areas the weight of steam passing is proportional to the area, so that if in any other turbine the area is one-half of the standard, then only one-half the weight of steam will pass at the same limiting temperature.

From *Fig. 2* the net available energy can be read off for every degree of vacuum temperature for different weights of steam passing per hour, and these figures are given for every 5° C. in Table VI (p. 456).

By expressing the figures in Table VI in the form of percentages of the maximum net available energy for each weight, Table VII (p. 457), is obtained for every 5° C.

The percentages above 83 per cent. in Table VII are plotted in *Fig. 4* (p. 458), which therefore shows the percentage of loss that will be involved at any steam-consumption of the turbine at any vacuum, the loss being calculated from the net available energy at the temperature of the limiting vacuum.

This figure shows the limiting vacuum and the losses, on the assumption that the steam flows through the full area of the last row of blades; but as there is friction between the steam and the blades, the steam must issue through a contracted area whose ratio

TABLE VI.—NET AVAILABLE ENERGY.*

Lbs. per Hour.	20° C.	25° C.	30° C.	35° C.	40° C.	45° C.	50° C.	55° C.	60° C.
50,000	0.1242	0.1222	0.1197	0.1163	0.1127	0.1089	0.1050	0.1011	0.0974
60,000	..	0.1209	0.1188	0.1158	0.1124	0.1087	0.1049	0.1011	0.0974
70,000	..	0.1190	0.1177	0.1152	0.1120	0.1085	0.1048	0.1010	0.0973
80,000	..	..	0.1166	0.1145	0.1116	0.1082	0.1046	0.1009	0.0972
90,000	..	..	0.1152	0.1137	0.1111	0.1079	0.1043	0.1007	0.0971
100,000	..	..	0.1137	0.1128	0.1106	0.1076	0.1041	0.1006	0.0971
110,000	..	..	..	0.1118	0.1100	0.1072	0.1038	0.1004	0.0970
120,000	..	..	..	0.1107	0.1093	0.1068	0.1035	0.1002	0.0969
130,000	..	..	..	0.1095	0.1086	0.1064	0.1032	0.1000	0.0968
140,000	..	..	..	..	0.1078	0.1059	0.1028	0.0997	0.0967
150,000	..	..	..	..	0.1070	0.1054	0.1024	0.0994	0.0965
160,000	..	..	..	..	0.1061	0.1049	0.1021	0.0992	0.0964
170,000	..	..	..	..	0.1051	0.1043	0.1016	0.0989	0.0962
180,000	..	..	..	..	..	0.1037	0.1012	0.0986	0.0961
190,000	..	..	..	..	..	0.1029	0.1007	0.0983	0.0959
200,000	..	..	..	..	..	0.1022	0.1002	0.0980	0.0958

TABLE VII.

Loads per Hour.	20° C.	25° C.	30° C.	35° C.	40° C.	45° C.	50° C.	55° C.	60° C.
50,000	99.6	98.1	96.0	93.3	90.4	87.4	84.2	81.1	78.2
60,000	..	99.6	97.8	95.4	92.6	89.5	86.4	83.3	80.2
70,000	..	100.0	98.9	96.8	94.1	91.2	88.1	84.9	81.7
80,000	..	..	99.5	97.8	95.3	92.4	89.3	86.2	83.0
90,000	..	..	99.9	98.6	96.3	93.6	90.5	87.3	84.2
100,000	..	..	100.0	99.2	97.3	94.6	91.5	88.5	85.4
110,000	..	..	..	99.6	98.0	95.5	92.5	89.5	86.4
120,000	..	..	..	99.9	98.6	96.4	93.4	90.4	87.4
130,000	..	..	..	100.0	99.1	97.1	94.2	91.3	88.4
140,000	..	..	..	..	99.5	97.8	94.9	92.0	89.3
150,000	..	..	..	..	99.8	98.3	95.5	92.7	90.0
160,000	..	..	..	..	100.0	98.8	96.2	93.5	90.8
170,000	..	..	..	..	100.0	99.2	96.7	94.1	91.5
180,000	..	..	..	..	..	99.6	97.2	94.7	92.3
190,000	..	..	..	..	..	99.8	97.6	95.3	93.0
200,000	..	..	..	..	..	99.9	97.8	95.7	93.5

to the full area may be termed the "friction coefficient" of the blades. If, for example, this coefficient is 90 per cent. for any turbine, then the limiting vacuum calculated for the full area at, say, 150,000 lbs. per hour, will be reached when the load is only 90 per cent. of this or 135,000 lbs. per hour.

The determination of the limiting vacuum temperature is a matter of considerable experimental difficulty in a power-station, when the turbo-generators have to be run on circuit, and not on artificial load. In the neighbourhood of the minimum, the rate of change of the steam-consumption is much less than the experimental errors introduced by changes in conditions. The condition that should be changed is that of the temperature of the circulating water, but this is experimentally impossible otherwise than by

Fig. 4.

natural annual changes. Direct attempts to find the limiting vacuum have not been successful, but an analysis of the weekly station returns for a year seem to show that the limiting vacuum is about 45° C. instead of 35° C. for certain machines. Fig. 3 would then show that the coefficient of the blades is between 0.60 and 0.70, or that the blades behave as "thin plate" orifices.

The use, as above, of temperature for expressing the degree of vacuum sweeps away all the lumber of expressing vacuum as fractions of an atmosphere, English or Continental, or as percentages of a nominal mean barometer, or in kilograms per square centimetre, or in pounds per square inch, or in inches or millimetres of mercury. It is just as easy to remember that the temperature of the vacuum is about 40° C. with a range of 10° C. on either side

as to remember that it is about 1 lb. per square inch, and may be as low as 0.5 lb. per square inch or as high as 2 lbs. per square inch.

The measurement of the performance of a condensing-plant is also much simpler when the result of the operation, i.e., the vacuum produced, is measured by its temperature instead of its pressure. Incidentally, too, the Author has found that the measurement of the temperature is simpler and more accurate than the pressure-measurements. In large turbines the thermometer pocket can easily be made of sufficient depth to eliminate the effects of the exhaust pipe itself.

The performance of a condensing-plant is well expressed by means of the conductivity constant K , which is generally defined as the number of thermal units transmitted per square foot of cooling surface per hour for each degree of temperature difference between T_v , the vacuum temperature and t_m , the mean temperature of the circulating water. The number of thermal units transmitted is the product of the thermal units per pound and the number of pounds of condensate per hour. There is no necessity to use the thermal unit as the energy unit, since the kilowatt-hour can be used as before, and since kilowatt-hours per hour are kilowatts, the definition of K becomes the capacity of the condenser in kilowatts per square foot of cooling surface per 1°C . of mean temperature difference across the tubes.

The numerical value of the energy rejected from the turbine into the condenser can be found from the Mollier diagram, when the condition of the steam in the exhaust pipe is known. One fact about the steam is readily ascertained, namely, its temperature, but in order to locate the point on the Mollier diagram one other fact must also be known. The lines showing the dryness of the steam at exhaust have been drawn on the diagram, and if there were some ready means of estimating the dryness these lines might be used. Again, the entropy might be used if there were such a thing as an entropy meter. But since neither of these figures is obtainable, another means must be found. If the efficiency ratio of the turbine is known, then the fraction of the available energy that has been used by the turbine is known. For example, if the energy per pound of steam on admission is 0.3917 kilowatt-hour, and for a perfect turbine the energy for the exhaust at 40°C . is 0.2784, then the available energy at the disposal of the turbine is 0.1133. Suppose, further, that the efficiency ratio of the turbine is 70 per cent., then this means that 70 per cent. of 0.1133, or 0.0793 kilowatt-hour per pound, has been turned into work. Therefore

0.3917 - 0.0793, or 0.3124 kilowatt-hour per pound, has not been converted into work, and is still heat in the exhaust steam. This gives the second fact that is required about the exhaust steam, the first being that the temperature is 40°C. , and the second that the energy is 0.3124 kilowatt-hour per hour. The intersection of these lines on the Mollier diagram gives the condition of the steam at exhaust.

In this particular example it is seen that the "dryness fraction" is 0.965. This energy of 0.3124 kilowatt-hour per pound sent into the condenser is measured from 0°C. , but the condensate leaves the condenser at a temperature a few degrees lower than the vacuum temperature. If, for example, the temperature of the condensate is 38°C. , then the circulating water has not to abstract the heat corresponding with this temperature. Each 1°C. that the condensate is cooled requires 1 Th.U. or 1/1,900 kilowatt-hour. The 38°C. represent, therefore, 0.02 kilowatt-hour of work that the condenser has not to do and may be called the energy sent to hot-well, and it would be more convenient if the thermometer were graduated directly in kilowatt-hours instead of in degrees C.

Subtracting 0.02 from the energy (0.3124) rejected by the turbine gives 0.2924 kilowatt-hour per pound as the energy that has to be abstracted by the condenser.

If W is the weight of condensate in pounds per hour,
and e ,, energy abstracted in kilowatt-hours per pound,
then We ,, "power" of the condenser in kilowatts,
and if S ,, cooling surface in square feet,
and $T_v - t_m$,, mean temperature difference, then by definition

$$K = We/S (T_v - t_m).$$

But if t_1 and t_2 are the temperature of the inlet and outlet circulating water, with sufficient accuracy

$$t_m = (t_1 + t_2)/2;$$

or if t_r is the rise in temperature across the condenser,

$$t_r = t_2 - t_1$$

and

$$t_m = t_1 + \frac{1}{2}t_r.$$

Re-arranging the definition of conductivity, it becomes

$$T_v = t_1 + \frac{1}{2}t_r + We/SK.$$

This equation shows that the vacuum temperature obtained is greater than the temperature level of the inlet circulating-water by half the rise in temperature of the circulating-water together with

the term We/SK , which is dependent upon the load and the value of K . This form of equation is especially useful in cases where t_1 is the greatest term and not We/SK , which is the case at the works of the Victoria Falls and Transvaal Power Company. The inlet temperature of the circulating-water is generally above 20°C . and even exceeds 30°C ., and on an average t_1 may be taken as 25°C . and t_r as 8°C . or 10°C ., thus $\frac{1}{2} t_r$ can be taken as 4°C . or 5°C . The value of e is nearly 0.3 and the value of K is in the neighbourhood of 0.15 ; further, the ratio of W/S is about 7.5 , so that the value of the term We/SK is about 15°C ., or about one-third of the value of T_v . Changes in the value of K will therefore only produce one-third the relative change in T_v . In situations where the value of t_1 is nearly 0°C . the term We/SK is the all-important one and the condenser problem may be treated somewhat differently.

The value of the conductivity constant K depends on the velocity of the circulating-water in the tubes, on the cleanliness of the tubes on both sides, and on the freedom from air. A series of tests was made on one of the condensers with air admitted through orifices so that the weight of air admitted could be calculated. The results are given in Table VIII, wherein T_v represents the temperature of the vacuum and T_c that of the condensate.

TABLE VIII.—EFFECT OF AIR LEAKAGE ON K .

Test.	Air.	T_v .	T_c .	K .	$T_v - T_c$.
	Lbs. per Sec.				
I	0.000	47.8	44.2	0.153	3.6
II	0.026	49.7	33.6	0.136	16.1
III	0.042	53.4	30.8	0.113	22.6
IV	0.066	60.2	30.6	0.079	29.6

The increase in the difference $T_v - T_c$ with increase of air leakage is also interesting.

With this form of the conductivity expression it is easy to calculate to what extent the condensing-plant is suitable to the station for which it was designed.

If, for example, the 9,600-kilowatt generators are run at a load-factor of 90 per cent. with a steam-consumption of 14.4 lbs. per kilowatt-hour, then the weight of condensate is $9,600 \times 0.90 \times 14.4$, or $124,000$ lbs. per hour; and if the cooling surface of the condenser is $17,750$ square feet, the ratio of W/S is 7.0 . Further, if $e = 0.30$ and $K = 0.15$, then the term We/SK is 14°C ., and if the temperature-

rise across the condenser is taken to be 10° C., the value of T_v becomes

$$T_v = t_1 + 19.$$

If, then, the monthly variation in the temperature of the inlet circulating water is known, the monthly value of T_v is also known and the value of the net available energy for the load can be read off from Table VI. The average monthly temperature is given in the second column of Table IX, from which is deduced the annual saving that might be effected by improving the vacuum up to its limiting value, as shown below.

TABLE IX.—LOSS DUE TO BAD VACUUM.

$$T_v = t_1 + 19^{\circ} \text{ C.}$$

Month.	t_1 .	T_v .	Net Available Energy.	Percentage of Maximum $(0.1103)^1$
	$^{\circ} \text{ C.}$	$^{\circ} \text{ C.}$	Kilowatt-Hours per Lb.	
January	29	48	0.1047	94.9
February	29	48	0.1047	94.9
March	28	47	0.1053	95.5
April	27	46	0.1059	96.0
May	26	45	0.1066	96.6
June	22	41	0.1086	98.5
July	19	38	0.1096	99.4
August	20	39	0.1093	99.1
September	22	41	0.1086	98.5
October	24	43	0.1077	97.6
November	26	45	0.1066	96.6
December	28	47	0.1053	95.5
Average	25	44	0.1069	96.9

¹ See Table III.

There would therefore be a difference of 3 per cent. between the fitness of the condensing-plant to meet the average annual conditions and the requirements due to the limiting vacuum. The coal-cost of this energy would have to be debited with the cost of obtaining the better vacuum with improved pumps or larger condensers in order to obtain the net saving due to improving the vacuum up to the limit.

If, however, the vacuum-conditions were improved so that $T_v = t_1 + 15^{\circ} \text{ C.}$, then the loss would be 1.5 per cent. instead of

3 per cent. The reduction in the value of T_v by 4°C. might be obtained partially by increasing the quantity of circulating water so as to reduce t_r from 10°C. to 8°C. and so reduce $\frac{1}{2} t_r$ by 1°C. The remaining 3°C. would have to be obtained either by increasing S the surface of the condenser or K the conductivity constant. In Table IX with $T_v = t_1 + 19^\circ \text{C.}$ the value of We/SK was 14°C. with $K = 0.15$. In order to reduce this to 11°C. , the value of K would have to be 0.19 , a rather high but not impossible figure.

It would therefore be possible with the existing plant to reduce the losses due to bad vacuum to little more than 1 per cent. without enlarging the condensers.

The figures given for the monthly temperatures of the circulating-water are those for a single and rather rainless year, so that with

Fig. 5.

better seasonal rainfall which occurs in the summer the loss might be wiped out by Nature. It is also to be remembered that this calculation is based on a coefficient of discharge through the blades, of unity, whereas it seems probable that the coefficient is nearer two-thirds, making the limiting vacuum higher, and the condensing-plant a better fit to the conditions.

This form of the conductivity equation is also useful in maintaining a check from week to week over the ordinary station operation. Where the machine load-factor varies little the term We/SK will be nearly constant, and as the operation of the circulating-pumps is also nearly constant, the term $\frac{1}{2} t_r$ will also be constant, in which case

$$T_v = t_1 + C.$$

Fig. 5 shows the result at one of the stations during the last

23 weeks of the year 1913. Here t_1 is taken as the base and T_v as the ordinate. It will be seen that for the greater number of the weeks the value of C lies between 10°C. and 11°C. , but that there are exceptional weeks, notably weeks 1 to 5 and 19 and 20, when some portion of the plant had been out of order. All these high values of T_v have been traced to air leakages into the condensers and the operations during subsequent weeks show that they have been remedied.

ROSHERVILLE COMPRESSORS.

Tests have been carried out on all the Rosherville compressors as opportunity offered during May, June and July, 1915, and the tabular statement of the results obtained with one of the smaller and one of the larger is given in Table X.

The results of the tests so far as the turbine is concerned can be expressed as follows:—

Compressor No.	Steam-Kilowatts.
I	$600 + 1.80 \text{ Air-Unit.}^1$
II	$600 + 1.70$ „
III	$600 + 2.03$ „
IV	$600 + 1.88$ „
V	$600 + 1.70$ „
VI	$600 + 1.62$ „
VII	$500 + 1.48$ „
VIII	$500 + 1.45$ „

These tests would indicate that the highest output is the most efficient. It is, however, necessary to examine the combined effect of the turbine and the condenser together.

A condenser is generally considered to be an apparatus for condensing the steam from the exhaust of the turbine so that the condensate is brought to a condition in which it can be pumped back into the boilers from the hot-well.

From this point of view the cyclic order of the plant is, boiler—turbine—condenser—hot-well—boiler. But the condenser may be considered as having another function, which is that of rendering part of the energy supplied by the boiler available for use in the turbine, and that condenser is the most efficient that will convert the greatest fraction of the energy into available energy. The total energy supplied at the stop-valve less the energy of the water in

¹ The "Air-Unit" is 0.641 of a kilowatt-hour of the work of isothermal compression of the air. The coefficient, 0.641, was determined by tests on compressors in order to satisfy certain conditions with regard to payment.

TABLE X.—ROSHERVILLE POWER-STATION.
TESTS ON COMPRESSORS.

Test No.	Compressor No. 1. 10th May, 1915.				Compressor No. 7. 30th June, 1915.			
	1	2	3	4	1	2	3	4
Load in Air-Units	2,870	2,490	2,680		6,560	5,960	5,290	4,750
Steam pressure (kg/cm ² absolute)	13.8	14.4	14.6		14.3	14.6	14.5	14.9
<i>t_s</i> (steam temperature) ° C.	339.0	326.0	301.0		348.0	340.0	335.0	337.0
<i>t_v</i> (vacuum temperature) ° C.	35.7	33.1	44.2		32.9	31.5	29.8	28.3
Intermediate steam pressure	3.54	2.93	3.25		3.36	2.89	2.45	2.05
Energy at stop-valve	0.3942	0.3907	0.3841		0.3963	0.3943	0.3930	0.3936
Energy at condenser	0.2764	0.2715	0.2692		0.2740	0.2716	0.2695	0.2685
Available energy	0.1178	0.1192	0.1149		0.1223	0.1227	0.1235	0.1251
Water (lbs. per hour)	49,820	42,600	47,620		83,950	75,000	68,030	59,940
Steam kilowatts	5,680	5,090	5,460		10,280	9,200	8,410	7,495
Efficiency ratio per cent.	50.5	48.9	49.0		63.9	64.8	62.9	63.8
Lbs. per Air-Unit	17.7	17.1	17.8		12.8	12.6	12.9	12.6
<i>t_i</i> (inlet circulating water) ° C.	18.2	18.2	18.0		16.5	16.0	16.0	15.5
<i>t_o</i> (outlet circulating water) ° C.	31.1	29.2	30.2		24.7	24.5	23.4	22.4
<i>t_c</i> (condenser temperature) ° C.	32.9	29.9	31.7		29.8	29.2	27.6	26.3
<i>t_v</i> - <i>t_c</i> ° C.	2.8	3.2	2.5		3.1	2.3	2.2	2.0
<i>t_v</i> - <i>t_m</i> ¹ ° C.	11.0	9.4	10.1		12.8	11.2	10.1	9.3
K (coefficient of conductivity)	0.168	0.169	..		0.131	0.133	0.134	0.129
Pipe-line pressure lbs. per sq. inch	115.0	117.0	109.0		121.0	119.0	119.0	117.0

$$^1 t_m = \frac{1}{2} (t_1 + t_2).$$

the hot-well (this may be denoted by e_{vw} when the energy is considered per pound of steam and by E_{vw} when multiplied by pounds per hour) should therefore be the smallest possible multiple of the available energy, which is the stop-valve energy less the isentropic energy at T_v , the temperature of the vacuum. In order to see how this ratio varies, a stop-valve condition of fourteen atmospheres absolute pressure and 330° C. may be assumed; this will give a stop-valve energy of 0.3917 kilowatt-hour per pound of steam. It may also be assumed that the temperature of the vacuum and of the condensate are the same.

TABLE XI.

T_v or t_c .	e_{vw} .	Available Energy per Lb.	$E_{vw}/\text{St. kw.}$
20	0.3810	0.1315	2.90
25	0.3783	0.1270	2.99
30	0.3757	0.1230	3.08
35	0.3731	0.1185	3.17
40	0.3704	0.1145	3.26
45	0.3678	0.1100	3.35
50	0.3652	0.1065	3.44

The last column of Table XI is the ratio of the second and third columns, but as numerator and denominator may each be multiplied by W , the weight of steam used in pounds per hour, without altering the ratio, the last column has been headed $E_{vw}/\text{St. kw.}$

When a turbo-compressor or a generator is run at different loads, the value of T_v will generally be lower at the light loads and higher at heavy loads. The value of T_v can thus be expressed as a function of the load, and from Table XI it is seen that the value of the ratio $E_{vw}/\text{St. kw.}$ can be expressed as a function of T_v . It is not, however, necessary to take these two steps, as the ratio of $E_{vw}/\text{St. kw.}$ can be expressed directly as a function of the load. Over the range at which the compressors and generators were used, and with the general method of operating the auxiliaries, it is found that the value of the ratio $E_{vw}/\text{St. kw.}$ is a linear function of the load—that is, the relation is of the form—

$$E_{vw}/\text{St. kw.} = a + bx$$

where x is the load either in air-units or electric kilowatt-hours, and a and b are constants.

But from the tests on the turbines the following relation obtains—

$$\text{St. kw.} = c + dx,$$

where x is again the load and c and d are two other constants. Hence by multiplying together—

$$E_{vw} = (a + bx)(c + dx),$$

or—

$$E_{vw}/x = (A + Bx + Cx^2)/x,$$

which is the expression for the total energy per air-unit or per kilowatt-hour, whichever x represents. Every equation of this form has a minimum value when $x = \sqrt{A/C}$, and the value of that minimum is $B + 2\sqrt{AC}$. This shows that for the minimum to occur at a high load A must be large compared with C .

In the event of a turbine being run at constant vacuum at all loads, then—

$$E_{vw}/\text{St. kw.} = a,$$

in which case—

$$E_{vw}/x = a(c + dx)/x,$$

and there is no minimum, but the economy increases with the load, tending to the limit ad .

If E_{vw} is divided by e_{vw} —that is, by the energy per pound—then the number of pounds of steam per hour is obtained, so that these formulas give the form of the curve that will be obtained when “pounds of steam” are used instead of “steam-kilowatts.”

For compressors Nos. VII and VIII the tests give the following data (Tables XII and XIII), from which the constants a and b have been calculated:—

TABLE XII.—Compressor No. VII.

Load.	e_v .	e_{vw} .	Available Energy.	$E_{vw}/\text{St. kw.}$
6,560	0.3963	0.3806	0.1223	3.11
5,960	0.3943	0.3789	0.1227	3.06
5,290	0.3930	0.3785	0.1235	3.06
4,750	0.3936	0.3798	0.1251	3.04

$$E_{vw}/\text{St. kw.} = 2.85 + 0.00004 \text{ AU.}$$

TABLE XIII.—Compressor No. VIII.

Load.	e_v .	e_{vp} .	Available Energy.	$E_{vw}/\text{St. kw.}$
6,520	0.3975	0.3786	0.1165	3.25
6,160	0.3975	0.3792	0.1179	3.22
5,150	0.3925	0.3761	0.1198	3.14
4,640	0.3912	0.3756	0.1187	3.16
4,250	0.3930	0.3783	0.1226	3.09

$$E_{vw}/\text{St. kw.} = 2.79 + 0.00007 \text{ Air-Units.}$$

Fig. 6.

Combining these equations for the performance of the condenser with the equations already given for the performance of the turbine, denoting Air-Units by A U :—

$$\text{No. VII St. kw.} = 500 + 1.48 \text{ AU.}$$

$$\text{No. VIII St. kw.} = 500 + 1.45 \text{ AU.}$$

the overall performance is found to be—

$$\text{No. VII } E_{vw}/\text{AU} = (1,425 + 4.24 \text{ AU} + 0.0000592 \text{ AU}^2)/\text{AU.}$$

$$\text{No. VIII } E_{vw}/\text{AU} = (1,395 + 4.08 \text{ AU} + 0.0001015 \text{ AU}^2)/\text{AU.}$$

From which

Compressor.	Minimum at	Minimum Value	Water lbs. per Air-Unit.
VII	4,900 Air-Units.	4.82 E_{vw}	12.7
VIII	3,700 Air-Units.	4.83 E_{vw}	12.7

From these equations it is a very easy matter to trace the complete curve beyond the range of the tests, as shown in *Fig. 6*. Turbo-compressors however become unstable at about one-third of full load, so that this extrapolation is merely for the sake of showing the complete form of the curve. This Figure also gives the curves for $E_{vw}/\text{St. kw.}$ for Nos. VII and VIII and the observed points are shown to indicate the agreement between them and the straight line.

ROSHERVILLE GENERATORS.

During March, 1915, a series of tests were carried out on the five generators by Mr. C. J. M. Constançon, Assistant to the Author, in order to determine the steam-consumption under definite standard conditions. From other tests made during the last year it appeared that the steam-consumption of these sets followed the Willans Law—that is, there was a straight-line relation between the kilowatts of available energy supplied by the steam (called steam kilowatts for the sake of brevity) and the output of electrical kilowatts. In some of the previous tests, however, there were departures from this straight-line relation, and one of the objects of these tests was to clear up the nature of these exceptions.

As regards the measurement of the condensate, there had been already many comparisons made of the discharge as measured by orifices in the pipes leading to the hot-wells, and the discharge as measured by means of tanks, so that it was decided to be unnecessary to install the tanks again for these tests. It was, however, necessary to be certain that the orifices in the pipes and their manometers were in good order, and each orifice was taken out and examined for condition, and every manometer was cleaned and refitted. In using these orifices¹ it is necessary to ensure (1) that they have a really sharp edge less than .05 millimetre in thickness, (2) that they have the correct diameter, (3) that they are correctly placed in the pipe-line with the chamfered side down-stream, (4) that the packing does not overlap the orifice beyond the part covered by the pipe-flanges. In the use of the manometers it is important to see (1) that the pressure connections are made close to the orifice plate, (2) that the pipes are clean, unobstructed, and free from leakage at the joints and connections, (3) that the down-stream side of the orifice is running full bore of the pipe, (4) that no air is lodging in any of the pipes, but that there is a continuous water column

¹ A description of similar orifices will be found in Mr. Holbrook Gaskell's Paper, "The Diaphragm Method of Measuring the Velocity of Fluid Flow in Pipes." *Minutes of Proceedings Inst. C.E.*, vol. xciv, p. 249.

between the orifice and the mercury of the manometer. This is a point that requires continual watching, as air is always brought over by the condensate. It is best overcome by blowing out the air from the pipes through the cocks provided for the purpose on every shift. (5) The mercury column of the manometer should be steady without throttling, such fluctuations as occur being those due to real changes of load only.

A manometer that oscillates much indicates either air in the connections, which may be overcome by blowing them out, or else is due to a badly running air-pump that allows the condensate to pile up in the condenser and thus gives an irregular flow. Preliminary tests have shown that it is an important matter to keep as few nozzles open in the turbine as possible and that neglect of this precaution may in some cases introduce errors.

In the calculation of the final results the relationship between the steam-kilowatt and the electric-kilowatt was calculated by the statistical method,¹ so that no personal judgment was introduced in the fitting of the line to the observations. These tests show that for every machine Willans's Law is followed, and there seems to be no indication of any more complicated curve than a straight line being required to fit the observations.

In calculating out the equation to the Willans Law line, the coefficient of correlation between the steam and electric kilowatts is first found, and this is 0.999 for all the tests, showing that there is a very high order of agreement between the observed points and the assumption that these points lie on a straight line, for the coefficient of correlation can only be unity when the points actually lie on a straight line. From the equation to the Willans line the steam kilowatts can be calculated for each value of the observed electric kilowatts, and from the differences between the observed and calculated values of the steam kilowatts the coefficients of variation are found.

Table XIV shows that these coefficients are all very close together, and their average value is just over 0.5 per cent. This is a most satisfactory figure, and one that could not well be improved upon without elaborating the methods very considerably. The Table also gives the values of (1) the coefficient of correlation, as calculated for each machine, (2) the equation to the Willans line, (3) the coefficient of variation, and (4) the steam kilowatts required for 10,000 electric kilowatts calculated from the equations.

¹ See, for example, Karl Pearson, *Phil. Mag.*, November, 1901, "On Lines and Planes of Closest Fit to Systems of Points in Space."

TABLE XIV.

Generator.	Coefficient of Correlation.	Steam Kilowatts.	Coefficient of Variation.	Steam Kilowatts at 10,000 Electric Kilowatts.
I	0.999	1,050 + 1.46 El. kw.	0.53	15,650
II	0.999	2,010 + 1.31 El. kw.	0.58	15,110
III	0.999	1,770 + 1.42 El. kw.	0.39	15,970
IV	0.999	2,100 + 1.35 El. kw.	0.64	15,600
V	0.998	2,080 + 1.29 El. kw.	0.49	14,980
Mean	0.999		0.53 per cent.	

From these figures it is seen that generators Nos. II and V may be taken as giving the same result of 15,000 steam-kilowatts at 10,000 electric-kilowatts, but that No. IV takes about 4 per cent. more steam. These three machines have practically the same value of "No-load" steam-consumption, namely, 2,060 steam kilowatts, and the rate of increase in consumption with the load may for Nos. II and V also be considered to be the same—that is, the condition of the fixed and moving blading is probably about the same. One difference is shown by the tests, and that is, that the loading on No. V machine for the same number of nozzles open is higher than on No. II machine. With two large nozzles the loads were 11,400 and 11,000 respectively. This would point to the nozzle-box of No. V being in bad condition, giving more nozzles open than would be accounted for by the position of the regulators.

The cause of the 4-per-cent. increase in consumption on No. IV generator is not shown by the tests. This machine had one Rateau stage removed.

Generator No. III, although it had just been thoroughly overhauled and put back into commission, showed at 10,000 kilowatts an increase over the average of Nos. II and V of 6 per cent. The no-load consumption was some 10 per cent. lower than that of generators Nos. II, IV and V, which would be in itself a good feature, if the machine were operated at very light loads. But as this was accompanied by a rate of increase with load 10 per cent. higher than that of Nos. II and V, the machine became more and more uneconomical as the load increased. Generator No. I gave a still more exaggerated case of the same nature, but as the no-load consumption was reduced much further, the steam-consumption at

10,000 electric-kilowatts was only 4 per cent. worse than Nos. II and V. Since the test was carried out No. I generator has been handed over to the Maintenance Department and opened up; it was found that the blading was in a bad condition. This may account for the difference in No. I, but not in No. III, as this machine had only been running a few days after complete overhaul.

The efficiency ratio at 10,000 kilowatts for the best machines Nos. II and V is 66·5 per cent. If the efficiency of the generator at this load and power-factor is 95 per cent., then the efficiency-ratio of the turbine is 70 per cent., including auxiliaries.

Table XV gives a summary of the different tests for generator No. V, from which, and from similar tests for the other generators, the foregoing results have been calculated.

TABLE XV.—SUMMARY OF TESTS.—GENERATOR NO. V.

Test No.	1	2	3	4	5
Date	26/4/15	26/4/15	26/4/15	26/4/15	26/4/15
Load . . . kilowatts	11,400	10,380	9,570	9,445	7,760
Steam-pressure absolute : kilograms per square centimetre	13·1	13·3	14·2	14·2	14·2
Steam temperature ° C.	342	344	335	325	331
Intermediate steam-pres- sure atmospheres (abso- lute)	5·10	4·49	3·95	3·97	3·01
T_v (Vacuum temperature) } ° C. }	47·3	45·0	42·6	42·1	38·0
Available energy : kilo- watt-hours per pound . }	0·1080	0·1102	0·1123	0·1115	0·1156
T_c (Temperature of con- densate) . . . ° C. }	43·4	40·7	38·1	37·7	37·7
Weight of water : Lbs. per hour	151,140	139,500	129,050	128,800	103,980
Steam kilowatts	16,700	15,500	14,500	14,350	12,000
Efficiency ratio	68·3	67·0	66·0	65·9	64·7
Power-factor	0·83	0·83	0·81	0·82	0·78
Lbs. per kilowatt-hour .	13·5	13·5	13·5	13·6	13·4
$T_v - T_c$	3·9	4·3	4·5	4·4	0·3
K (Coefficient of conduc- tivity)	0·129	0·129	0·129	0·130	..
Nozzles	2 large { 1 large 1 small }	1 large 1 small	2 small	1 large	1 small

$$t_1 = 20^\circ \text{ C.}$$

The equations obtained for the condensers are as follows:—

Generator.	$E_{ev}/\text{St. kw.}$
I	$3\cdot00 + 0\cdot00004 \text{ El. kw.}$
II	$3\cdot05 + 0\cdot00004 \text{ El. kw.}$
III	$3\cdot00 + 0\cdot00004 \text{ El. kw.}$
IV	$3\cdot20 + 0\cdot00002 \text{ El. kw.}$
V	$2\cdot80 + 0\cdot000055 \text{ El. kw.}$

Combining the condenser and turbine equations, as in the case of the compressors, the following is obtained for the over-all performance of the sets:—

Generator.	E_{ev} .
I	$3150 + 4\cdot42 \text{ El. kw.} + 0\cdot0000584 \text{ El. kw}^2.$
II	$6130 + 4\cdot07 \text{ El. kw.} + 0\cdot0000524 \text{ El. kw}^2.$
III	$5310 + 4\cdot33 \text{ El. kw.} + 0\cdot0000568 \text{ El. kw}^2.$
IV	$6720 + 4\cdot36 \text{ El. kw.} + 0\cdot0000270 \text{ El. kw}^2.$
V	$5820 + 3\cdot73 \text{ El. kw.} + 0\cdot0000709 \text{ El. kw}^2.$

From these equations the position of the minimum value of $E_{ev}/\text{El. kw.-hr.}$ can be found. These are—

TABLE XVI.

Generator.	Minimum at	Minimum Value.	Water. Lbs. per Kilowatt-Hour.
I	7,350 El. kw.	5·28	13·9
II	10,800 El. kw.	5·21	13·7
III	9,700 El. kw.	5·43	14·3
IV	15,800 El. kw.	5·21	13·7
V	9,050 El. kw.	5·01	13·2

The water in pounds per kilowatt-hour in the last column of Table XVI is obtained by taking $e_{ev} = 0\cdot38$. The point of maximum economy in No. IV generator falls beyond the working range of the generator. The equation for the condenser of No. I generator has been deduced from only two points on the curve, but it is probably nearly correct. The point of maximum economy is low on this machine owing to the small value of the first term in the turbine equation.

The curves corresponding to the equations are shown in *Fig. 7* for generators I, II, and IV.

For Generator V the minimum value of steam kilowatts between the stop-valve and hot-well is about 5·0 per electric kilowatt

generated, hence the overall thermal efficiency is $1/5$, or 20 per cent. This includes both the generator and the auxiliaries, so that, making an allowance for these, the thermal efficiency of the turbine is about 21.5 per cent. to 22.0 per cent.

Fig. 7.

The investigations were carried out on behalf of The Victoria Falls and Transvaal Power Company by the Research Department of the Company, for which department the Author is responsible. The results are published with the permission of the Company.

The Paper is accompanied by seventeen sun-prints, from which the Figures in the text have been prepared.