260. Pascal's Theorem

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Source: *The Mathematical Gazette*, Vol. 4, No. 72 (Jun., 1908), pp. 284-286

Published by: Mathematical Association

Stable URL: <http://www.jstor.org/stable/3603383>

Accessed: 14-12-2015 06:08 UTC

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is thus true for any rational x . And the truth follows for irrational values of x by the continuity of $\exp ix$ on the one side (which has to be proved), and of $\cos x$ and $\sin x$ on the other.

5. In conclusion, I wish to add a few words. The Multiplication Theorem of two absolutely convergent series is undoubtedly an important theorem, leading as it does to proofs of the Binomial and Exponential Theorems and of the sine and cosine series, not to mention its importance, otherwise, in theory. But there is a theorem of far greater importance in the Theory of Infinite Series and Infinite Products which appears neglected; and to this theorem, I think, we should turn for any substantial improvement in the elementary treatment of Infinite Series and Infinite Products. And I am of opinion it is high time we should give this theorem its fundamental place and relegate the Multiplication Theorem to second rank. I hope to return to this point as soon as possible.

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Gooty, India,

24th January, 1907.

260. [L¹. 1. c.] *Pascal's Theorem*.

Proved for the conic and line-pair by the methods of Euclid and Apollonius.

a, b, c and a', b', c' are any six points on a conic, or by threes on the pair of lines L and L' .

α, β, γ are the intersections of $(bc', b'e), (ca', c'a), (ab', a'b)$.

It is required to prove that α, β, γ are collinear.

Through a draw ae parallel to ca' . In the conic, O is the centre $bst, b's't'$, and OA are parallel to ca' , and OB is parallel to aa' . Complete the figures by joining points as required.

I. For the conic. By similar triangles,

$$\begin{aligned} eh : b's' &= hc : cb' \\ &= am : mb' \\ &= aa' : a't \end{aligned}$$

and

$$\begin{aligned} a'm : b't' &= aa' : at'. \\ \therefore eh . a'm : aa'^2 &= b's' . b't' : at' . at' \\ &= OA^2 : OB^2. \end{aligned} \quad \dots\dots\dots(1)$$

Again,

$$\begin{aligned} ck : sb &= ec : es \\ &= aa' : at. \end{aligned}$$

and

$$\begin{aligned} \therefore ck : aa' &= sb : at \\ al : aa' &= bt : ta'. \\ \therefore ck . al : aa'^2 &= sb . bt : at . ta' \\ &= OA^2 : OB^2. \end{aligned} \quad \dots\dots\dots(2)$$

$\therefore$ by (1) and (2),

$$ck . al = eh . a'm.$$

II. For the line-pair. By similar triangles,

$$\begin{aligned} ck : ae &= kb : be \\ &= ka' : le. \end{aligned}$$

$$\therefore ck : ka' = ae : le.$$

$$\therefore ck : ca' = ae : al.$$

$$\therefore ck . al = ca' . ae. \quad \dots\dots\dots(3)$$

Again, from the similar triangles $b'eh, b'a'c$,

$$\begin{aligned} eh : a'c &= b'e : b'a' \\ &= ae : a'm. \end{aligned}$$

$$\therefore eh . a'm = a'c . ae = ck . al, \text{ by (3).}$$

$\therefore$ in both the conic and the line-pair,

$$eh : ck = al : a'm.$$

$$\therefore eg : gk = l\gamma : \gamma a'.$$

$$\therefore ek : gk = la' : \gamma a' \quad \dots\dots\dots(4)$$

and

$$bk : be = ba' : bl.$$

$$\therefore bk : ek = ba' : a'l. \quad \dots\dots\dots(5)$$

$\therefore$ by (4) and (5),

$$bk : gk = ba' : \gamma a'.$$

$$\therefore \gamma g \text{ is parallel to } a'c \text{ or } ae.$$

Now γ is the intersection of $(ab', a'b)$ and g of $(eb, b'c)$.

$\therefore$ if we introduce the point c' in the place of b' , and if n be the intersection of $(ac', a'b)$, and n' that of $(eb, c'e)$, nn' will be parallel to $a'c$ or ae , and therefore also to γg .

$$\begin{aligned} \therefore \beta p : pc &= nr : rn' \\ &= \gamma q : qg. \end{aligned}$$

$$\begin{aligned} \therefore \beta p : \gamma q &= pc : qg \\ &= ap : aq. \end{aligned}$$

$$\therefore \beta p : pa = \gamma q : qa.$$

$$\therefore a, \beta, \gamma \text{ are collinear.}$$

JOHN J. MILNE.

261. [x. 4. b. β .] Graphical solution of a biquadratic.

The roots of the quartic

$$x^4 + ax^3 + bx^2 + cx + d^2 = 0$$

are the abscissae of the meets of the hyperbola $xy = d$ and the circle

$$x^2 + y^2 + ax + \frac{c}{d}y + b = 0.$$

The last term of the quartic can always be made positive by increasing the roots, each by the same amount, so that the method always applies. We get the same values of x by using the hyperbola $xy = -d$.

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