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XXVIII. *Capacity and Eddy Current Effects in Inductometers.*

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ABSTRACT.

1. A theoretical investigation has been made upon the effects of self and intercapacities and of eddy-current losses in mutual inductances for which the coils have a common point. The following are the main conclusions arrived at.

(a) Small capacities introduce variations in the effective self and mutual inductances which change as the square of the frequency.

(b) Corresponding variations are introduced in the effective resistances of the coils.

(c) The mutual inductance is no longer "pure," that is, the secondary E.M.F. is no longer in exact quadrature with the primary current. The primary and secondary circuits behave as if they have a resistance in common which varies as the square of the frequency.

(d) Eddy currents produce impurity and changes of effective resistance which are initially of the same nature as those due to capacity.

(e) Eddy current and capacity effects may be so combined that the variations of mutual inductance and of impurity cancel simultaneously, so that it is theoretically possible to obtain a "pure" mutual inductance having no variation with frequency over the whole range of audio frequencies.

2. The conditions of balance of three alternating-current bridges, viz. :—

(a) The Heaviside inductance bridge.

(b) The Carey-Foster bridge.

(c) A modified Campbell frequency bridge

have been worked out on the basis that the mutual inductance is not pure and the mutual and self-inductances are not invariable with frequency. It is shown how combined observations on these bridges may be used to determine the frequency corrections of a given inductometer.

3. An experimental study has been made upon an inductometer of the Campbell type. The results show that if such an inductometer is used at the higher audio-frequencies for the measurements of the effective resistance of coils or of condensers, the inductometer corrections may in certain cases be larger than the quantities measured.

4. A method for automatically correcting an inductometer is suggested in order that it may be used for the measurement of power factors of condensers without having recourse to tables of corrections.

1.

It is usually assumed when mutual inductances are employed in alternating-current bridge measurements that the E.M.F. developed in the secondary coil is in exact quadrature

with the primary current. This assumption is practically correct at low frequencies, but at the higher telephonic frequencies a component of secondary E.M.F. which is in phase with the primary current begins to become appreciable.

Mutual inductances in which this in-phase component is developed are said to be "impure," and the magnitude of the in-phase component when unit current flows in the primary may be taken as the measure of the impurity.* Impurity, as defined in this way, is of the dimensions of a resistance, and if it is denoted by σ , the vector relation connecting secondary E.M.F. E_s with primary current I_p must be written

$$E_s = (\sigma + j\omega M)I_p, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

M being the mutual inductance, $\omega/2\pi$ the frequency and j the operator rotating through a right angle.

2. Factors Producing Impurity.

In the most general case, the factors bringing about impurity in a mutual inductance composed of coils with air cores are self and intercapacities, leakage, and eddy-current losses. If the coils have a common point, impurity might also arise due to a small resistance in common with the primary and secondary circuit, unless proper precautions are taken in regard to the mode of connection. The Carey-Foster bridge used in measurements of the capacity and effective resistance of condensers is particularly liable to error owing to this form of impurity, as the common point of the mutual inductance is a four branch point. (See Fig. 7.) Leakage is usually of little importance with proper insulation. However, a non-uniform distribution of resistance or inductance, combined with distributed capacity, may cause an inductance to behave as if it possessed an end to end capacity having leakage. This is readily seen by considering the case of a pure inductance, L , shunted by a capacity, C , and connected in series with a resistance, R . If C and $R/\omega L$ are small, then the capacity C across L alone is equivalent to a capacity C , having leakance $-2CR/L$, and acting as a shunt across both R and L .

The effects of self and intercapacity are treated in section 4 for the case where the two coils have a common point. It is there shown that small capacities produce impurities varying as the square of the frequency, and this variation is accom-

* Silsbee, Bull. Bureau of Standards, Vol. XIII., p. 414, 1916.

panied by variations of effective resistance and inductance following the same law.

The effects due to eddy currents are considered in section 7 *et seq.* The effects are mainly variations in impurity and effective resistance, the changes in inductance due to eddy currents being small. If the frequency is not too high, the changes in impurity and effective resistance are proportional to the square of the frequency. Thus it is impossible by effective resistance measurements alone to separate the effects due to capacity from those due to eddy currents, but some estimate of the relative importance of the two effects might be obtained by determining the frequency coefficients of the inductances and assuming these to be due mainly to capacity.

3. Preliminary Network Transformations.

The theory to be given is much simplified if use is made of the following network transformations.*

(A) Two inductive coils (Fig. 1a), with resistance operators

FIG. 1a.

FIG. 1b.

ξ , η and mutual operator m , and having a common point, may be replaced by three inductive coils having resistance operators $\xi - m$, $\eta - m$, m , and no mutual operators radiating as a "star" combination from the common point. (Fig. 1b.)

In this transformation, m must be taken as positive when the directions of winding are such that if current is applied to the combination at the open ends the mutual inductance will oppose the self-inductances.

* G. A. Campbell, Trans. Am.I.E.E., Vol. XXX., p. 891, 1911.

(B) Three arms, OA , OB , OC , having resistance operators a , b , c respectively, and forming a "star" combination

may be replaced by a triangular "mesh,"

provided that the resistance operators X , Y , Z of the arms BC , CA , AB have the values $(bc+ca+ab)/a$, b , c .

Conversely, the mesh may be replaced by a star, and then

$$a, b, c = (YZ, ZX, XY) / (X + Y + Z).$$

If the symbols represent conductance operators, the above relations are interchanged.

4. The Effect of Self and Intercapacities in Mutual Inductances.

Let the two coils have a common point. Let their resistance operators, apart from capacity, be ξ , η , and let their mutual operator be m . Suppose the self capacities to be sufficiently represented by condensers of capacities c_1 , c_2 in parallel with the coils ξ , η , and let the intercapacity be represented by a condenser of capacity c_{12} , joining the open ends of the coils. The system is then as shown in Fig. 2a.

By application of transformation (A), the system is converted to that of Fig. 2b, in which

$$x = m, y = \xi - m, z = \eta - m \quad \dots \quad (2)$$

and α , β , γ are the conductance operators of the capacity arms c_{12} , c_2 , c_1 .

By application of transformation (B) to the star combination x , y , z , the system of Fig. 2b is converted to that of Fig. 2c, in which

$$\begin{aligned} \alpha' &= \alpha + x/\mu, \beta' = \beta + y/\mu, \gamma' = \gamma + z/\mu \\ \mu &= yz + zx + xy \quad \dots \quad (3) \end{aligned}$$

By the converse of transformation (B), the system of Fig 2d is then obtained, in which

$$\left. \begin{aligned} X, Y, Z &= \frac{x + \alpha\mu, y + \beta\mu, z + \gamma\mu}{1 + \alpha(y + z) + \beta(z + x) + \gamma(x + y) + \lambda\mu} \\ \lambda &= \beta\gamma + \gamma\alpha + \alpha\beta \end{aligned} \right\} \quad \dots \quad (4)$$

Finally, by the converse of transformation (A), the system of Fig. 2*d* is converted to that of Fig. 2*e*, in which

$$\xi' = X + Y, \eta' = Z + X, m' = X \quad . \quad . \quad . \quad (5)$$

The result is thus a system in which the capacities have been suppressed, so that ξ' , η' , m' are the modified operators to employ to include the effects of capacity. If c_1 , c_2 , c_{12} are

FIG. 2.

small, quantities involving their squares or products may be neglected, and in this case it is found from (2), (4) and (5) that

$$\left. \begin{aligned} m' &= m + \alpha(\xi - m)(\eta - m) - \beta m \eta - \gamma m \xi \\ \xi' &= \xi - \alpha(\xi - m)^2 - \beta m^2 - \gamma \xi^2 \\ \eta' &= \eta - \alpha(\eta - m)^2 - \beta \eta^2 - \gamma m^2 \end{aligned} \right\} \quad . \quad . \quad . \quad (6)$$

Now, in the case of currents alternating with frequency $\omega/2\pi$, resistance operators may be replaced by vector impedances, so that we may then write

$$\begin{aligned} \xi &= R_1 + j\omega L_1, & \eta &= R_2 + j\omega L_2, & m &= j\omega M \\ a &= j\omega C_{12}, & \beta &= j\omega C_2, & \gamma &= j\omega C_1 \end{aligned} \quad (7)^*$$

in which R_1, R_2 are the resistances of the coils, L_1, L_2, M the inductances.

Applying (7) to (6) and separating the real and imaginary parts, the following formulæ are obtained.

$$m_c = C_{12}R_1R_2 + \omega^2 \{C_1L_1M + C_2L_2M - C_{12}(L_1 - M)(L_2 - M)\} \quad (8)$$

$$\sigma_c = \omega^2 [C_1R_1M + C_2R_2M - C_{12}\{R_1(L_2 - M) + R_2(L_1 - M)\}] \quad (9)$$

$$l_{1c} = -(C_1 + C_{12})R_1^2 + \omega^2 \{C_1L_1^2 + C_2M^2 + C_{12}(L_1 - M)^2\} \quad (10)$$

$$l_{2c} = -(C_2 + C_{12})R_2^2 + \omega^2 \{C_1M^2 + C_2L_2^2 + C_{12}(L_2 - M)^2\} \quad (11)$$

$$r_{1c} = 2\omega^2 R_1 \{C_1L_1 + C_{12}(L_1 - M)\} \quad (12)$$

$$r_{2c} = 2\omega^2 R_2 \{C_2L_2 + C_{12}(L_2 - M)\} \quad (13)$$

In these formulæ m_c, l_{1c}, l_{2c} are the increases in mutual and self inductance, σ_c is the impurity introduced by the capacities, r_{1c}, r_{2c} are the added resistances due to capacity.

5. Measurement of Self and Intercapacity.

If it may be assumed that the variations of self and mutual inductance are due to capacity only, equations (8), (10), and (11) may be used to determine the capacities C_1, C_2, C_{12} .

With both coils in position and having a common point, the effective self inductances of the two coils separately and in series are determined at a number of different frequencies. Then, if for brevity we denote the coefficients of ω^2 in (8), (10), (11) by k_{12}, k_1, k_2 and express C_1, C_2, C_{12} in terms of these coefficients, we obtain

$$\left. \begin{aligned} (L_1L_2 - M^2)^2 C_1 &= k_1 L_2 (L_1 - M) - k_2 M (L_1 - M) \\ &\quad + k_{12} (L_1 L_2 - 2ML_1 + M^2) \\ (L_1L_2 - M^2)^2 C_2 &= -k_1 M (L_2 - M) + k_2 L_1 (L_2 - M) \\ &\quad + k_{12} (L_1 L_2 - 2ML_2 + M^2) \\ (L_1L_2 - M^2)^2 C_{12} &= k_1 M L_2 + k_2 M L_1 - k_{12} (L_1 L_2 - M^2) \end{aligned} \right\} \quad (14)$$

* If leakage is also present α, β, γ are of the forms $G + j\omega C$ in which G is the leakage associated with the capacity C .

These equations determine C_1 , C_2 , C_{12} as k_1 , k_2 , k_{12} and may be deduced from the frequency variations of the self and mutual inductances.

The method fails when the coupling is very close, as, for perfect coupling, the three relations employed are no longer independent.

The values of C_1 , C_2 obtained by this method are not necessarily the same as would be obtained when the coils are separated from each other, as the capacities C_1 , C_2 include any intercapacity that may be regarded as acting from the inner end of one coil to the outer end of the other. Also if the common point is altered the values of C_1 , C_2 , C_{12} may

FIG. 3.

also change because of the redistribution of the intercapacities.

However, by a systematic alteration of the common points the assumption that distributed intercapacities may be replaced by end capacities may be put to an experimental test.

Thus, when the coils are not connected the system is as in Fig. 3, in which the broken lines represent the capacity arms. Four common points are possible, so that 12 combinations of the six capacities—I, II, III, IV, V, VI—may be measured. These are given in the following table:—

Common Point.	C_1		C_2		C_{12}
PR	I+VI	...	II+V	...	IV
QS	I+V	...	II+VI	...	III
QR	I+III	...	II+IV	...	V
PS	I+IV	...	II+III	...	VI

Since the values of C_{12} give the four intercapacities, the remaining observations give four independent determinations of the self capacities I and II respectively. If the end to end capacity theory is adequate, these should be consistent and in agreement with the self capacities as determined when the coils are separated.

6. *Conditions for Zero Impurity and for No Variation of Mutual Inductance with Frequency.*

By equation (8) the frequency coefficient of mutual inductance is zero when

$$C_{12} = M(C_1 L_1 + C_2 L_2) / (L_1 - M)(L_2 - M). \quad (15)$$

By equation (9) the impurity due to capacity is zero when

$$C_{12} = M(C_1 R_1 + C_2 R_2) / \{R_1(L_2 - M) + R_2(L_1 - M)\}. \quad (16)$$

If condition (15) is inserted in equation (9) an essentially negative value is obtained for σ_e , so that a mutual inductance invariable with frequency is always associated with a negative capacity impurity. As, however, the whole impurity includes both capacity and eddy current terms, and as the latter may be positive or negative it is theoretically possible to construct a mutual inductance invariable with frequency and free from impurity. This conclusion only holds when the common point is such that the two coils in series have their minimum inductance.

The procedure indicated by theory to bring about this result is to secure the condition of invariability of mutual inductance by artificial addition of self or intercapacity and then the condition of zero impurity by introduction of artificial eddy losses.

EFFECTIVE RESISTANCES AND IMPURITIES DUE TO EDDY CURRENT LOSSES.

7. *Eddy Current Losses in a Long Cylinder.*

Let a long non-magnetic cylinder of diameter d and of material of resistivity ρ be placed in a uniform field alternating with frequency $\omega/2\pi$. Let H' and H be the maximum values of the components of the field along and perpendicular to the axis of the cylinder. Also write

$$z^2 = \pi \omega d^2 / \rho. \quad (17)$$

Then it may be shown that the rate of dissipation of energy in unit length of the cylinder due to the eddy currents set up by the alternating field is

$$W_0 = \frac{\rho}{2\pi} G(z)(H^2 + \frac{1}{2}H'^2) \quad . \quad . \quad . \quad (18)$$

in which for brevity $G(z)$ is written for

$$\frac{1}{4}z \frac{\text{ber } z \text{ ber}' z + \text{bei } z \text{ bei}' z}{\text{ber}^2 z + \text{bei}^2 z} \quad . \quad . \quad . \quad (19)$$

When z is less than 1

$$G(z) \doteq z^4/64, \quad . \quad . \quad . \quad (20)$$

When z is greater than 4

$$G(z) \doteq (\sqrt{2}z - 1)/8, \quad . \quad . \quad . \quad (21)$$

while generally the following table may be used:—

$z=0.5$	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$G(z)=0.000975$	0.01519	0.0691	0.1724	0.295	0.405	0.499	0.584
$z=4.5$	5.0						
$G(z)=0.669$	0.755						

8. Eddy Current Losses in a Coil.

Equation (18) may be applied to determine the eddy current losses in a coil placed in an alternating magnetic field, provided that the values of H and H' are practically constant over the section of an individual wire, that their variation is slow along the axis of the wire and that the radius of curvature of the wire is large compared with the radius of the wire itself. Then assuming for simplicity that the field is acting transversely so that $H'=0$, the rate of energy dissipation in the whole coil is

$$W = \frac{\rho}{2\pi} G(z) \int_0^l H^2 dl, \quad . \quad . \quad . \quad (22)$$

where the integration is taken throughout the length l of the wire of the coil.

If H_m^2 is the mean square value of the transverse field acting on the coil and R is the direct current resistance of the coil, equation (22) may be written

$$W = \frac{1}{8} d^2 R G(z) H_m^2 \quad . \quad . \quad . \quad (23)$$

9. *Coil in Presence of Two Other Coils.*

Let the field H be due to two currents of vector values, $\mathbf{I}_1, \mathbf{I}_2$, and of phase difference ψ , flowing in two neighbouring coils. The vector fields due to $\mathbf{I}_1$ and $\mathbf{I}_2$, which act on an element of the coil, will be of the forms $a_1\mathbf{I}_1, a_2\mathbf{I}_2$, and will be inclined in space at an angle φ .

The components of the field along and perpendicular to $a_1\mathbf{I}_1$ are

$$\mathbf{H}_1 = a_1\mathbf{I}_1 + a_2\mathbf{I}_2 \cos \varphi$$

$$\mathbf{H}_2 = a_2\mathbf{I}_2 \sin \varphi,$$

so that if H_1, H_2 are the amplitudes of $\mathbf{H}_1, \mathbf{H}_2, I_1, I_2$ the amplitudes of $\mathbf{I}_1, \mathbf{I}_2$, the value of H^2 is

$$\begin{aligned} H^2 &= H_1^2 + H_2^2 \\ &= a_1^2 I_1^2 + a_2^2 I_2^2 \cos^2 \varphi + 2a_1 a_2 I_1 I_2 \cos \varphi \cos \psi + a_2^2 I_2^2 \sin^2 \varphi \end{aligned}$$

and its mean value throughout the whole coil is

$$H_m^2 = \frac{I_1^2}{l} \int_0^l a_1^2 dl + \frac{I_2^2}{l} \int_0^l a_2^2 dl + 2 \frac{I_1 I_2}{l} \cos \psi \int_0^l a_1 a_2 \cos \varphi dl. \quad (24)$$

The rate of dissipation of energy in the coil is therefore obtained by using this value of H_m^2 in (23).

10. *Resistance System Imitating Eddy Current Losses.*

Let the two coils (I and II) which carry the currents I_1 and I_2 have their circuits connected by the resistance system shown in Fig. 4. The rate of dissipation of energy in this resistance system is

$$W' = \frac{1}{2} r_{1e} I_1^2 + \frac{1}{2} r_{2e} I_2^2 - \sigma_e I_1 I_2 \cos \psi, \quad \dots \quad (25)$$

and the system will dissipate the same energy as the eddy currents in the coil just considered provided that

$$\left. \begin{aligned} r_{1e} &= \frac{1}{4} d^2 R G(z) \left\{ \frac{1}{l} \int_0^l a_1^2 dl \right\} \\ r_{2e} &= \frac{1}{4} d^2 R G(z) \left\{ \frac{1}{l} \int_0^l a_2^2 dl \right\} \\ \sigma_e &= -\frac{1}{4} d^2 R G(z) \left\{ \frac{1}{l} \int_0^l a_1 a_2 \cos \varphi dl \right\} \end{aligned} \right\} \quad \dots \quad (26)$$

If by the converse of transformation (A) we convert σ_e to a mutual operator, the common point may be removed and $r_{1e},$

r_{2e} , r_e will then represent the increments of resistances and the impurity introduced by the eddy current losses in a neighbouring coil. If there are a number of neighbouring coils, then since the losses are additive, the eddy current resistances and impurities are sums of expressions such as (26). In particular the neighbouring coils may simply be the coils *I* and *II* and the eddy current resistances and impurities are got by performing the integrations in (26) throughout the lengths of wire in the two coils.

The increments in resistance obtained in this way are to be added to the ordinary "skin" increments of resistance, the

FIG. 4.

latter being due to the currents flowing in the individual elements of the coil.

11. Conditions Governing Sign of σ_e .

In applying equation (26) some care is required in order to avoid the wrong sign for σ_e . In Fig. 4, from which the equations have been deduced, the directions of the currents are such that if $I_1 = I_2 = I$ the current I will flow only through the resistances $r_{1e} - \sigma_e$, $r_{2e} - \sigma_e$, so that when the coils are connected in series the total eddy current resistance is $r_{1e} + r_{2e} - 2\sigma_e$. In order therefore to make σ_e consistent with the defining equation (1) the mutual inductance M between the coils *I* and *II* must, under the same conditions, oppose the self-inductances. Now a_1 , a_2 are fields due to unit currents in coils *I* and *II* respectively, so that in order to obtain the correct sign for σ_e the directions of these unit

currents must be so chosen that the mutual inductance opposes the self-inductances. With this convention σ_e will be positive if the sum of the eddy current resistances is less than the eddy current resistance of the coils in series and helping.

12.

It is readily seen from equations (26) that the inequalities

$$r_{1e} + r_{2e} > 2\sigma_e$$

$$r_{1e}r_{2e} > \sigma_e^2$$

hold for the eddy current resistances, relations analogous to those between self and mutual inductances.

13. *Systems of Parallel Wires.*

The theory finds its simplest application when the coils *I* and *II* consist of systems of parallel wires. The results in these cases may be applied to coaxial coils provided that winding

FIG. 5.

sections of the coils and their distance apart are small compared with their radii.

In order to illustrate the properties of the impurity σ_e , consider three parallel wires *A*, *B*, *C* (Fig. 5). Let *A* be the coil *I* and *B* the coil *II*. Let *C* carry no current. The impurity introduced by the presence of *C* will be negative, zero or positive according as the angle of inclination of the fields at *C* produced by opposite currents in *A* and *B* is less than, equal to, or greater than $\pi/2$. If a circle be drawn with *AB* as diameter, then since the two fields are normal at all points on this circle, the impurity due to the presence of *C* changes sign from negative to positive as *C* is moved out

of the circle. Along the line AB , the fields either help or oppose each other, so that when C is collinear with AB the condition $r_{1e}r_{2e}-\sigma_e^2=0$ is satisfied. This is a case of perfect eddy current coupling.

The above conclusions still apply when the wires A and B are replaced by bundles of wires, provided that the distance of C is sufficient to warrant the assumption that the currents in A and B act as if concentrated at a point. It is thus possible to counteract an already existing impurity of any sign by introducing a third coil carrying no current in the proper position with respect to the two coils forming the mutual inductance. An example of this use of eddy currents to correct impurity is given in Section 19.

14. System of Four Wires.

Let there be four coplanar wires, two constituting the coil I and two constituting the coil II , spaced at equal distances D . Numbering the wires in order 1, 2, 3, 4, there are three possible combinations, (1, 2), (3, 4); (1, 3), (2, 4); (1, 4), (2, 3), each giving different effective resistances and impurities.

By a simple application of the formula $H=\frac{2I}{r}$ we obtain the following values for a_1 , a_2 , $\cos \varphi$ for the combination (1, 2), (3, 4).

Wire	$a_1 D$	$a_2 D$	$\cos \varphi$
1	2	$5/3$	-1
2	2	3	+1
3	3	2	+1
4	$5/3$	2	-1

Using these values in (26) we find

$$r_{1e}=2.472 \frac{d^2}{D^2} G(z)R=r_{2e}, \sigma_e=-0.667 \frac{d^2}{D^2} G(z)R$$

in which R is the direct-current resistance of one pair of wires. By similar methods for the combination (1, 3), (2, 4) we find

$$r_{1e}=1.139 \frac{d^2}{D^2} G(z)R=r_{2e}, \sigma_e=+0.667 \frac{d^2}{D^2} G(z)R$$

and for the combination (1, 4), (2, 3)

$$r_{1e}=0.361 \frac{d^2}{D^2} G(z)R, r_{2e}=3.250 \frac{d^2}{D^2} G(z)R, \sigma_e=0.$$

In all three cases $r_{1e}+r_{2e}+2\sigma_e$ has the same value, viz., $3.611 \frac{d^2}{D^2} G(z)R$, which is, of course, the proper value for the proximity resistance of a system of four parallel wires carrying currents in the same direction. The example also shows that the sign and magnitude of the impurity depends upon the interlacing of the wires of the two coils, the tendency being for the impurity to increase positively as the interlacing increases. This point may be further illustrated by a system of 12 equidistant wires, of which six constitute one circuit and six the other circuit. If we write

$$\sigma^e = u_{12} \frac{d^2}{D^2} G(z)R,$$

u_{12} takes the following values for different arrangements :—

Arrangement	u_{12}
1 1 1 1 1 1 2 2 2 2 2 2	—0.80
1 1 1 1 1 2 2 2 2 2 2 1	—0.70
1 1 1 1 2 2 2 2 2 2 1 1	—0.66
1 1 1 2 2 2 2 2 2 1 1 1	—0.65
1 1 1 2 2 2 1 1 1 2 2 2	—0.35
1 1 2 2 1 1 2 2 1 1 2 2	—0.00 ₃
1 2 1 2 1 2 1 2 1 2 1 2	+0.66

Interlacing the coils will increase intercapacity effects, so that if the coils are used with their common point such that the inductance in series is a minimum, the effect of increasing intercapacity will cause a capacity impurity acting oppositely to the eddy current impurity.

15. *Short Single Layer Coils in Contact.*

The eddy current impurity between two short coils of equal radius in end contact may be calculated if their separate effective resistances r_{1e} , r_{2e} and their effective resistance r_s in series are known for $r_s = r_{1e} + r_{2e} + 2\sigma_e$.

The value of r_s has been shown by the author in another Paper to be of the form

$$r_s = u_n \frac{d^2}{D^2} G(z)R. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

Where R is the direct-current resistance of the whole combination, $G(z)$ is defined in Section 7, d is the diameter of the wire, D the distance of consecutive wires and u_n is a

factor depending on the number of wires n in the two coils. The following values when plotted against $1/n$ enable u_n to be determined in any case:—

$$\begin{array}{cccccc} n = & 1 & 2 & 3 & 5 & 10 & \text{inf.} \\ u_n = & 0.000, & 1.000, & 1.500, & 2.014, & 2.514, & 3.290 \end{array}$$

The values of r_{1e} , r_{2e} are, however, greater than would be estimated by the above method because of the eddy currents induced in one coil by the current in the other. If the number of wires is finite in both coils, the additive term due to the presence of the other coil may be calculated by methods similar in principle to those employed in the case of the four-wire system. When the number of wires is large, this method becomes laborious, but the extreme case of an infinite number of wires may be obtained by integration. Treating the narrow coils as straight-wire systems, the field at any point in the plane of a strip of width a is $2 \frac{I}{D} \log \frac{a+x}{x}$ where I is the current in each wire and x the distance of the point from the edge of the strip. If the strip representing the other coil has width b , the mean square value of the field over this coil is

$$\frac{4I^2}{D^2} \cdot \frac{1}{b} \int_0^b \log^2 \frac{a+x}{x} dx = \frac{4I^2}{D^2} \frac{a}{b} f\left(\frac{b}{a}\right)$$

in which
$$f\left(\frac{b}{a}\right) = \int_0^{\frac{b}{a}} \log^2 \frac{1+\mu}{\mu} d\mu.$$

The values of $f(\mu)$ may be obtained in the form of series. Thus suitable series are:

$$\mu < 2$$

$$f(\mu) = -\mu \log^2 \nu + 2 \log \nu \log(1-\nu) + 2 \sum_{r=1}^{\infty} \frac{\nu^r}{r^2} \quad \dots \quad (28)$$

in which

$$\nu = \mu/(1+\mu);$$

$$\mu > 2$$

$$f(\mu) = \frac{\pi^2}{3} - \sum_{r=1}^{\infty} \frac{(-1)^{r+1}}{r \mu^r} \sum_{s=1}^{s=r} \frac{1}{s(s-r+1)} \quad \dots \quad (29)$$

By (26) the term in r_{1e} for the coil (a), which is due to the eddy currents in (b) is

$$\frac{d^2}{D^2} G(z) \frac{a}{b} f\left(\frac{b}{a}\right) R = \frac{d^2}{D^2} G(z) f\left(\frac{b}{a}\right) R_a.$$

where R_a , R_b are the direct-current resistances of R_a and R_b .
Hence

$$\left. \begin{aligned} r_{1e} &= \left\{ u_n(a) + f\left(\frac{b}{a}\right) \right\} \frac{d^2}{D^2} G(z) R_a \\ r_{2e} &= \left\{ u_n(b) + f\left(\frac{a}{b}\right) \right\} \frac{d^2}{D^2} G(z) R_b \end{aligned} \right\} \dots (30)$$

These equations, together with (27) and the tables of u_n and $f(b/a)$ enable the eddy current resistances and impurities of short single layer coils in end contact to be calculated. In the tables for $f(b/a)$ the values for the case b/a infinite but n finite are calculated from the formula

$$f\left(\frac{b}{a}\right) = \frac{\pi^2}{6} + 1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots + \frac{1}{(n-1)^2} - \frac{1}{n} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n-1} \right).$$

Where n is the number of wires in a .

When n is also infinite $f(b/a) = \pi^2/6$.

Values of $f(b/a)$ in Equations (30).

n_a = number of wires in system a .

n_b = " " " " " b .

$b/a < 1$.

n_a	n_b	$f(b/a)$	n_a	n_b	$f(b/a)$
2	1	1.125	5	1	1.042
3	2	1.511	10	4	1.704
3	1	1.120	10	3	1.523
5	4	1.858	10	2	1.265
5	3	1.701	10	1	0.858
5	2	1.462			

$n_a = n_b = \text{infinity}$.

$b/a = 0.2 \quad 0.4 \quad 0.6 \quad 0.8$

$f(b/a) = 1.375 \quad 2.089 \quad 2.334 \quad 2.492$

$b/a > 1$.

n_a	$b/a = 1$	2	3	4	5	inf.
1	1.000	1.250	1.361	1.423	1.463	1.645
2	1.472	1.743	1.859	1.923	1.963	2.145
3	1.715	1.992	2.108	2.172	2.209	2.395
5	1.970	2.248	2.365	2.429	2.470	2.652
10	2.218	2.497	2.613	2.674	2.710	2.901
inf.	2.606	2.886	3.002	3.067	3.119	3.290

For interpolation $f(b/a)$ may be plotted against b/a for different values of n_a , and against $1/n$ for different values of b/a .

16.

As an illustration of the application of these results let the coil (*a*) have 20 turns, and the coil (*b*) 30 turns.

Then by graphical interpolation from the tables we find for coil (*a*).

$$f(b/a, n_a) = f(3/2, 30) = 2.55$$

$$u_n(a) = u_{20} = 2.84,$$

so that by (30)

$$r_{1e} = 5.39 \times 20 G(z) \frac{d^2}{D^2} R, \quad \dots \quad (A)$$

R being the direct-current resistance of one turn of wire.

For coil (*b*)

$$f(a/b, n_b) = f(2/3, 30) = 2.24$$

$$u_n(b) = u_{30} = 2.98$$

and

$$r_{2e} = 5.22 \times 30 G(z) \frac{d^2}{D^2} R. \quad \dots \quad (B)$$

For coils (*a*) and (*b*) in series, helping

$$f = 0 \quad u_n = u_{50} = 3.10$$

and

$$r_{1e} + r_{2e} + 2\sigma_e = r_e = 3.10 \times 50 G(z) \frac{d^2}{D^2} R. \quad \dots \quad (C)$$

From (A), (B) and (C)

$$r_{1e} : r_{2e} : \sigma_e = 10.8 : 15.7 : 5.5$$

σ_e being negative.

The absolute magnitudes will depend upon the frequency, the diameter of the wire and the spacing. If we assume the wire to be of copper of resistivity 1,600 C.G.S. units, and of diameter 1 mm., then by (17) the value of *z* is unity when the frequency is approximately 8,000 cycles per second, and then $G(z) = 0.01519$. Therefore if we take $d/D = 0.8$

$$\frac{r_{1e}}{20R} = 5.39 \times 0.01519 \times (0.8)^2 = 0.0523,$$

so that at this frequency the value of r_{1e} is 5.2 per cent. of the direct-current resistance of coil (*a*).

The ordinary skin effect for copper wire of diameter 1 mm.

at this frequency causes an increase of 0.52 per cent., and this must be added to the above value. If coil (*b*) had been absent the value of r_{1e} would be reduced in the ratio 2.84/5.39, so that slightly less than half of the above increase is due to the presence of the coil (*b*).

For frequencies less than 8,000 cycles per second the eddy current resistances would fall as the square of the frequency, as by section (7) z^2 is proportional to frequency, and $G(z)$ is proportional to z^4 .

At greater frequencies the increase is not so rapid, and may be deduced from the values of $G(z)$ given in section (7), z being proportional to the square root of the frequency.

In certain applications the values of $r_{1e}/L_1\omega$, $r_{2e}/L_2\omega$, $\sigma_e/M\omega$ are of importance, the latter quantity giving the want of quadrature of the mutual inductance. In the above example if we suppose the coils such that R/L for coil (*a*) is about 1,000, we have $r_{1e}/L_2\omega=0.001$, and this also gives the order of $\sigma_e/M\omega$. Hence at a frequency of 8,000 cycles per second the secondary E.M.F. will lead the primary current by an angle which exceeds a right angle by about 0.001 radian, i.e., about 3'.

This angle does not increase indefinitely with frequency, as from (27) and (17) $\sigma_e/M\omega$ is proportional to $G(z)/z^2$. From the table of values for $G(z)$ it is seen that $G(z)/z^2$ is maximum when $z=2.6$, and (for the above example) the maximum value for the want of quadrature is about 10'.

It should be noted that for a given spacing factor this maximum angle is independent of the diameter of the wire, but is pushed further up the scale of frequency as the diameter of the wire diminishes.

17. *Short Many-layered Coils Wound Over Each Other.*

The investigation will be confined to the case in which the winding depth is small compared with the winding breadth, and the latter small compared with the coil radius. The results apply whether the coils are solenoidal or flat.

For a single coil of winding breadth, b , having m layers and n turns per layer of wire of diameter d , and direct-current resistance R_1 , the term to be added to the effective resistance due to the increased eddy currents produced by coiling, has been shown by the author to be

$$r_e = \frac{1}{8}\pi^2 \left(\frac{nd}{b}\right)^2 (2m^2 - 1)G(z)R_1 \quad . \quad . \quad . \quad (31)$$

If a second coil of M layers is wound over this coil there is a further increase in resistance due to the eddy currents induced in the wire of the second coil.

The components of the field due to the current I in the first coil acting along and perpendicular to the winding breadth have the values $2\pi mnI/b$, and $2mnI \log \frac{b-x}{x}/b$ respectively at a point on its surface distant x from the edge. If the second coil has very small winding depth, and is close on the first, the mean square value of the field having the above components may be used in (23) to determine the required additive term.

Since

$$\frac{1}{b} \int_0^b \log^2 \frac{b-x}{x} dx = \frac{\pi^2}{3},$$

the result is

$$r'_e = \frac{4}{3} \pi^2 \left(\frac{nd}{b} \right)^2 MmG(z)R_1, \quad \dots \quad (32)$$

remembering that R in (23) refers to the second coil, and is R_1M/m .

Combining with (31) we have

$$r_{1e} = \frac{1}{3} \pi^2 \left(\frac{nd}{b} \right)^2 G(z)R_1(2m^2 + 4Mm - 1). \quad \dots \quad (33)$$

Similarly

$$r_{2e} = \frac{1}{3} \pi^2 \left(\frac{nd}{b} \right)^2 G(z)R_2(2M^2 + 4Mm - 1), \quad \dots \quad (34)$$

and for the two coils in series helping (31) immediately gives

$$r_{1e} + r_{2e} + 2\sigma_e = \frac{1}{3} \pi^2 \left(\frac{nd}{b} \right)^2 G(z)(R_1 + R_2) \{2(M+m)^2 - 1\} \quad (35)$$

Solving for σ_e with $\frac{R_1}{m} = \frac{R_2}{M}$,

$$\sigma_e = \frac{1}{3} \pi^2 \left(\frac{nd}{b} \right)^2 G(z)(R_1 + R_2)Mm. \quad \dots \quad (36)$$

Comparing with (31) $\sigma_e/r_e = M(M+m)/(2m^2 - 1)$ from which it is seen that in the case of equal coils the impurity is slightly greater than the term due to the coiling when the two coils are separate. Since the electromagnetic coupling is fairly close, the value of $r_e/L\omega$ for either coil may be taken as a measure also of $\sigma_e/M\omega$, the want of quadrature.

It should be noted also that σ_e is positive for coils with their widest edges in contact, thus differing from the coils in end contact treated in section 15.

18.

Some notion of the magnitude of the eddy current impurities in coils of the type just considered may be obtained by taking wire of a diameter of 1 mm. and a frequency of 8,000 cycles per second. Suppose also that for a single layer $R_1/L_1=1,000$. By the example of section (16) the value of $r_e/L_1\omega$ for a single layer is of the order 5/10,000. Now as the layers increase L increases as m^2 , R_1 as m , so that by (31) $r_e/L_1\omega$ will increase as $(2m^2-1)/m$. If m is large the value of $r_e/L_1\omega$ will therefore be $m/1,000$, and this may also be taken as a measure of $\sigma_e/M\omega$. Errors in quadrature due to this cause will therefore amount to one per cent. (34') at a frequency of 8,000 cycles per second. For lower frequencies and diameters the error is proportional to the frequency and the fourth power of the diameter, so that the formula

$$\frac{\sigma_e}{M\omega} = \frac{mf}{8 \times 10^9} \cdot \frac{R}{L} d^4$$

may be used to give a very rough estimate of the want of quadrature. In this formula R/L is the time constant of a single layer, d the diameter of the wire in mm., f the frequency in cycles per second, m the number of layers in one of the two equal coils.

Separation of the coils will not necessarily determine the value of σ_e/M as both σ_e and M will diminish.

19. *Balancing of Impurity and Frequency Co-efficient of a Mutual Inductance.*

It will now be shown that it is possible to introduce inter-capacity and eddy current losses of such a magnitude as to balance simultaneously an already existing frequency co-efficient and impurity in a mutual inductance, provided that the inductance is used with a common point in such a way that the coils are in opposition when in series. Let the two coils be equal and coaxial. Let the radius of each coil be 10 cm., their separation 3 cm. and let there be 240 turns with a square winding section of 1 cm. side. Each coil would then have an inductance of approximately 24 millihenries

and their mutual inductance would be approximately 10 millihenries. The frequency co-efficient of mutual inductance will be assumed to be positive and of amount $4/1,000$ at a frequency of 2,000 cycles per second.* The impurity at the same frequency will be taken to be $+0.08$ ohm.*

If intercapacity is introduced then by (8) and (9) with $L_1 = L_2 = L$, the frequency co-efficient of M is reduced by $\omega^2 C_{12}(\bar{L} - M)^2/M$ and the impurity by $2R\omega^2 C_{12}(\bar{L} - M)$. Using the above values of L , M the value of C_{12} required to wipe out the already existing frequency co-efficient is approximately $1,280 \mu\mu F$. This will also introduce negative impurity of amount $0.0057R$ ohms. If the resistance of the coils is less than 14 ohms resistance may be added until $0.0057R = 0.08$, and this will be sufficient to balance the impurity.

If R is greater than 14 ohms, the impurity has become negative by the addition of capacity, and in this case an auxiliary coil of the same radius as the other coils can be placed coaxially with them to introduce positive impurity. Thus suppose the auxiliary coil to consist of wire 1 mm. diameter placed at 1.5 cm. separation from the nearer coil, the positive impurity

introduced by the auxiliary coil is $\frac{N^2 d^2}{D_1 D_2} G(z) R'$,

where N = No. of turns in the inductometer coils = 240.

a = diameter of wire = 0.1 cm.

D_1, D_2 = separations of coil from inductometer coils.
= 1.5, 4.5 cm.

R' = resistance of auxiliary coil.

Using these values and remembering that $G(z) = 0.01519$ at a frequency of 8,000 cycles per second and falls as the square of the frequency, the value of R' to give 0.01 ohm impurity at 2,000 cycles per second is

$$R' = 0.124 \text{ ohm.}$$

This corresponds to about 10 turns of wire of radius 10 cm.

This balancing will hold over the range of frequencies, for which the square law holds. If the diameter of any wire used does not exceed 1 mm. the range is up to 8,000 cycles per second.

* These are the order of magnitude of these quantities in the Campbell type of inductometer.

20. *Effect of Impurity, &c., in Alternating Current Bridges.*

The effect of impurity in alternating current bridge measurements in which mutual inductances are employed may be included by replacing the mutual impedance $j\omega M$ in the vector equations of balance of these bridges by $\sigma + j\omega M$. In interpreting these equations account should of course be taken of the variations of inductance and resistance of the various arms with frequency. Three bridges will be considered.

- (1) The Heaviside Inductance bridge with equal arms.
- (2) The Carey Foster Inductance-capacity bridge.
- (3) A modified Campbell frequency bridge.

21. *The Heaviside Inductance Bridge.**

The bridge is shown in Fig. 6. P, P are two similar coils of small residual inductance. L, R is the inductive coil to be

FIG. 6.—HEAVISIDE INDUCTANCE BRIDGE.

measured by means of the inductometer M . R is a constant inductance rheostat.

Let the procedure be as follows:—

With the source connected to a and $\mathcal{L}, \mathcal{R}$ removed, L_1 and R are adjusted for balance. Then with $\mathcal{L}, \mathcal{R}$ in position and the source connected to b , M and R are adjusted for balance. Let the two values of R be R', R'' .

If residuals and impurities are ignored $\mathcal{L} = 2M$ and $\mathcal{R} = R' - R''$.

As regards residuals, if observations are repeated with the

* Heaviside, "Phil. Mag.," p. 173, Vol. XXIII., 1887.

arms P , P reversed the residuals of P , P cancel in the mean. Also, R being of constant inductance the residuals of the other arms cancel when the above procedure is used.

If the mutual inductance has impurity σ a simple application of transformation (*A*) shows that the two conditions of balance are $\mathcal{L}=2M$, $\mathcal{R}=2\sigma+R'-R''$

It should be noted that the value of R_2 for the coil L_2 only remains the same in the two adjustments provided that the inductive action of M is introduced by the method of transference of the source from a to b keeping the primary coil of the inductometer in position. This is the procedure used in the Campbell form of inductometer, the fine adjustment only being made by a moving coil. In inductometers in which the whole of the adjustment is made by a moving coil, the eddy current and capacity conditions may be different in the two balances, so that R_2 will have different effective values.

Since for an inductometer of 10 millihenries σ may be of the order 0.1 ohm at a frequency of 2,000 cycles per second, neglect of σ in the relation between the resistance may introduce errors of order two per cent. in measurements of the effective resistance of 20 millhenry inductances of a nominal resistance of 10 ohms.

22. The Carey-Foster Inductance Capacity Bridge.*

The elements of this bridge (Fig. 7) are :—

(a) The condenser arm having effective capacity C and effective series resistance S .†

Let its vector impedance be $Z=S-j/\omega C$.

(b) The inductive arm P , L having vector impedance $\xi=P+j\omega L$;

(c) The resistance arm R having residual inductance l and therefore vector impedance $Y=R+j\omega l$;

(d) The mutual inductance M having impurity σ and acting on the inductive arm. Its mutual impedance is $m=\sigma+j\omega M$.

The condition of balance is readily found by applying transformation (*A*) to the mutual inductance. The result is a Wheatstone bridge of which the upper arms have vector

* A. Campbell, Proc. Phys. Soc., Vol. XX., p. 626 (1907). Also Phil. Mag., October, 1907.

† The effective capacity and resistance refer to the whole arm. Thus C includes the equivalent capacities of the leads and any residual inductance in S , while S includes the effective resistance of the condenser and leads as well as any added resistance.

impedances m , $\xi - m$, and the lower arms have vector impedances Y , Z . The vector equation of balance is therefore

$$mZ=Y(\xi-m) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

Inserting the values of m , ξ , Y , Z and separating real and imaginary parts,

$$M/C + \sigma S = (P - \sigma)R - \omega^2 l(L - M). \quad (38)$$

$$MS - \sigma / \omega^2 K = R(L - M) + Pl. \quad (39)$$

In these equations M, K, σ, P, L, S may vary with frequency. Also σ, l and the frequency variations may be taken to be so small that their squares or products may be ignored. Thus in

FIG. 7.—THE CAREY-FOSTER INDUCTANCE CAPACITY BRIDGE.

the small terms of (38) and (39) we may assume with sufficient accuracy that the relations

$$M/C=PR, S=R(L-M)/M \quad . \quad . \quad . \quad . \quad (40)$$

will hold.

In regard to equation (38) if we write $P=P_0+r$, where P_0 is the direct current resistance of P and $C_0=M/P_0R$ we obtain to the first order of small quantities

$$C=C_0\left\{1-\frac{r-L/M}{P_0}+\frac{\omega^2 l(L-M)}{P_0 R}\right\}, \quad (41)$$

which shows that if we calculate C_0 with the value of M proper to the frequency of measurement there is a further frequency correction due not only to residual inductance in R , but also due to the change of the effective resistance of P and to the change of impurity with frequency. The latter correction is less the greater the value of P_0 , so that different values of C_0 may be obtained if measurements are made with different

values of P_0 keeping P_0R constant. Even if the values of C_0 are consistent with various values of P_0 , this is no proof that r and σ are negligible, but merely that they are of the same sign and that r/L and σ/M are approximately equal.

Since r and σ normally vary as the square of the frequency it is probably preferable to write (41).

$$C = \frac{M_0}{P_0 R} (1 + \gamma \omega^2), \quad (42)$$

in which $\gamma \omega^2$ is the frequency correction peculiar to the particular arrangement of capacity inductance bridge used, and M_0 is the geometrical mutual inductance.

It is possible by means of Anderson's bridge to determine the true variation of the capacity of a condenser with frequency so that the value of γ may be determined.

Further, by taking observations with different values of P_0 keeping P_0R constant, the first correction term in (41) may be separated, and therefore since the residual inductance l of R may be measured we can determine by means of the capacity balance of the Carey-Foster bridge the true variation of M with frequency and the value of $r - \sigma L/M$.

Turning now to the second equation of balance (39) the resistance S includes the series resistance, (s) representing loss in the condenser, so that putting $S = S_0 + s$ equation (39) may be written

$$s - \sigma/\omega^2 MC = R \left(\frac{L}{M} - 1 \right) + \frac{Pl}{M} - S_0 \quad (43)$$

Now L/M may vary with the frequency, but otherwise the quantities on the right-hand side of (43) may be regarded as known.

$$\text{Writing } L/M = L_0/M_0 + \mu \omega^2 \quad (44)$$

$$\text{and} \quad s_0 = R(L_0/M_0 - 1) + \frac{Pl}{M} - S_0, \quad (45)$$

s_0 being capable of being precisely determined, we have

$$s - \sigma/\omega^2 MC - \mu \omega^2 R = s_0. \quad (46)$$

Thus the second equation in the Carey-Foster bridge balance merely gives a relation between three unknown quantities.

However, by taking observations with different values of R keeping PR constant, the value of μ may be determined.

If σ varies as the square of the frequency equation (46) shows that even if the correction $\mu\omega^2R$ is applied the apparent series resistance of a condenser as measured by the Carey Foster method is still in error by a constant amount.

As regards the above determination of the correction μ , a check determination may be made if the variation of the series resistance of a condenser with frequency can be determined independently. This may be done by a proper use of Anderson's bridge*. A comparison of the variations of s_0 in equation (46) with those of s then gives the value of $\mu\omega^2R$, the assumption being that σ/ω^2MC is constant, or that σ varies as the square of the frequency. As has already been pointed out, the four-point contact where the galvanometer and R are connected to the inductometer may cause trouble in regard to the impurity. If this causes a true resistance coupling it will introduce a term varying as $1/\omega^2$ in the apparent series resistance of the condenser, and is thus of importance at the lower frequencies.

23. *The Modified Campbell Frequency Bridge.*†

This bridge is shown in Fig. 8, the modification being the addition of an adjustable condenser C' across the open ends of the inductometer M .

If the mutual inductance and capacity C are both perfect the only condition of balance is

$$\omega^2MC=1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

If these two elements are not perfect a second balance is required. Equation (9) shows that intercapacity may be used to cause the impurity of a mutual inductance to assume a negative value. This suggests the introduction of the condenser C' in Fig. 8. The addition of C' will modify the original condition (47). The new conditions are found as follows:—Apply transformation (*A*) to the mutual inductance to convert it to a "star" combination without mutual inductance. Then apply transformation (*B*) to the mesh formed by C' and the two arms of the star which it connects. The result is that the generating and detecting circuits are connected only by a single arm and the conditions of balance follow from the

*The use of Anderson's Bridge for determining the true variation of the series resistance and capacity of a condenser will form the subject of a separate Paper.

† A. Campbell, Proc. Phys. Soc., Vol. XXI., p. 69 (1908).

condition that the vector impedance of this arm shall be zero. Thus after transformation (A) the circuits are as shown in Fig. 8a in which the vector impedances are

$$\begin{aligned} \alpha &= R_1 - \sigma + j\omega(L_1 - M), & \beta &= R_2 - \sigma + j\omega(L_2 - M), \\ \gamma &= -j/\omega C', & \delta &= \sigma + s + j(\omega M - 1/\omega C). \end{aligned}$$

Applying transformation (B) to the mesh α, β, γ the arm common to the generating and detecting circuits will then have vector impedance $\alpha\beta/(\alpha+\beta+\gamma) + \delta$, and the condition of balance is that

$$\alpha\beta/(\alpha+\beta+\gamma) + \delta = 0. \quad (48)$$

Now for small values of σ and s , δ will be small compared with α and β in the neighbourhood of balance. Hence γ

FIG. 8.—MODIFIED CAMPBELL FREQUENCY BRIDGE.

must be large, so that to the first order of small quantities (48) becomes

$$\alpha\beta/\gamma + \delta = 0. \quad (49)$$

Inserting the values of $\alpha, \beta, \gamma, \delta$ the two conditions are

$$\omega^2 MC = 1 - \omega^2 CC' \{R_1 R_2 - \omega^2 (L_1 - M)(L_2 - M)\} \quad (50)$$

$$\sigma + s = \omega^2 C' \{R_1 (L_2 - M) + R_2 (L_1 - M)\}. \quad (51)$$

σ being ignored where it is multiplied by the small quantity C' .

In regard to equation (50) it should be noted that if the relation

$$R_1 R_2 = \omega^2 (L_1 - M)(L_2 - M). \quad (52)$$

is satisfied then to the first order of small quantities, the original condition

$$\omega^2 MC = 1$$

is the condition for this balance. At the usual frequencies for which the bridge is suitable (1,000 cycles per second upwards) the relation (52) is practically L_1 or $L_2 = M$. When this condition is satisfied the balances do not react on each other if adjustments are made by C and C' . Otherwise the adjustments may be made independent by inserting a small adjustable resistance ($=S$, say) in series with the condenser C , and adjusting by C and S .

As regards equation (51) it is not necessary to know the separate values of R_1 , R_2 , &c., if the following procedure is adopted. Using the adjustments C and S repeat the balances with different values of C' . Then since $\sigma + s + S$ is proportional to C' , the latter when plotted against S will give a straight line cutting the negative side of the axis of S at the value $\sigma + s$. This method may be used if C' is so large that the approximation used in obtaining equations (50) and (51) is no longer valid. For then by expansion of $a\beta/(\alpha + \beta + \gamma)$ in powers of $1/\gamma$ and extracting the real part we obtain instead of (51) an expression of the form

$$\sigma + s + S = AC' + BC'^2 + \dots \quad (53)$$

By using a sufficient number of values of C' the quantities $\sigma + s$, A , $B \dots$ may be found. The same procedure holds in regard to (50), the equation being of the form

$$\omega^2 MC = 1 + A'C' + B'C'^2 + \dots \quad (54)$$

The modified frequency bridge will therefore yield the values of $\sigma + s$ and MC at a known frequency.

24. *Determination of Impurity of a Mutual Inductance and Series Resistance of a Condenser by two sets of Bridge Measurements.*

It has been shown in section 22 that the value of $s - \sigma / \omega^2 MC$ may be determined by means of the Carey-Foster bridge, and in section 23 that the value of $\sigma + s$ may be found by means of the modified Campbell frequency bridge if the frequency is such that $\omega^2 MC \doteq 1$. Hence the two sets of determinations yield $s - \sigma$ and $s + \sigma$ respectively at this frequency and enable

s and σ to be determined separately. An essential point in this and similar methods is that the common point of the mutual inductance shall be the same in both bridges, as a change in common point will alter the intercapacity relations and, therefore, alter σ .

25. *Measurement of Impurity and Effective Resistance by Combined Observations on the Heaviside Bridge and the Carey-Foster Bridge.*

If observations of the capacity balance are taken on the Carey-Foster bridge with different values of P , keeping PR constant, the results enable $(r - \sigma L/M)$ to be determined, where r is the added resistance in the arm P due to the alterations of the current. Again, from the Heaviside bridge, if r' is the added resistance of another inductive coil (the coil L of Fig. 6), the measurements give $r' - 2\sigma$. Hence, if two inductometers are used we can measure $r - \sigma L/M$, $r' - \sigma' L'/M'$, $r' - 2\sigma$, $r - 2\sigma'$, the accented letters referring to the second inductometer. These four relations will, therefore, give the required quantities r , r' , σ , σ' .

26. *Experimental Study of the Frequency Variations of a Mutual Inductometer.*

The inductometer studied was one of the Campbell type* having a range of 10 millihenries sub-divided by tappings in steps of 0.1 millihenry for coarse adjustment, while fine adjustments were secured by a moving coil having a range somewhat greater than 0.1 millihenry. The scale of the moving coil was capable of being read to 0.1 microhenry, so that an accuracy of setting of one part in 100,000 of the whole range could be secured.

The method of sub-division of the fixed coils is important. This is obtained by twisting 10 equal insulated strands together, connecting in series through tappings, winding as one coil (the primary coil) and adjusting the *whole* to have a mutual inductance of 10 millihenries with regard to the fixed secondary coil. By this method the sub-division into millihenries is found to be correct to a fraction of a microhenry.

This mode of sub-division, however, introduces large capacity effects in the coil because the first and last sections are very close together. When one section only is used, the presence of the remaining nine sections, almost perfectly

* A. Campbell, Proc. Phys. Soc., Vol. XXI., p. 69 (1908).

coupled, causes the system to behave as a 9 to 1 transformer with a capacity load, and thus introduces a large effective self capacity in the coil in use.

27.

The frequency variations to be determined are those of the mutual inductance and the self inductance of the secondary coil, the impurity, and the effective resistance of the secondary coil. These quantities are not usually required for the primary coil as this coil is in the source arm in alternating current bridges. The methods to be given are independent of any assumption in regard to the perfection of inductive coils and condensers. All that is assumed is that "non-inductive" resistances and the residual inductances of these resistances are invariable with frequency.

28. *Source Employed.*

The source employed was the simple valve circuit, shown in Fig. 9.

The filament current of the valve was about 4 amperes, and the filament voltage 12 volts, while the anode voltage was 400

FIG. 9.—SOURCE OF ALTERNATING CURRENT.

volts. The anode inductance was 20 millihenries, and the anode capacity was adjustable continuously to 5 mfd., so that frequency could be varied from 500 ~ per second upwards. The grid inductance was about 1 henry and was not usually tuned.

A number of experiments were made using the modified Campbell frequency bridge, partly to test the modification, but mainly to form some notion of the steadiness of the valve source and its reaction to various factors.

With a frequency bridge for which $M \doteq 10$ millihenries, $C = 0.5$ mfd., so that the frequency was about 1,600~ per second, it was found that by placing an air condenser (c) in parallel with C , and making the final adjustments by this condenser, the sensitivity with a 60 ohms Brown telephone was such as to enable a change of $1 \mu\mu F$ to be detected. This means that the arrangement is capable of holding a frequency constant to one part in a million if the other elements of the bridge can be kept constant to this accuracy.

The following observations illustrate a test of the drift in frequency which occurred during a three hours' continuous run of the source, the bridge being kept on continuously :—

Time. h. m.	Temperatures		M Setting $\mu H.$	Readings. $\mu\mu F.$					
	Source Table	Bridge Table							
	$^{\circ}C.$	$^{\circ}C.$							
5 15	18.8	20.5	...	...					Valve on Bridge on minute readings. $\frac{1}{2}$ minute readings
5 30	...	...	...	...					
5 43	...	...	10010.0	129	135	141	147	159	
5 52				165	170	178	185	190	
6 15	18.7	20.5	10012.0	132	134	136	138	139.5	
6 20				142	143.5	146	148	150	
6 47	18.8	20.5	10013.0	124	125	127	128	129	
6 52				129.5	131	131.5	134	134	
7 15	18.7	20.5	10013.0	166.5	167	167.5	167.5	168	
7 20				168.5	169	170	170.5	171	
7 45	18.65	20.5	10013.0	187.5	188	188	189	189.5	
7 50				189	189	189.5	190.5	190	
8 13	18.6	20.5	10013.0	197	197	196	196.5	196	
8 18				196.5	196.5	196.5	197	197	

From the groups of observations of c the values of the inductance M which would have secured a balance without change of c were deduced for the moments given in the following table, which also gives the drifts at these moments expressed in terms of M . Since for the frequency bridge $\omega^2 MC = 1$, the changes in adjustments of M with time are proportional to $1/\omega^2$.

Time. h. m.	Corrected M . $\mu H.$	Drift per min.
5 45	10011.42 ₁	0.071
6 15	13.32 ₀	0.040
6 45	14.20 ₀	0.022
7 15	14.66 ₂	0.010
7 45	14.87 ₉	0.004 ₃
8 15	14.96 ₄	0.000 ₃

The results are plotted in Fig 10, the tangential lines representing the observed drifts. The curve *B* shows the observed drift on a Carey-Foster bridge under similar conditions. The balance for this bridge is practically independent of frequency, so that this curve gives some indication as to what portion of the drift is contributed by the elements of the bridge.

The curve shows the extremely regular behaviour of a valve source when working under favourable conditions.

The whole change in $2\frac{1}{2}$ hours is only 2 parts in 10,000 when expressed in terms of frequency, the change in the last hour

FIG. 10.—DRIFT IN FREQUENCY OF A VALVE SOURCE AS INDICATED BY THE MODIFIED FREQUENCY BRIDGE.

being 15 in a million, and during the whole period there was no detectable departure from a regular drift. Other tests were made showing that the source was susceptible to various factors. A change of valve condenser after the circuit had been brought to a steady state immediately produced a further drift in frequency towards a new state of equilibrium (apart altogether from that due to any change in the value of the capacity). Some condensers produced a drift of opposite sign.

An increase in condenser current of 2 per cent. produced an

increase in frequency of two parts in 100,000, and a reduced filament current also increased the frequency by a few parts in 100,000, these changes taking place immediately. By running the valve on full filament current for some time it was found possible to bring about a steady state for a reduced filament current much earlier than is indicated by Fig. 10. The source frequency is also susceptible to variations in load on the bridge, the change of frequency being such as would be expected from the reduced effective inductance of the valve circuit when coupled to the bridge circuit. It is therefore necessary when bridge measurements are being taken which involve extreme steadiness of frequency and also considerable changes in the effective resistance of the bridge to have an auxiliary frequency bridge to hold the frequency constant. The modified frequency bridge has been found very useful for this purpose.

29. *Earth Capacities.*

It was found to be necessary to make corrections for earth capacities in certain of the bridge measurements. In bridges of the Wheatstone bridge type (the Heaviside bridge and the Carey-Foster bridge) the Wagner earthing device may be used,* but actually a method of correction was employed involving two measurements with different earth points. This method will form the subject of a separate Paper.

In the case of the frequency bridge, the bridge was connected to earth at the point common to the source, detector and condenser arm as the earth capacities (assumed to be condenser arms connecting the branch points to earth) will either have no effect or act as an added capacity to C (Fig. 8). As this bridge was simply used to measure *changes* in frequency or to determine quantities in which C was involved, only to the first order of small quantities, no appreciable error could arise from this cause.

30. *Frequency Coefficient of Mutual Inductance.*

For this purpose a Carey-Foster bridge method was employed, using a condenser of nominal value 0.5 mfd., whose variation with frequency had been determined by Anderson's bridge. The curve of variation for this condenser is shown in Fig. 11 in which the variations ΔC in micro-microfarads are plotted against frequency. Since the leads to the condenser and

* See Laws, "Electrical Measurements."

the resistance S are included in the condenser arm (Fig. 7) the effect of their residual inductances must be included in the value of C in equation (41). Let the residual inductance of the condenser arm be λ . Then if C in (14) is replaced by $C(1+\omega^2\lambda C)$ the equation becomes

$$C = \frac{M}{P_0 R} \left\{ 1 - \frac{r - \sigma L/M}{P_0} + \frac{\omega^2 l(L-M)}{P_0 R} - \omega^2 \lambda C \right\}. \quad (55)$$

Writing $M = M_0(1 + \gamma'\omega^2)$, (55) becomes

$$C = \frac{M_0}{P_0 R} (1 + \gamma\omega^2), \quad (56)$$

in which

$$\gamma\omega^2 = \gamma'\omega^2 - \frac{r - \sigma L/M}{P_0} + \frac{\omega^2 l(L-M)}{P_0 R} - \omega^2 \lambda C, \quad (57)$$

and is the frequency correction of the whole bridge outside the condenser.

γ may be determined by the variations in the M settings

FIG. 11.—CHANGE OF CAPACITY OF $\frac{1}{2} \mu F$ CONDENSER WITH FREQUENCY.

required to balance the known variations in C at different frequencies. Actually coarse adjustments only were made by the moving coil of M , the fine adjustments being made by

when the values of s_0 are determined with different values of R , keeping $P_0 R$ constant, the values of a and b being by (46)

$$a = s - \sigma / \omega^2 MC,$$

$$b = \mu\omega^2,$$

s being the true series resistance of the condenser C , σ the impurity of the mutual inductance M , and $\mu\omega^2$ the frequency correction of L/M . The values of a and b were determined using the condenser already employed in section 30 for three different frequencies. The results are given in the following table, the columns marked " s_0 calculated" being obtained from (58), using the best values of a and b for the observations.

TABLE.

Carey-Foster Bridge.

Variation of P and R .

$$M_0 \doteq 10 \text{ mh.}$$
$$L_0 = 26.110 \text{ mh.}$$
$$C = 0.5 \mu F$$
$$PR = 20,000.$$

f = frequency $\sim$ per second.

R ohms.	$f=1,000.$		$f=1,600.$		$f=2,000.$	
	s_0^o observed ohms.	s_0^o calculated.	s_0^o observed ohms.	s_0^o calculated.	s_0^o observed ohms.	s_0^o calculated.
50	0.098	0.098	0.096	0.096	0.128	0.130
100	0.128	0.127	0.167	0.165	0.143	0.241
200	0.185	0.185	0.302	0.303	0.464	0.462
$a=$	0.069		0.027		0.019	
$-b=$	0.00058		0.00138		0.00222	
$-\frac{b}{\omega^2}=$	1.42×10^{-11}		1.38×10^{-11}		1.40×10^{-11}	

Hence, as a mean value

$$\mu = -1.40 \times 10^{-11}.$$

A check on this determination may be obtained by combining the results of observations on the Carey-Foster bridge with observations on Anderson's bridge.*

For the Carey-Foster bridge we have equation (46), viz.,

$$s_0 = s - \sigma / \omega^2 M C - \mu \omega^2 R, \quad (46)$$

and for Anderson's bridge,* if s' is the series resistance used with the condenser,

$$s' = 4 - s,$$

* The use of Anderson's bridge for determining the true variation of the series resistance and capacity of a condenser will form the subject of a separate Paper.

where A is a quantity independent of frequency involving the remaining residuals in Anderson's bridge.

Hence

$$s_0 + s' = A - \frac{\sigma}{\omega^2 MC} - \mu \omega^2 R,$$

or, since σ is proportional to ω^2

$$s_0 + s' = A' - \mu \omega^2 R, \quad . \quad . \quad . \quad . \quad . \quad (59)$$

from which μ may be found by observations at different frequencies.

The results obtained by this method for three different condensers are given in the following table :—

TABLE.

Anderson's Bridge and Carey-Foster Bridge.

For the Carey-Foster Bridge—

$M = 10$ mh. $C = 0.5 \mu F$. $L_0 = 26.110$ mh. $P = 400$ ohms. $R = 50$ ohms.

$$s_0 = R \left(\frac{L_0}{M_0} - 1 \right) + \frac{Pl}{M} - S.$$

s' = series resistance in Anderson's Bridge.

Frequency per second.	—	Condenser I.	Condenser II.	Condenser III.
1,000	s_0	0.100	0.077	-0.016
	s'	0.625	0.650	0.715
	$s_0 + s'$ { observed	0.725	0.727	0.699
	{ calculated	0.726	0.726	0.700
1,600	s_0	0.096	0.072	+0.001
	s'	0.675	0.700	0.740
	$s_0 + s'$ { observed	0.771	0.772	0.741
	{ calculated	0.771	0.771	0.745
2,000	s_0	0.127	0.096	0.042
	s'	0.685	0.715	0.751
	$s_0 + s'$ { observed	0.812	0.811	0.793
	{ calculated	0.812	0.812	0.786

The calculated values of $s_0 + s'$ were obtained from formula (59), using as the values A' for the three condensers.

$$A' = 0.698 \quad 0.698 \quad 0.672,$$

and for μ the common value $-1.42_5 \times 10^{-11}$, which are the values agreeing best with the observations. This value of μ differs from the previous determination by 2 per cent.

As regards A' , the capacity of condenser III. was a few per

cent. higher than those of condensers I. and II., which differed only by 2 or 3 parts in 10,000. The differences in A' are sufficiently accounted for by the changes in the residuals in Anderson's bridge brought about by rebalancing for condenser III.

Determination of σ .

Using the known value of μ in equation (46) the values of $s - \sigma' \omega^2 MC$ at a frequency of 1,600 cycles per second for the three condensers are

$$+0.025, \quad +0.001, \quad -0.070.$$

Since $\omega^2 MC = \frac{1}{2}$ at this frequency, these are the values of $s - 2\sigma$.

Another relation between s and σ can be found by the modified frequency bridge (section 23). For this bridge $\omega^2 MC = 1$, and this condition may be secured by paralleling two of the condensers, say, I. and II.

By the usual laws of combination, and by (53) the modified frequency bridge measures

$$(s_1 + s_2)/4 + \sigma.$$

The value obtained with Condensers I. and II. was

$$(s_1 + s_2)/4 + \sigma = 0.096.$$

Now,

$$s_1 - 2\sigma = 0.025$$

$$s_2 - 2\sigma = 0.001,$$

so that, solving for s_1 , s_2 , σ , $s_1 = 0.115$, $s_2 = 0.091$, $\sigma = 0.045$ ohms. Using this value of σ for the Condenser III.,

$$s_3 = 0.020.$$

Since σ varies as the square of the frequency, the value of σ for the inductometer studied is 0.017₅ ohm, at a frequency of 1,000 cycles per second.

32. Summary of Inductometer Corrections.

From the results obtained in sections 30 and 31 the corrections for frequency for the inductometer studied are as follows:

At a frequency of 1,000 $\sim$ per second the mutual inductance

has a positive correction of 9.8 parts in 10,000,* the self-inductance of the secondary coil has a positive correction of 7.6 parts in 10,000, the impurity is 0.017₅ ohm, and the effective resistance of the secondary coil is greater than its direct current value by 0.042 ohm.

All the corrections vary as the square of the frequency.

These corrections only hold when the mutual inductance has a common point such that when the coils are in series, the mutual inductance opposes the self-inductances.

33. *Determination of Impurity by Measurements of Effective Resistance.*

If an inductive coil L is placed in series with a condenser C , non-inductive resistance and an alternating-current ammeter so as to form a simple circuit, the current induced in the circuit by an E.M.F. E , alternating with frequency $\omega/2\pi$, is $I = E / \{R^2 + (\omega L - 1/\omega C)^2\}^{\frac{1}{2}}$, so that if C is adjusted to resonance with L , the current is a maximum and has the value $I_{\max.} = E/R$.

By observing the maximum current with different values of the non-inductive resistance the resistance of the remainder of the circuit may be deduced. The method therefore gives the resistance of the coil and condenser in series. Assuming the condenser to have the same power-factor at all frequencies, then remembering that the value of the capacity is altered for different frequencies the series resistance of the condenser will increase as the frequency, while if the coil is wound with thin wire the resistance of the coil increases as the square of the frequency, so that the losses in the condenser will become less and less important as the frequency increases. Moreover, at the higher frequencies the resonating capacity may be made so small that an air condenser may be used and thus the condenser losses may be still further reduced.

In order to determine the impurity of a mutual inductance by this method, the two coils are connected at one point (B in Fig. 7A) and the effective resistances measured across the points AB , BC , CA . If at a particular frequency the values of the effective resistances are R_1 , R_2 , R_3 respectively, then since $R_3 = R_1 + R_2 - 2\sigma$ we have $\sigma = \frac{1}{2}(R_1 + R_2 - R_3)$.

The following values were obtained for a fixed mutual in-

* This result is in good agreement with an earlier determination on the same inductometer by Campbell. (Proc. Roy. Soc., Vol. LXXXVII, 1912, p. 406.)

ductance of 10 millihenries, wound on marble with No. 26 d.s.c. wire.

Primary coil $L=21.9$ millihenries. D.C.R.=12.54 ohms.

Secondary coil $L=21.5$ millihenries. D.C.R.=12.25 ohms.

Alternating Current Resistances.

Frequency. ~ per second.	Primary.	Secondary. ohms.	Series opposition.
17,300	36	34	59
25,000	62	59	99
33,300	105	98	154

The resistances are plotted against the square of the fre-

FIG. 12.—EFFECTIVE RESISTANCE OF COILS OF INDUCTOMETER.

quency in Fig. 12, and from the mean straight lines we find at $f=1,000$ ~ per second, the following corrections :—

Primary 0.081 ohm,
 Secondary 0.075 ohm,
 Series opposition 0.118 ohm,

from which the impurity is 0.020 ohm.

The same inductance was tested at a frequency of 1,600 ~ per second upon the modified frequency bridge, using a $1\mu F$ condenser of known power-factor. The deduced impurity at

a frequency of 1,000~ per second by this method was 0.0207 ohm.

34. *Errors in Power Factor Measurements due to Imperfections of Mutual Inductance.*

Equation (46) may be written

$$\omega s_0 C = \omega s C - \sigma / \omega M - \mu \omega^2 M / P$$

and since $\omega s C$ is the power factor of the condenser, $\omega s_0 C$ will be apparent power-factor of the condenser as deduced from Carey-Foster bridge observations upon the assumption of a perfect mutual inductance. It is seen that the apparent power-factor is subject to two errors, one due to impurity and proportional to the frequency, and the other due to the change of L/M with frequency, and proportional to the cube of the frequency. The latter error depends also on the resistances employed in the bridge. For the inductometer studied, the error due to impurity is $2.8/10^4$ at a frequency of 1,000, and tends to make the apparent power-factor low, while the error due to the change of L/M is $0.9/10^4$ (when $P=400$ ohms), tending to make the apparent power-factor too high. These values hold whatever capacity is being measured. Since the power-factor of a good condenser is of the order of a few parts in 10,000, it is clear that the Carey-Foster bridge measures the imperfections of its own mutual inductance, rather than the power-factor of the condenser at the higher telephonic frequencies. However, by a proper use of intercapacity the imperfections of the inductance may be compensated, the compensation holding over the whole range of frequencies for which the eddy current and capacity effects vary as the square of the frequency. A method of compensating a mutual inductance for change of mutual inductance with frequency and for impurity has already been given in section 19. This compensation is suitable for a mutual inductance used for the measurement of self-inductance upon a Heaviside bridge, but when using the inductance upon a Carey-Foster bridge for power-factor measurements, what is required is that the impurity shall be zero, and the variation of L/M shall be zero.

Let a condenser C' connect the open ends of the inductometer. Then by equations (8), (9), (11), the change in the frequency coefficient of L_2/M is $\omega^2 C' (L_2 - M)(L_1 L_2 - M)^2 / M L_2$ and the change in σ is

$$-\omega^2 C' \{R_2(L_1 - M) + R_1(L_2 - M)\}.$$

For the inductometer studied the values of L_1 , L_2 , R_1 , R_2 , M were

$$L_1 \doteq 7 \text{ millihenries, } L_2 \doteq 26 \text{ millihenries, } M = 10 \text{ millihenries,} \\ R_1 \doteq 10 \text{ ohms, } R_2 \doteq 18 \text{ ohms.}$$

Using these values we find that if $C' = 1,100 \mu\mu F$ the change of L_2/M due to C' is $+2.2/10^4$ at a frequency of 1,000 $\sim$ per second, and this will just balance the existing variation of $-2.2/10^4$. Further with this value of C' a negative impurity of amount 0.0175 ohm will be introduced if R_1 is made equal to 21.5 ohms, so that by adding 11.5 ohms to the primary coil and using an added intercapacity of 1,100 $\mu\mu F$ across the whole combination the corrections for this inductometer will be automatically compensated.

The experimental work was carried out at the National Physical Laboratory. I wish to thank a number of my colleagues, particularly Mr. D. W. Dye, for valuable suggestions during the course of the experimental investigation.

DISCUSSION.

Dr. A. RUSSELL said he presumed the Paper was on an experimental basis, as the author had mixed theory and experimental approximation throughout, *e.g.*, in treating the variation of mutual and self-inductance due to what he terms the mutual capacity and self-capacity of the two coils. He thought G. A. Campbell's method was probably not good for high frequencies. The author neglected skin effects. He supposed he was familiar with Curtis's Paper on skin effects in parallel wires in which formulæ are given for the eddy-current losses which are fully verified by experiment. Did these results agree with Mr. Butterworth's? He fully agreed with the author's remarks on the Carey Foster Bridge. In the equation for $\omega^2 C$ the equating of the last factor to zero is the condition that the phase difference between the two currents is a maximum.

Dr. D. OWEN said the Paper should prove of great value in inductance measurements of high precision. The first impression on reading it might perhaps be that alternate-current methods introduced complications: but this was really a measure of the great accuracy which these methods of measurement afforded. The Paper showed that many conditions have to be borne in mind in obtaining definite results; one would like to know whether the effects of capacities to earth had also been kept in view.

Mr. ALBERT CAMPBELL (communicated remarks): Mr. Butterworth's elaborate and thorough research will be of great value to all who are interested in precision measurements in which inductometers are used. The effects which he has investigated in general only give trouble in the rather extreme cases, *e.g.*, at the higher audio-frequencies, or when extremely minute power losses are being measured (in condensers of low power factor). In such cases it is probably better to use a more elaborate form of inductometer, wound with multiple pancake coils or otherwise so as to minimise the capacitance effects. Such construction would, of course, increase the volume of the instrument and add to its cost. The inductometer which Mr. Butterworth has studied is of early type, in which the higher subdivisions are all obtained by stranding the wires. In the more modern inductometers of this kind the higher coils are not stranded together; each subdivision consists of an individual coil separately adjusted. Stranding is

still employed for the lower sections where the capacitance effects are of much less relative importance. The methods described by the author for applying automatic corrections will be valuable in practice and the whole Paper contains much that may ultimately be of importance to the radio engineer.

Mr. BUTTERWORTH, in reply to Dr. Russell, said he was unaware of the criticism of G. A. Campbell's method which he had mentioned, but it was all right if the frequency was under 2,000 or so. He had seen Curtis's Paper. As far as skin effects are concerned it should be taken in conjunction with a Paper by himself (at present in the course of publication), in which a much shorter formula was obtained.

In reply to Dr. Owen the question of earth capacities is dealt with in a Paper originally included as an appendix to the present one, but to be published later as a separate Paper.