The Vibrations of a Film in reference to the Phoneidoscope

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solution, or the thickness of the plate of glass inserted in the cell.

This arrangement, as far as it has been tested, gives as good results as the previous one, while it is much more easily constructed and adjusted.

VI. *The Vibrations of a Film in reference to the Phoneidoscope.*

By WALTER BAILY, M.A.*

[Plate I.]

THE object of this paper is to consider the superposition of several systems of waves in a plane film, when the vibrations are perpendicular to the film, and the wave-lengths of all the systems are the same, with the view of discovering (1) what combinations of systems give simple results in an infinite film, (2) which of these combinations can exist in a finite film, and (3) which of the latter can account for appearances presented in the phoneidoscope.

Let us examine the case of a plane uniform film of infinite extent traversed by three systems of waves with straight fronts, the vibrations being perpendicular to the film, and the wave-length of each system being the same.

Draw (fig. 1) Ac , Bc , Cc meeting in c ; and let $\angle BcC = 2\alpha$; $\angle CcA = 2\beta$; $\angle AcB = 2\gamma$. Take $cB = cC = \lambda$. Draw $QcR \perp Ac$; $RBP \perp Bc$; $PCQ \perp Cc$. Join Pc , and draw Qb and Rb , bisecting PQR and QRP . Draw $ca \parallel Qb$, $cm \perp Rb$, $an \perp Rc$. It may be easily shown that $\angle cab = \alpha$, $\angle abc = \beta$, $\angle bca = \gamma$;

$$cm = \frac{\lambda}{2 \sin \gamma}, \quad an = \frac{\lambda \cos \beta}{2 \sin \gamma \sin \alpha}, \quad ca = \frac{\lambda}{2 \sin \gamma \sin \alpha}.$$

Let λ be the wave-length; Ac , Bc , Cc the directions of the three systems of waves; h , k , l their amplitudes; and d , e , f their phases at some point. The values of $(e-f)$, $(f-d)$, $(d-e)$ are independent of time, and depend only on the position of the point chosen. QR , RP , PQ are wave-fronts; for they are $\perp$ to the directions of the waves.

Consider the line Pc . Every point on this line is equidis-

* Read June 12.

tant from the wave-fronts RP and PQ , so that $(e-f)$ is constant along Pc . Similarly $(d-e)$ is constant along ab , and $(f-d)$ is constant along Rb ; and as we may shift the point c , it follows that along any lines $\parallel Pb$, Qb , and Rb we have $(e-f)$, $(d-e)$, and $(f-d)$ constant respectively. Hence $(f-d)$ is constant along ca .

The value of e at c differs by 2π from its value along PQ , since $Bc = \lambda$. Hence the value of $(d-e)$ at c differs by 2π from its value along ab . Similarly the values of $(f-d)$ at b , and of $(e-f)$ at a differ by 2π from the corresponding values along ca and bc .

Now the wave-fronts Rc , cB , BR will reach a simultaneously after crossing distances $= an$. Hence the difference of phase of the whole vibration at c and that at a

$$= \frac{an}{\lambda} 2\pi = \frac{\pi \cos \beta}{\sin \gamma \sin \alpha}.$$

Draw (fig. 2) a series of straight lines $\parallel ab$ at distances $\frac{\lambda}{2 \sin \gamma}$ apart. Along each of these lines $(d-e)$ is constant; and its value changes by 2π in passing from any line to the next. Draw another series $\parallel bc$ at distances $\frac{\lambda}{2 \sin \alpha}$ apart. These lines will have similar properties with respect to $(e-f)$. Straight lines drawn through the intersections of these two series will form a series $\parallel ca$, at distances $\frac{\lambda}{2 \sin \beta}$ apart, and having similar properties with respect to $(f-d)$. At all the intersections $(d-e)$, $(e-f)$, $(f-d)$ have the same values respectively; and therefore at all the intersections the amplitude of the vibration of the film is the same.

We can find a line such that $(d-e)$ is a multiple of 2π , and another line such that $(e-f)$ is a multiple of 2π . At their intersection $(f-d)$ will be also a multiple of 2π . Such a point is a ventral segment. Suppose (fig. 2) the lines to be moved until one of the intersections lies on a ventral segment; then all the intersections are ventral segments.

Now in a Δ with ventral segments at its $\perp\perp$, let p, q, r be the distances of a point from the sides of the Δ . Let p', q', r' be the differences between the values of $(e-f)$, $(f-d)$, $(d-e)$

at the point and on the respective sides. Then

$$p' : 2\pi = p : \frac{\lambda}{2 \sin \alpha};$$

hence

$$p = \frac{p'\lambda}{4\pi \sin \alpha}, \quad q = \frac{q'\lambda}{4\pi \sin \beta}, \quad r = \frac{r'\lambda}{4\pi \sin \gamma}.$$

The displacement of the film at the point at any time may be expressed as

$$h \cos (d + \tau) + k \cos (e + \tau) + l \cos (f + \tau),$$

where τ is proportional to the time after the given moment. The maximum displacement will be

$$\sqrt{\{h^2 + k^2 + l^2 + 2kl \cos p' + 2lh \cos q' + 2hk \cos r'\}};$$

that is,

$$\sqrt{\left\{4s^2 - 4kl \sin^2 \frac{p'}{2} - 4lh \sin^2 \frac{q'}{2} - 4hk \sin^2 \frac{r'}{2}\right\}},$$

where

$$2s = h + k + l.$$

This expression vanishes when

$$\sin^2 \frac{p'}{2} = \frac{s(s-l)}{kl}, \quad \sin^2 \frac{q'}{2} = \frac{s(s-k)}{lh}, \quad \sin^2 \frac{r'}{2} = \frac{s(s-l)}{hk}.$$

These values show that there is a node in each triangle the position of which depends on the relative amplitudes of the waves. If one of the amplitudes (say l) is very nearly equal to the sum of the other two, $(s-l)$ is very small, and therefore r' is very small, and consequently r is very small; the result is that the nodes lie in pairs very near together. When one amplitude equals the sum of the other two, the pairs of nodes coalesce; and when one amplitude is greater than the sum of the other two, there can be no node.

In the particular case in which the directions of the waves are inclined at angles of 120° to one another, and the amplitudes are equal, the triangles in fig. 2 become equilateral, with their sides $= \frac{2\lambda}{3}$, and the nodes are equidistant from the ventral segments. Also the difference of phase between successive angles of the triangles is 120° . In fig. 3 let the dots represent the nodes and the numbers the ventral segments.

Then all the *ones* move together, and so do all the *twos*, and also all the *threes*, the difference of phase between the different sets being 120° .

This last case may be investigated algebraically. Let h = the amplitude of each wave, v the wave-velocity, t the time, x, y and r, θ the coordinates of a point in the film, and z the displacement at that point, at the time t . Take a ventral segment as origin, and the direction of one of the waves as axis of x . Then

$$\begin{aligned} h^{-1}z &= \cos [2\pi\lambda^{-1}\{vt-r\cos\theta\}] \\ &+ \cos [2\pi\lambda^{-1}\{vt-r\cos(\theta-120^\circ)\}] \\ &+ \cos [2\pi\lambda^{-1}\{vt-r\cos(\theta+120^\circ)\}] \\ &= \cos \{2\pi\lambda^{-1}(vt-x)\} \\ &+ 2 \cos \left\{ 2\pi\lambda^{-1} \left(vt + \frac{x}{2} \right) \right\} \cos (\pi\lambda^{-1}y\sqrt{3}). \end{aligned}$$

When z is a maximum we must have

$$\cos \{2\pi\lambda^{-1}(vt-x)\} = 1;$$

and either

$$\cos \left\{ 2\pi\lambda^{-1} \left(vt + \frac{x}{2} \right) \right\} = 1, \text{ and } \cos (\pi\lambda^{-1}\sqrt{3}) = 1,$$

or

$$\cos \left\{ 2\pi\lambda^{-1} \left(vt + \frac{x}{2} \right) \right\} = -1, \text{ and } \cos (\pi\lambda^{-1}\sqrt{3}) = -1.$$

These conditions are satisfied by the sets of values given in the following table; l, m, n being any integers:—

$\lambda^{-1}vt$	$l - \frac{1}{3}$	$l - \frac{1}{3}$	l	l	$l + \frac{1}{3}$	$l + \frac{1}{3}$
$\lambda^{-1}x$	$2m + \frac{2}{3}$	$2m - \frac{1}{3}$	$2m$	$2m + 1$	$2m - \frac{2}{3}$	$2m + \frac{1}{3}$
$\lambda^{-1}y\sqrt{3}$	$2n$	$-2n$	$2n$	$-2n$	$2n$	$-2n$

Let Z be the amplitude of the vibration at any point, then Z is the maximum value of z with respect to t . Obtaining this we get

$$h^{-2}Z^2 = \sin^2 (\pi\lambda^{-1}3x) + \{ \cos (\pi\lambda^{-1}3x) + 2 \cos (\pi\lambda^{-1}y\sqrt{3}) \}^2.$$

At the nodes $Z=0$, and therefore

$$\sin (\pi\lambda^{-1}3x) = 0,$$

and

$$\cos (\pi\lambda^{-1}3x) + 2 \cos (\pi\lambda^{-1}y\sqrt{3}) = 0.$$

These conditions are satisfied when

$$\lambda^{-1}3x = 2m \quad \text{and} \quad \lambda^{-1}y\sqrt{3} = 2n \pm \frac{1}{3},$$

or

$$\lambda^{-1}3x = 2m + 1 \quad \text{and} \quad \lambda^{-1}y\sqrt{3} = 2n \pm \frac{2}{3}.$$

Putting $y=0$ in the above equations, we get

$$h^{-1}z = \cos \{2\pi\lambda^{-1}(vt-x)\} + 2 \cos \left\{ 2\pi\lambda^{-1} \left(vt + \frac{x}{2} \right) \right\},$$

$$h^{-2}Z^2 = 5 + 4 \cos (\pi\lambda^{-1}3x).$$

The former of these equations gives a section of the film at time t , through a ventral segment in the direction of any one of the waves; and the latter gives a similar section of the surface which encloses the space within which this film vibrates. By putting $x=0$, we get

$$h^{-1}z = \cos (2\pi\lambda^{-1}vt) \{1 + 2 \cos (\pi\lambda^{-1}y\sqrt{3})\},$$

$$h^{-2}Z^2 = \{\cos (\pi\lambda^{-1}3x) + 2\}^2;$$

and these equations give similar sections to the former ones, but in directions parallel to the fronts of the waves.

The next case we will examine is that of six waves of the same amplitude meeting each other in pairs, the directions of the pairs being inclined to one another at angles of 120° , with the condition that at some one point all the vibrations shall be in the same phase. These may be divided into two sets of three waves each; and the position of the ventral segments of the first set may be represented as in fig. 2. The position of the ventral segments of the second set may be represented by a similar figure, except that we should have to put 2 instead of 3, and 3 instead of 2, in numbering the ventral segments. In superposing the one figure on the other, we must make a ventral segment of the one figure coincide with a ventral segment of the same numeral of the other. Let a *one* of each figure coincide, then all the *ones* will coincide, and will indicate the points of maximum vibration of the film. On the other numerals the film will not have its maximum vibration, as one set of vibrations will partly destroy the other.

We can get a simple algebraic expression for the form of the film.

Divide the waves into two sets as before, and let z_1 be the displacement due to one set, z_2 that due to the other. Then

we have

$$h^{-1}z_1 = \cos \{2\pi\lambda^{-1}(vt-x)\} \\ + 2 \cos \left\{ 2\pi\lambda^{-1} \left(vt + \frac{x}{2} \right) \right\} \cos (\pi\lambda^{-1}y\sqrt{3}).$$

By changing the sign of x and y we turn the whole figure through 180° , and so reverse the motions of the waves; hence we get

$$h^{-1}z_2 = \cos \{2\pi\lambda^{-1}(vt+x)\} \\ + 2 \cos \left\{ 2\pi\lambda^{-1} \left(vt - \frac{x}{2} \right) \right\} \cos (\pi\lambda^{-1}y\sqrt{3});$$

$$\therefore h^{-1}z = h^{-1}z_1 + h^{-1}z_2 \\ = \cos (2\pi\lambda^{-1}vt) \{ 2 \cos (2\pi\lambda^{-1}x) \\ + 4 \cos (\pi\lambda^{-1}x) \cos (\pi\lambda^{-1}y\sqrt{3}) \},$$

$$h^{-1}Z = 2 \cos (2\pi\lambda^{-1}x) + 4 \cos (\pi\lambda^{-1}x) \cos (\pi\lambda^{-1}y\sqrt{3}).$$

The results of this equation are represented in fig. 4. The large dots represent the points at which Z is at its maximum, viz. $6h$. They occur when

$$-\cos \pi\lambda^{-1}x = \cos \pi\lambda^{-1}y = \pm 1;$$

that is, when

$$x = 2m\lambda, \quad y\sqrt{3} = 2n\lambda,$$

and when

$$x = (2m+1)\lambda, \quad y\sqrt{3} = (2n+1)\lambda.$$

The small dots represent points at which $Z = -3h$. Putting this value for Z , the equation becomes

$$0 = \sin^2 (\pi\lambda^{-1}y\sqrt{3}) + \{ \cos (\pi\lambda^{-1}y\sqrt{3}) + 2 \cos (\pi\lambda^{-1}x) \}^2.$$

This is satisfied when

$$2 \cos (\pi\lambda^{-1}x) = -\cos (\pi\lambda^{-1}y\sqrt{3}) = \pm 1;$$

that is, when

$$y\sqrt{3} = 2n\lambda, \quad x = (2m \pm \frac{2}{3})\lambda,$$

and when

$$y\sqrt{3} = (2n+1)\lambda, \quad x = (2m \pm \frac{1}{3})\lambda.$$

The dotted lines give the locus of points at which $Z = -2h$. Putting this value into the equation, we get

$$0 = \cos (\pi\lambda^{-1}x) \{ \cos (\pi\lambda^{-1}x) + \cos (\pi\lambda^{-1}y\sqrt{3}) \}.$$

This equation is satisfied when

$$x = (m + \frac{1}{2})\lambda,$$

and when

$$x \pm y\sqrt{3} = (2n+1)\lambda;$$

so that the locus consists of three sets of parallel straight lines.

The nodal lines are obtained by putting $Z=0$. The equation to them is

$$0 = \cos(2\pi\lambda^{-1}x) + 2\cos(\pi\lambda^{-1}x)\cos(\pi\lambda^{-1}y\sqrt{3}).$$

It is obvious from the loci already obtained, that these lines must be closed curves surrounding the points for which $Z=6h$; and that they must approximate to an hexagonal form, the greatest radii being towards the corners, and the least perpendicular to the sides of the hexagons formed by the locus of $Z=-2h$.

Putting $y=0$, we have

$$x = \frac{\lambda}{\pi} \cdot \cos^{-1} \cdot \frac{\sqrt{3}-1}{2} = .381 \cdot \lambda.$$

Putting $x=0$, we have

$$y = \frac{2\lambda}{3\sqrt{3}} = .385 \cdot \lambda.$$

These are the values of the greatest and least radii; and therefore the nodal lines are very nearly circles, with radii $=.383 \cdot \lambda$, and centres at the points for which $Z=6h$. The nodal lines are represented by the circles in fig. 4.

We will next consider the case of four waves meeting two and two, the angle between these directions being 2α , the amplitude of all the waves being the same, with the condition that at one point all the waves shall be in the same phase. We have

$$\begin{aligned} h^{-1}z &= \cos[2\pi\lambda^{-1}\{vt-r\cos(\theta-\alpha)\}] \\ &\quad + \cos[2\pi\lambda^{-1}\{vt+r\cos(\theta-\alpha)\}] \\ &\quad + \cos[2\pi\lambda^{-1}\{vt-r\cos(\theta+\alpha)\}] \\ &\quad + \cos[2\pi\lambda^{-1}\{vt+r\cos(\theta+\alpha)\}] \\ &= 4\cos(2\pi\lambda^{-1}vt)\cos(2\pi\lambda^{-1}x\cos\alpha)\cos(2\pi\lambda^{-1}y\sin\alpha). \end{aligned}$$

The ventral segments occur when the quantities have the values given in the following table:—

$\lambda^{-1}vt$	l	l	$l + \frac{1}{2}$	$l + \frac{1}{2}$
$\lambda^{-1}x\cos\alpha$	m	$m + \frac{1}{2}$	$m + \frac{1}{2}$	m
$\lambda^{-1}y\sin\alpha$	n	$n + \frac{1}{2}$	n	$n + \frac{1}{2}$

The ventral segments are divided into two sets: all one set

vibrate together ; and all the other set vibrate in the opposite direction.

The nodal lines are two sets of straight lines, whose equations are

$$4x \cos \alpha = (2m + 1)\lambda$$

and

$$4y \sin \alpha = (2n + 1)\lambda.$$

Hence the nodal lines divide the film into oblongs whose length and breadth are

$$\frac{\lambda}{2 \cos \alpha} \quad \text{and} \quad \frac{\lambda}{2 \sin \alpha} ;$$

and a ventral segment lies in the centre of each oblong. When the directions of the waves are $\perp$ each other, $\alpha = 45^\circ$, and the oblongs become squares. This form is shown in fig. 5.

When two waves of equal amplitude meet at an angle $= 2\alpha$, the equation is

$$\begin{aligned} h^{-1}z &= \cos [2\pi\lambda^{-1} \{vt - r \cos (\theta - \alpha)\}] \\ &\quad + \cos [2\pi\lambda^{-1} \{vt - r \cos (\theta + \alpha)\}] \\ &= 2 \cos \{2\pi\lambda^{-1}(vt - x \cos \alpha)\} \cdot \cos (2\pi\lambda^{-1}y \sin \alpha). \end{aligned}$$

The nodal lines are the straight lines

$$4y \sin \alpha = (2n + 1)\lambda.$$

When $\alpha = 90^\circ$, we get two waves meeting each other in the same direction. The equation becomes

$$h^{-1}z = 2 \cos (2\pi\lambda^{-1}vt) \cos (2\pi\lambda^{-1}y).$$

Hitherto we have dealt only with infinite films ; we will now consider what forms of vibration can be maintained in films of limited size, the waves undergoing reflection from the boundaries of the film.

When the film is an equilateral triangle, suppose a set of waves to be started having their fronts $\perp$ to one of the sides of the triangle. These waves will be reflected so as to have their fronts $\perp$ to another side, and again reflected so as to have their fronts $\perp$ to the remaining side, and by another reflection they will assume their first direction. If the wavelength is such that the time a wave takes to return to the same position is an integral number of wave-periods, we shall have the case of three sets of equal waves meeting each other at

angles of 120° . Fig. 8 shows the form of vibration when the wave-length is $\frac{1}{3}$ of the side of the triangle (no allowance being made for the change of phase at the reflections). The thin lines show the position of the maximum displacement due to each wave at one of the instants at which these lines all pass through certain points, and the numerals 1, 1 show the ventral segments which are then at their maximum displacement, the ventral segments 2, 2 and 3, 3 come to their maximum at different times, as already explained.

Again, suppose two sets of waves to be started in opposite directions, each set having the front $\perp$ to a side of the triangle, and the phases of both sets being the same along a perpendicular from an angle on the opposite side; we shall with a suitable wave-length have six sets of waves meeting each other in pairs, the directions of the pairs making with one another angles of 120° , and all the phases being the same at certain points.

Again, suppose waves to start simultaneously from each of the sides of the equilateral triangle; the waves will be reflected so as to produce other three sets of waves also with their fronts parallel to the sides, by having their direction of motion reversed. Here we again get the six sets of waves above considered, but in a different position. See fig. 6, in which this form of vibration is shown, without allowing for any change of phase at the reflections. The continuous lines show the coincidence of the maximum displacement in one direction due to the waves meeting; and the dotted lines show the same coincidence in the opposite direction. At the black spots we get the coincidence of all the maximum displacements in the same direction; so that these spots show the ventral segments. The wave-length is equal to the distance between the ventral segments $\times \frac{\sqrt{3}}{2}$.

With the number of ventral segments shown in the figure the wave-length equals $\frac{1}{3}$ of the height of the triangle when no allowance is made for change of phase at the reflections. The form of vibration of the film is shown in fig. 4, which has been already explained.

We may obtain a similar set of waves in a rhombus having one of its angles $= 60^\circ$, if we suppose sets of waves to start

simultaneously from the four sides, and in each direction from the shorter diagonal.

If in such a rhombus a set of waves starts from the longer diagonal, we get the three sets; and if two sets of waves in opposite directions start from the longer diagonal, we get the six sets.

With a right-angled isosceles triangle we may start a set of waves from the hypotenuse, and so get two opposite sets of waves $\parallel$, and two opposite sets $\perp$ to the hypotenuse; and we may get four similar sets of waves in another position by starting waves simultaneously from the two sides of the triangle.

With a square we can get four sets of waves meeting two and two, the directions being $\perp$ to each other, by starting waves simultaneously from all the sides. In fig. 7 the continuous lines show the coincidence of the maximum displacement in one direction of two waves, and the dotted lines show a similar coincidence in the other direction. The black spots show the ventral segments which move together, and the small circles those which move in the opposite directions. The wave-length = the shortest distance between the ventral segments $\times \sqrt{2}$; and with the number of ventral segments shown in the figure, the wave-length, not allowing for change of phase at the reflections, is $\frac{1}{3}$ of the side of the square.

A similar arrangement of waves in another position may be obtained from a square by starting two sets of waves in opposite directions from one of the diagonals.

To obtain four sets of waves meeting each other two and two, the angles between their directions being 2α , take a rectangle having its diagonals inclined at an angle 2α , and start two sets of waves from one of the diagonals; these will by reflection give two sets of waves with fronts parallel to the other diagonal.

With any rectangle two sets of waves meeting each other can be obtained by starting a set of waves from one side of the rectangle.

The case of two sets of waves meeting each other not in the same direction is impossible in a limited film; and I have not been able to discover any form of film which could maintain three sets of waves not making equal angles with one another,

or six sets meeting in pairs whose directions do not make equal angles with one another.

In the phoneidoscope we have a soap-film thrown into a state of vibration by a musical note. The effect is to send the matter of the film towards the ventral segments, and to make them the thickest part of the film. The consequence is that the colours of thin plates are seen less at the ventral segments than at other parts of the film; and we can recognize the ventral segments in this manner. This effect on the film may be illustrated by M. Decharmé's experiments on Chladni's plates*, in which he shows that if a thin layer of water instead of sand be spread over the plate, the water covers the ventral segments and the nodes are left bare.

The two following experiments may, I think, be explained by what has been said.

(1) A square film, $\frac{1}{2}$ of an inch in side, was thrown into vibration by a note having 92 vibrations in a second. The position of the ventral segments was that shown in figs. 5 and 7. Hence the vibration of the film was the result of four sets of waves starting simultaneously from the four sides of the film; and the wave-length was approximately $\frac{1}{3}$ of the side; and the wave-velocity was approximately $\frac{1}{3} \times \frac{1}{2} \times 92$ inches per second—that is, about 28 inches per second.

(2) An equilateral triangle, one inch in height, was thrown into vibration by a note having 152 vibrations in a second. The position of the ventral segments was that shown in figs. 4 and 6. Hence the vibration of the film may be the result of three sets of waves starting simultaneously from the three sides of the Δ , and giving by reflection three other sets moving in the opposite directions. And the wave-length would then be approximately $\frac{1}{3}$ of the height; and the wave-velocity was approximately $\frac{1}{3} \times 1 \times 152$ inches per second, or about $30\frac{1}{2}$ inches per second. Or the figure may be the result of *one* set of waves starting perpendicular to one of the sides of the triangle (see fig. 8). In this case the wave-length would be $\frac{3}{10}$ of the side of the triangle; and the wave-velocity would be $\frac{3}{10} \times \frac{2}{\sqrt{3}} \times 152$ inches per second, or about $52\frac{1}{2}$ inches per second. As this wave-velocity differs very much from the

* *Ann. de Chim. et de Phys.* sér. 5. vol. xvi. pp. 338-376.

wave-velocity derived from the experiment with the square film, we must reject this latter explanation.

The two experiments may be made to give the same wave-velocity by supposing a change of phase equal to half a period to take place at each reflection. In the first experiment the side of the square has to be taken equal to $\frac{1}{2}$ wave-lengths, and in the second the height of the triangle as equal to $\frac{1}{2}$ wave-lengths. The two expressions for the wave-velocity become $\frac{2}{3} \times \frac{1}{2} \times 92$, and $\frac{2}{3} \times 1 \times 152$, both of which expressions are equal to $33\frac{1}{3}$. Hence we are perhaps justified in inferring that the edges are stationary, and that the wave-velocity in the soap-film is nearly 34 inches in a second.

VII. *On a Means to determine the Pressure at the Surface of the Sun and Stars, and some Spectroscopic Remarks.* By EILHARD WIEDEMANN*.

IN a former paper I tried to show that we can calculate, at least approximately, the time elapsing between two encounters of the molecules of a gas by measuring the greatest difference in the length of path at which interference of two rays of light coming from the same source of light is possible. I may be allowed here to quote a passage out of that paper†:—

“Two rays of light can only interfere if they emanate from the same source of light, and if there is no sudden change of phase in the source of light during the time elapsing between the two instants at which the first and second ray leave the source. That difference in phase at which interference is still possible is therefore a measure of the time during which no sudden change in phase has taken place at the source of light. A luminous body sends out light coming from a great number of atoms or molecules. Each molecule will only vibrate regularly as long as it does not come within the sphere of action of another molecule; that is, it will only vibrate regularly in the time elapsing between two encounters. For the different molecules, the time elapsing between two encounters may, according to the molecular theory of gases, be either zero or

* Read June 12th.

† Wied. Ann. v. p. 503 (1878); Phil. Mag. [5] vii. pp. 79, 80.