PROCEEDINGS
OF THE
LONDON MATHEMATICAL SOCIETY.

VOL. V.

TENTH SESSION, 1873-4.

November 13th, 1873.

ANNUAL GENERAL MEETING, held at 22, Albemarle Street.

Prof. CAYLEY, F.R.S., in the Chair.

The Reports of the Treasurer and Secretaries were read and adopted. From the Report of the Secretaries, it appeared that the number of Members was the same (113) as at the last General Meeting, held November 14th, 1872.

The Obituary of the Society, during the Session, comprised the names of Mr. Archibald Smith, F.R.S., Mr. Jacob Waley, M.A., and Mr. G. O'Hanlon.

The communications made to the Society during the past Session had been as follows:

"Remarks on some Recent Generalisations of Algebra:" valedictory Presidential Address by Dr. W. Spottiswoode, F.R.S.

"On a Theorem relating to Polyhedra analogous to Mr. Cotterill's Theorem on Plane Polygons:" by Prof. W. K. Clifford.

"Investigation of the Disturbance produced by a Spherical Obstacle on the Waves of Sound:" by the Hon. J. W. Strutt.

"Sur l'Intégration des Fonctions Circulaires:" by M. Hermite.

"On the Mechanical Description of a Cubic Curve:" by Prof. Cayley.

"On the Mechanical Description of certain Quartic Curves by a Modified Oval Chuck:" by the same.

"On Geodesic Lines, especially those of a Hyperboloid:" by the same.

- "On a Deduction from Von Staudt's Property of Bernoulli's Numbers:" by Mr. J. W. L. Glaisher.
- "Geometry on an Ellipsoid:" by Prof. Clifford.
- "On Parallel Surfaces:" by Mr. S. Roberts.
- "On the Greatest Common Divisors of the Minor Determinants of a Rectangular Matrix of which the Constituents are Integral Numbers:" by Prof. H. J. S. Smith.
- "On an Arithmetical Demonstration of a Theorem in the Integral Calculus:" by the same.
- "Summation of certain Series:" by Prof. Wolstenholme.
- "On the Higher Singularities of Plane Curves;" and "On Systems of Linear Congruences:" by Prof. H. J. S. Smith.
- "On an Extension of the term 'Area' to any Closed Circuit in Space:" by Mr. R. B. Hayward.
- "On the Evaluation of a Class of Definite Integrals involving Circular Functions in the Numerator and Powers of the Variable only in the Denominator:" by Mr. J. W. L. Glaisher.
- "Note on Normals and the Surface of Centres of an Algebraical Surface:" by Mr. S. Roberts.
- "On the Calculation of the Value of the Theoretical Unit-Angle to a Great Number of Decimal Places:" by Mr. J. W. L. Glaisher.
- "On Systems of Porismatic Equations, Algebraical and Trigonometrical:" by Prof. Wolstenholme.
- "Note on Epicycloids and Hypocycloids:" by the same.
- "Locus of Point of Concourse of Perpendicular Tangents to a Cardioid:" by the same.
- "Elliptic Motion under Acceleration Constant in Direction:" by the same.
- "Locus of Point of Concourse of Tangents to an Epicycloid, inclined to each other at a Constant Angle:" by the same.
- "On the Theory of a System of Electrified Conductors:" by Prof. J. Clerk Maxwell.
- "On the Focal Lines of a Refracted Pencil:" by the same.
- "On an Application of the Theory of Unicursal Curves:" by M. Hermite.
- "On a Rational Quintic Correspondence of Two Points in a Plane:" by Prof. Cayley.
- "Bicursal Curves:" by the same.
- "Plan of a Curve-Tracing Apparatus:" by the same.
- "Note on the Plückerian Characteristics of Epi- and Hypo-trochoids, &c.:" by Mr. S. Roberts.
- "Some General Theorems relating to Vibrations:" by Hon. J. W. Strutt.
- "Invariant Conditions of Multiple-Concurrence of Two Conics:" by Mr. J. J. Walker.

"On a New Form of Biquaternion, being the Ratio of Two Systems of Forces:" by Prof. W. K. Clifford.

"Further Note on Geodesic Lines:" by Prof. Cayley.

Shorter communications were also made by Prof. Henrici, Mr. J. J. Walker, Hon. J. W. Strutt, Mr. M. Jenkins, Mr. J. Macleod, and Prof. W. K. Clifford.

The same Mathematical Journals had been subscribed for as in the preceding Session.

The Proceedings had been sent to the Mathematical Society of Paris, and to the University Library, Cambridge, in addition to the persons and Societies named in the last Report.

The Society had received the Bulletin of the Mathematical Society of Paris, in addition to those mentioned in the last Report.

The following presents had been made to the Society:—

"Ueber Systeme von Functionen mehrer Variabeln," von L. Kronecker, 2 parts (extracted from "Monatsbericht"): from the Author.

"On the Law of Facility of Errors of Observations;" and "On the Method of Least Squares," by J. W. L. Glaisher: from the Author.

"The Theory of Screws, a Geometrical Study of the Kinematics, Equilibrium, and Small Oscillations of a Rigid Body," by R. Stawell Ball: from the Author.

"Nautical Almanac for 1876," presented by authority of Lords Commissioners of the Admiralty.

"Vergleichende Betrachtungen über neuere geometrische Forschungen," von Dr. Felix Klein: from the Author.

"Royal Society's Catalogue of Scientific Papers," vol. vi.

"Catalogue of the Mathematical and Scientific Library of the late Charles Babbage."

"A Catalogue of a Collection of Models of Ruled Surfaces, constructed by M. Fabre de Lagrange, with an Appendix containing an account of the application of analysis to their investigation and classification," by C. W. Merrifield: from the Author.

"Annual of the Royal School of Naval Architecture, No. 3:" from Mr. Merrifield.

"On the Contact of Surfaces; and on some recent Generalisations of Algebra:" by W. Spottiswoode, F.R.S.: from the Author.

"A Complete Practical Treatise on the Nature and Use of Logarithms, and on Plane Trigonometry; with Logarithmic and Trigonometrical Tables;" by James Elliot (with Key): from the publishers.

"Salmon's Higher Plane Curves:" from the Author.

"Electricity and Magnetism" (2 vols.), by Prof. J. Clerk Maxwell: from the Author.

"Sur les trajectoires des points d'une droite mobile dans l'espace, et

démonstration géométrique d'une proposition due à M. Bertrand ;" by M. Mannheim : from the Author.

"An Easy Rule for filling up all Magic Squares;" by S. M. Drach : from the Author.

Memoirs, Notes &c., by the late Prof. De Morgan : from Mrs. De Morgan.

"Untersuchungen über Elasticität unter Berücksichtigung der Wärme, und über das Ellipsoid von kleinsten Volumen bei gegebenem Flächeninhalt einer Anzahl von Centralschnitten, und über die Transformation der Electricitätsgleichungen in allgemeine orthogonale Coordinaten;" by C. W. Borchardt : from the Author.

"Sur une classe remarquable de courbes et de surfaces algébriques, et sur la théorie des imaginaires," par G. Darboux : from the Author.

Memoirs from det Kgl. Norske Universitet à Christiania.

After the Reports had been read, and Mr. H. Perigal, F.R.A.S., elected Auditor, the Meeting proceeded to the election of the new Council.

The following gentlemen were declared by the Scrutators (Messrs. W. Marsham Adams and R. Wormell) duly elected :—

President, Dr. Hirst, F.R.S. ; Vice-Presidents, Prof. Cayley, F.R.S. ; *J. J. Sylvester, Esq., LL.D., F.R.S., and W. Spottiswoode, Esq., LL.D., F.R.S. ; Treasurer, Samuel Roberts, Esq., M.A. ; Secretaries, M. Jenkins, Esq., M.A., and R. Tucker, Esq., M.A. ; other Members of the Council, Prof. Clifford, M.A., Thomas Cotterill, Esq., M.A., J. W. L. Glaisher, Esq., B.A., F.R.A.S., Rev. R. Harley, F.R.S., Prof. Henrici, Dr. Phil., C. W. Merrifield, Esq., F.R.S., *The Right Hon. Lord Rayleigh, F.R.S., Prof. H. J. S. Smith, F.R.S., and J. J. Walker, Esq., M.A. (The gentlemen marked * were not on the Council for the Session 1872-73.)

Mr. G. S. Carr was elected a Member of the Society. Mr. R. R. Webb, B.A., Fellow of St. John's College, and Lecturer at Emmanuel College, Cambridge, Mr. Horace Lamb, B.A., Fellow of Trinity College, Cambridge, and the Rev. M. M. U. Wilkinson, M.A., late Fellow of Trinity College, Cambridge, were proposed for election.

The Chairman laid before the Meeting a printed letter received from the University of Christiania, informing the Society of the death of Prof. Hansteen, under whose superintendence the Trigonometrical Survey of Norway was made.

Mr. Sylvester then gave a description of a new instrument for converting circular into general rectilinear motion, and into motion in conics and higher plane curves, and was warmly applauded at the close of his address.

Mr. Merrifield and the Chairman asked several questions upon the subject of the communication.

Several instruments were submitted for inspection.

Prof. Cayley having to leave the Meeting, requested Mr. Sylvester to take his place.

Mr. W. Marsham Adams exhibited his *Mensurator* and *Cælometer*, showing their mode of operation, and gave a short account of the objects to which they could be applied.

Mr. S. Roberts read a short Note on the expression of the Arc of a Cartesian by Elliptic Functions.

The following presents were received:—

"*Rendiconti, serie ii. vol. v. fasc. viii.—xvi. (adunanza del Aprile 1872 to 1° & 7° Aug. 1872).*"

"*Memorie del Reale Istituto Lombardo, vol. xii. to iii. della serie iii. fasc. v.*"

"*Rappresentazione piana di alcune superficie algebriche dotate di curve cuspidali, nota del Prof. Luigi Cremona,*" Bologna, 1872, through Prof. Cremona.

"*Annali di Matematica,*" serie ii^a. tom. v. fasc. 4^a., May, 1873.

"*Monatsbericht,*" Feb. 1873 (2 parts); Mai (2 parts), 1873.

"*Bulletin des Sciences Mathématiques et Astronomiques,*" July to November, 1873; and "*Table des Matières,*" tome iv., 1^{re} semestre, 1873.

"*Journal of London Institution,*" No. 21.

"*Transactions of the Royal Irish Academy,*" vol. xxiv. pts. xvi., xvii.; vol. xxv., pts. i.—iv.

"*Proceedings of the Royal Irish Academy,*" vol. x. pt. iv. (1870); vol. i., series ii., Nos. 2 ('70-'71) to 8 ('72-'73): from the Academy.

"*Appendixes to the volumes of Astronomical and Meteorological Observations made at the United States Naval Observatory during 1870 and 1871;*" from Rear-Admiral B. F. Sands, Superintendent.

"*Crelle,*" 76 Band, zweites und drittes Heft, 1873.

"*Journal of the Institute of Actuaries,*" No. xcii. July, 1873, vol. xvii. pt. vi., and List of Members.

Carte de Visite of Lord Rayleigh, F.R.S., taken by himself.

Wilson's "*Elementary Geometry,*" 3rd edition, 1873: from the Author.

"*Proceedings of the Royal Society,*" Nos. 145, 146.

"*Note sur la différentiation et l'intégration d'une intégrale multiple par rapport à une constante;*" par D. Bierens de Haan. (*Extrait des Archives Néerlandaises, t. vi., 1871.*)

"*La Méthode d'Euler pour l'intégration de quelques équations différentielles linéaires démontrée à l'aide de l'équation intégrante;*" par D. B. de Haan. (*The same, t. vii., 1872.*)

"*On the Right Ascensions of the Equatorial Fundamental Stars, and the Corrections necessary to reduce the Right Ascensions of Different Catalogues to a Mean Homogeneous System;*" by Simon Newcomb (1872).

"Report on the Difference of Longitude between Washington and St. Louis," by William Harkness. Results of Washington Observations, 1853 to 1860. United States Naval Observatory, Washington, 1872.

"Catalogue of Stars observed at the United States Naval Observatory during the years 1845 to 1871;" Washington, 1873. From the Naval Observatory, Washington City.

"The Problem of Pythagoras;" by W. Marsham Adams, B.A.

"Handbooks to the Mensurator and Cœlometer" (in duplicate); by the same: from the Author.

Note on the expression of the length of the arc of a Cartesian by Elliptic Functions. By SAMUEL ROBERTS, M.A.

Having observed that the periphery of a Cartesian oval can be expressed by means of elliptic quadrants, when the axial foci are real,* I inferred that the general expression for a Cartesian arc might possibly be reduced to elliptic functions, by some real or imaginary transformation.

I propose now to show that this is really the case.

Let the equation of the Cartesian be

$$\rho^2 - 2(a + b \cos \omega) \rho + c^2 = 0,$$

then the arc element is

$$ds = \frac{(a + b \cos \omega) \sqrt{(a^2 + b^2 - c^2 + 2ab \cos \omega)} d\omega}{\sqrt{(a + b \cos \omega)^2 - c^2}} \pm \sqrt{(a^2 + b^2 - c^2 + 2ab \cos \omega)} d\omega;$$

and, if $2\theta = \omega$,

$$ds = \frac{2(a + b - 2b \sin^2 \theta)(1 - \kappa'^2 \sin^2 \theta) d\theta}{\sqrt{(1 - \kappa^2 \sin^2 \theta)(1 - \kappa'^2 \sin^2 \theta)(1 - \kappa''^2 \sin^2 \theta)}} \pm 2\sqrt{(a + b)^2 - c^2} \sqrt{(1 - \kappa'^2 \sin^2 \theta)} d\theta \dots\dots\dots (a);$$

where κ^2 , κ'^2 , κ''^2 are written for

$$\frac{2b}{a + b + c}, \quad \frac{2b}{a + b - c}, \quad \frac{4ab}{(a + b)^2 - c^2}.$$

The expression affected by the $\pm$ sign is already an elliptic arc-element.

As to the first part of the value of ds , I remark that $\kappa^2 + \kappa'^2 - \kappa^2 \kappa'^2 = \kappa''^2$, and the expression in question is, in fact, a transformation of Jacobi's

* Proceedings, May 12, 1870.