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could be fitted to microscopes, and more convenient than those at present employed.

NOTE ON THE SENSATION OF COLOUR. BY C. S. PEIRCE.

It may, perhaps, be worth while to notice a few consequences of three theories concerning colour which are usually regarded with some favour.

First Hypothesis.—The appearance of every mixture of lights depends solely on the appearances of the constituents, without distinction of their physical constitution. This I believe is established.

Second Hypothesis.—Every sensation of light is compounded of not more than three independent sensations, which do not influence one another. This is Young's theory. It follows that, if we denote the units of the three elementary sensations by i , j , and k , every sensation of light may be represented by an expression of the form

$$Xi + Yj + Zk.$$

Third hypothesis.—The intensity of a sensation is proportional to the logarithm of the strength of the excitation, the barely perceptible excitation being taken as of unit strength. Negative logarithms are to be taken as zero. This is Fechner's law. It is known to be approximately (and only approximately) true for the sensation of light. From this it follows that, if x , y , z be the relative proportions of a mixture of three lights giving the elementary sensations i , j , k , the sensation produced by the mixture is

$$I \log x \cdot i + J \log y \cdot j + K \log z \cdot k,$$

where I , J , K , are three constants.

From these principles it follows that, if a light giving any sensation such as that just written have its intensity increased in any ratio r , the resulting sensation will be

$$\begin{aligned} & I \log rx \cdot i + J \log ry \cdot j + K \log rz \cdot k \\ &= I \log x \cdot i + J \log y \cdot j + K \log z \cdot k + \log r (Ii + Jj + Kk). \end{aligned}$$

Thus the result of increasing the brilliancy of any light must be to add to the sensation a variable amount of a constant sensation, $Ii + Jj + Kk$; and all very bright light will tend toward the same colour, which may therefore be called the *colour of brightness*. Moreover, if the three primary colours be mixed in the proportions in which each by itself is just perceptible, the sensation produced will be

$$\log r (Ii + Jj + Kk),$$

and can only differ by more or less.

Now I find in fact that all colours are yellower when brighter. If two continuous rectangular spaces be illuminated with the same homogeneous light, uniformly over each, but unequally in the two, they will appear of different colours.

If both are	red,	the brighter will appear	scarlet ;
"	"	green	" yellowish ;
"	"	blue	" greenish ;
"	"	violet	" blue.

If we have the means of varying the wave-length of the light which illuminates the fainter rectangle, we can improve the match between the two, by bringing the fainter toward the yellow. Such motions will converge toward a certain point of the spectrum which they will never cross—a point a little more refrangible than D and having a wave-length of 582.10^{-8} mm., according to Ångström's map. If both rectangles be illuminated with this light, the fainter appears white or even violet ; but if it be varied in wave-length with a view of improving the match, it will be found to return to the same point with the utmost precision.

It appears, therefore, that, if our hypotheses are correct, the colour $\log r (I i + J j + K k)$ is like that of the spectrum at $\lambda = 582$, only that it contains less blue or violet, and is consequently of greater chromatic intensity.

It further follows from Fechner's law that, if any light be gradually reduced in brightness, one element of the sensation will disappear after another—and that when very faint it will exhibit only one primary colour, which is the one which it contains in greatest proportion relatively to the proportion in the light which has the colour of brightness. Now although this does not seem to be exactly the case, yet we do get some approximation to it. It is true that any light whatever, when sufficiently faint, appears white, owing to the self-luminosity of the retina. We cannot therefore, unfortunately, get sight of the primary colours by reducing the light of three parts of the spectrum. But we may, as has often been suggested, make use of the principle of contrast. If any red spectral light be sufficiently reduced, it will perfectly match any less-refrangible light. We may therefore say that a faint spectral red in contrast with a bright light of the same kind, excites with approximate purity one of the elementary sensations. The same thing is true of the violet ; and therefore a rich violet may be taken as another primary colour. In my book entitled 'Photometric Researches,' the printing of which is nearly complete, I show reason to think that the pure green has a wave-length intermediate between E and b. A faint green of this sort contrasted with a bright one appears as a very bluish green ; and this may therefore be supposed to be the third primary colour.

We have seen that it results from the theory that an increase in the brilliancy of any light adds to the sensation nothing of the peculiar colour of that light, but only a certain amount of the colour of brightness. If this be the fact, then the photometric sensibility of the eye should be the same for all colours. In order to ascertain whether this is so or not, I have made a series of determinations of my photometric probable error. Each determination was based on twenty-eight comparisons of two parts of the same-coloured disk. Since there were two unknown quantities

(namely, the relative brightness of the two surfaces compared, and an instrumental constant), it follows that only twenty-six observations were effective for determining the probable error. Let R be my photometric probable error of a single comparison. Then the probable error of a single determination of R (which

we may denote by r) would be $\frac{\cdot 51}{\sqrt{26}} \times R$, or say $\frac{1}{10} R$. Having

made a considerable number of such determinations of R with different coloured disks, let us ascertain their probable error from their discrepancies, considering them as so many independent observations of the same unknown quantity, and denote this probable error by r' . If, then, R really is the same for all colours, we should have

$$r' = r;$$

or at least the difference should not exceed ρ , the probable error of r' , which may be calculated by the formula

$$\rho = \frac{\cdot 51}{\sqrt{mr'}},$$

where m is the number of sets of experiments diminished by 1. If, on the other hand, R varies with the different colours, and not merely accidentally, r' should have a larger value. The following are the values I obtained for R , the sum of the brightness of the two surfaces compared being taken as unity:—

	R .	Diff. from mean.
Feb. 6. White	$\cdot 0041$	$+\cdot 0001$
Red, just before C.....	$\cdot 0046$	$+\cdot 0006$
Chrome-yellow, A 2	$\cdot 0032$	$-\cdot 0008$
Feb. 7. Red, just before C.....	$\cdot 0040$	$\pm \cdot 0000$
Staat's emerald green	$\cdot 0046$	$+\cdot 0006$
Carmine, B	$\cdot 0044$	$+\cdot 0004$
Chrome-yellow, A 1	$\cdot 0037$	$-\cdot 0003$
Purple, Hoffmann's violet RRR	$\cdot 0033$	$-\cdot 0007$
Feb. 13. Red, just before C.....	$\cdot 0048$	$+\cdot 0008$
Green, complementary to carmine	$\cdot 0034$	$-\cdot 0006$
Blue-violet, No. 2	$\cdot 0048$	$+\cdot 0008$
Yellow, A 1, mixed with black..	$\cdot 0032$	$-\cdot 0008$
Mean	$\cdot 0040$	

After these experiments the method of observing was changed, and I obtained the following:—

	R .	Diff. from mean.
Feb. 14. White window-shade, ill. by sun	$\cdot 0030$	$-\cdot 0002$
Brown	$\cdot 0030$	$-\cdot 0002$
Greenish sky-blue.....	$\cdot 0037$	$+\cdot 0005$
Very reddish purple	$\cdot 0028$	$-\cdot 0004$
Yellow-orange	$\cdot 0032$	$\pm \cdot 0000$
Feb. 15. "Fundamental green of Müller"	$\cdot 0030$	$-\cdot 0002$
Vermilion, half between C and D	$\cdot 0034$	$+\cdot 0002$
Violet.....	$\cdot 2032$	$\pm \cdot 0000$
Yellow	$\cdot 0036$	$+\cdot 0002$
Mean.....	$\cdot 0032$	

We thus get from the

first twelve determinations, $r = \cdot 00040$, $r' = \cdot 00048$, $\frac{r'}{r} = 1\cdot 2$,

last nine determinations, $r = \cdot 00032$, $r' = \cdot 00019$, $\frac{r'}{r} = 0\cdot 6$,

and from the weighted mean, $\frac{r'}{r} = \cdot 96$; so that it appears from these

experiments that the photometric susceptibility of the eye is the same for all colours. The result, however, is uncertain, because it may be that R is chiefly due to other sources of error than the limitation of sensibility; still the experiments show as small a value of R as is usually obtained. I shall endeavour, by further observations, to obtain a conclusive result.

A further consequence of our hypotheses will be reached by differentiating the expression for a light-sensation. We have

$$d(I \log x \cdot i + J \log y \cdot j + K \log z \cdot k) = \frac{1}{x} dx \cdot Ii + \frac{1}{y} dy \cdot Jj + \frac{1}{z} dz \cdot Kk.$$

Now, as x , y , and z all exceed unity, the differential is greater the nearer unity x , y , and z are. Hence, since the variation of the proportions of the primary colours with a variation of position in the normal spectrum is uniform, it follows that the change of colour of the normal spectrum should be most rapid about $\lambda = 582$, as it of course is. It is also obvious that, if the total quantities of the three colours are nearly the same in different parts of the spectrum (I here refer to these colours not as really objective, but as measured in the usual objective way), then the part about $\lambda = 582$ must be the brightest—another familiar fact.

I may observe that there is a modification of our formula for a sensation of light, which probably better represents the relations of the sensations. Writing, in the first place,

$$i = Ii, \quad j = Jj, \quad k = Kk,$$

the formula is

$$\log x \cdot i + \log y \cdot j + \log z \cdot k.$$

This loses its validity when any of the logarithms become negative. If z is the smallest of the three quantities, we may substitute

$$X = \frac{x}{z}, \quad Y = \frac{y}{z};$$

and the formula becomes

$$\log X \cdot i + \log Y \cdot j + \log z (i + j + k).$$

When x or y is smallest there will be two other formulæ. Now, as the variation in the brilliancy of the light affects only the last term of the last formula, and not the first two depending on X and Y , it is more than probable that the eye is habituated to separating the element of sensation which this last term represents, and which

is continually changing its values, from the rest which remains constant. It is therefore likely that the classification of light into three kinds, according as the *violet*, the *red*, or the *green* is contained in the smallest proportion, is one which has a relation to the natural powers of discrimination.—Silliman's *American Journal*, April 1877.

ON ACCIDENTAL DOUBLE REFRACTION. BY J. MACÉ.

The phenomena of accidental double refraction produced by compression have, since their discovery by Brewster, given rise to numerous researches, while those produced by tempering have been but little studied. Since the discovery of the latter by Seebeck, we find but little concerning them except the papers of M. de Luynes, in which the author studies chiefly the mechanical properties acquired through an energetic tempering, and those of M. Mascart on the tempering of cast plates of glass (*Journal de Physique*, 1876), in which the author applies himself especially to the practical side of the question.

The aim of the investigation which I commenced some months since was, on the contrary, to discover what are the laws which govern this phenomenon, and to search out the analogies or the differences which might exist between that which results from tempering and the other cases of accidental double refraction. I therefore proposed to myself to study plates of simple geometrical forms, especially rectangular and square, differing in dimensions, thickness, and constitution. If it be remarked that Wertheim, by the study of the regular compression of glass, has been able to show that the differences in the course are proportional to the pressures exerted, it will be seen that this led me to investigate the distribution of the differences of the course in the various plates submitted to experiment.

The arrangement employed was very simple :—A car carrying the plate permitted it to be displaced parallel to a micrometer formed, as in M. Jamin's apparatus, of two parallel fine threads and, besides, of a third, horizontal thread, rendered necessary by the curved form often (and particularly in square plates) affected by the fringes to be studied. As always, the polarizer was placed at 45° ; the analyzer was alternately transverse and parallel, so as to exhibit fringes corresponding to differences of course varying by half wavelengths. The compensator was suppressed, it having the inconvenience of deforming the fringes and often rendering them difficult to observe; but it was made use of for the rectangular plates, in order to measure the central difference of course. For the illumination the burner of Laurent's saccharimeter was used. The position of each fringe was measured to within 0.1 of a millimetre at the least.

In studying the distribution of the course-differences along a line parallel to and equally distant from two of the sides of the plate, the phenomenon could in all cases be represented by a formula