315. Systems of Conics Whose Director Circles Have a Common Radical Centre

Author(s): E. J. Nanson

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MATHEMATICAL NOTES.

314. [v. i. a.] On $\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}$.

Certain algebraical theorems appear more difficult to some beginners than would have been anticipated beforehand.

Thus the proof of the proposition that if $\frac{a}{b} = \frac{c}{d}$ each ratio is equal to $\frac{a+c}{b+d}$ is apparently much easier than that of the proposition that the sum of the roots of $x^2+px+q=0$ is $-p$.

Yet it will, I think, be agreed that the former proposition is considered the more difficult by many boys and is least thoroughly comprehended or readily applied.

One hardly likes to offer such a suggestion as that which follows, but I am not sure that this is not a mistaken feeling. To put such ideas forward as being novel or original would indeed be foolish, but in teaching children a childish remark may be very much to the point.

Moreover, there are many people teaching mathematics who are very hard worked in term time and who experience the pressure of the holidays rather severely at other times, and so have little time for thinking things over quietly.

The suggestion is that some concrete illustrations of the proposition may render its simplicity manifest and prepare the way for an enunciation of the result in its most general form.

1. A regiment of 1000 men contains 130 Yorkshiremen : another of 1200 men contains 156.

The proportion of Yorkshiremen in each regiment is the same, for

$$\frac{130}{1000} = \frac{156}{1200}.$$

The proportion of Yorkshiremen in the total force is

$$\frac{130+156}{1000+1200};$$

and this is obviously the same as the proportion in either regiment.

2. A boy gets 34 out of 100 for arithmetic, 68 out of 200 for algebra, 51 out of 150 for geometry.

The proportion of marks on each paper is the same, for

$$\frac{34}{100} = \frac{68}{200} = \frac{51}{150};$$

and the proportion of the total marks on all three papers is

$$\frac{34+68+51}{100+200+150};$$

and as the boy got 34 per cent. on each paper, he must have got 34 per cent. on the total.

3. Let the student draw up a statement of a rule which will apply to both these examples. The obvious graphical illustration should of course also be given.

C. S. JACKSON.

315. [L. 17. e.] *Systems of conics whose director circles have a common radical centre.*

A conic is uniquely determined by five pairs of conjugate lines. Taking two of the pairs to be the isotropic lines through two fixed points, we see that the director circle of a conic having three assigned pairs of conjugate lines is uniquely determined when two of its points are given. Hence the

director circles of conics having three assigned pairs of conjugate lines have a common radical centre, and therefore also a common orthogonal circle which may be real or imaginary.

Two particular cases of this theorem are well known. The director circles of conics having a given self-conjugate triangle, or of conics inscribed in a given triangle, have a common orthogonal circle. The position of the orthogonal circle in either case is readily seen by a consideration of special conics of the system. Thus, in the former case, taking the conic to be a small ellipse having two of the sides of the self-conjugate triangle as conjugate diameters we see that the orthogonal circle is the circumscribing circle of the self-conjugate triangle. In the latter case taking the conic to be a thin ellipse with one end at a vertex and the other on the base of the circumscribing triangle we see that the orthogonal circle must be the self-polar circle of the triangle.

The general theorem is that the director circles of all conics harmonically inscribed in three given conics have a common orthogonal circle.

E. J. NANSON.

316. [K. 11. a.] *On orthogonal circles.*

If the diameter AA' of a circle cuts an orthogonal circle in B, B' , and cuts the common chord CD in L , then

$$\frac{AL}{LA'} = \left(\frac{AB}{BA'}\right)^2 = \left(\frac{AB'}{B'A'}\right)^2.$$

Proof. Let M be the mid-point of BB' , and N the mid-point of the common chord, and S, S' the centres of the circles.

Then

$$SL \cdot SM = SN \cdot SS' = SC^2;$$

$$\therefore (AA', LM) \text{ is harmonic}$$

and

$$\frac{AL}{LA'} = \frac{AM}{A'M} \dots\dots\dots(1)$$

Also, since (AA', BB') is harmonic, and M bisects BB' ,

$$\therefore AM \cdot A'M = BM^2;$$