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XXIII. *New analytic Formula and Table of an increasing Life Annuity; with Remarks on the Surrender of Life Assurance Policies at Proprietary Offices.* By Mr. J. B. BENWELL, Hoxton.

To Mr. Tillock.

SIR, — THE accompanying table, computed from real observations and the probabilities of life, is applicable to a description and species of life annuity transaction, by no means common, and scarcely, under any form whatever, practised in this country; and any thing new or novel of this kind I am inclined to imagine will be interesting to, and invite the attention of societies engaged and concerned in this particular and important branch of human œconomy. On a former occasion I recommended the subject to their notice; and I do it now with the same persuasion of its being, by an easy extension of application, as suitably adapted to the views and purposes of individuals, as any other mode of assurance, and of its becoming at some future period in more general recurrence. I do not expect any credit for the suggestion proposed, because we know that, in the commerce of life, according as interest and gain may impart a colouring to the medium an object is seen through, it is either abandoned, or pursued with avidity. Now it happens in the instance under review, that the influence of this ordinary spirit and idol of general adoration is to be perceived: for the Offices (I speak in reference to the companies constituted solely on the trading system) in rather indirect terms and manner convey the inference gathered, that it is, as we may suppose, not a profitable speculation to them, or, more figuratively, the sweets of the hive are not exactly found in this track; and since the legislature has anticipated their privileged means in this respect, what comparatively they can do, but poorly compensates for the incidents attendant; for it is to be remarked, that it is not among a few independent but general mass of lives combined, that the calculations of contingencies are founded and applied; and as an apparent consequence, the latter condition may be expected to partake in a higher degree favourable in the determination of the happening and failing of events generally, as well as in reference to the claims that may in probability of expectation successively terminate.

These reflections naturally lead me then to inquire into the motive, why most of the chartered companies display and hold forth their legislative powers enrolled and enabling them to speculate in the grant and purchase of annuities, on just and equitable principles, when at the same time they evince a coldness,

neglect and indifference to encourage and promote any emendation or extension devised in seeking out the ramifications of theory subordinate to practice. Where then is the consistency, in what is contained the policy of the immediate supporters of these schemes, or the engagements formed therein with them? If indeed they are to be identified with a character for disinterestedness, liberality and ingenuousness, and assuredly connive not at the intervention of stratagem, let them cease longer to hold out beacons deceptive, as well as those false lights that dazzle in the presence, but do not inform and direct the mind of the beholder, if all this be through a cause inadvertently, and not of design to impose on or delude the unwary. In glancing over these observations, it is likely they will be accounted matter of assertion unsupported by testimony of fact: that this may not supervene, I shall recognise, though among several, one circumstance only as of recent occurrence in exemplification, and to show that they have not sprung out of a feeling of invective or prejudice, and as in the sequel there will be cause still to condemn the selfish and mercenary spirit by which such of these public bodies are actuated.

In the course of reference made to me on the subject of contingent property, I was lately requested to report upon the value of three policies of assurance, effected in certainly a very popular and flourishing concern: I allude not to the Equitable, for there in all its affairs we find a conscientious and scrupulous regard paid to the equitable claims and interest of all its members indiscriminately alike: thus a true parent of nature marks no distinction in the being of a common family and stock.

We will now briefly state the terms and dates of these policies when effected; they are as under—the life now in forty-first year of age:

			Prem. paid.		
1.	..	500	..	12 <i>l.</i> 5 <i>s.</i> 5 <i>d.</i>	.. 1806
2.	..	500	..	12 15 5	.. 1808
3.	..	1000	..	26 2 6	.. 1809
		<u>2000</u>		<u>51 3 4</u>	

In the proposed consideration here it is necessary merely to remark, that upon a direct appeal of the party at the office to ascertain the sums that would be returned in lieu of the claims existing on a surrender being made conformably with the usual practice—they were the following nominally:

1 .. 30*l.* 2 .. 23*l.* 3 .. 40*l.*

However, it may be replied by those in the conduct and management of the offices, that the sum the directors may, in caprice, think proper and choose to offer when they cancel a policy,

is

is no question of specific relation to the mathematical value; but let them understand, that every person so conversant with this subject is enabled to say, and knows as well as they, what compensation should at any time be returned; and what as parties to a contract, at least construed in equity, they are bound by ties of ordinary faith to give the assured member. Let us now contrast the preceding sums, and equal but to 93*l.* as what would be allowed in a propriety concern: let us, I say, compare the extraordinary difference in the value of an assurance equal in amount effected with the Equitable under almost precisely similar circumstances; viz. on the same life for 2000*l.* in 1809, at an annual payment of 52*l.* 4*s.*; but it is right to add, there is on this a bonus of 600*l.* were it now a claim; as also of 150*l.* on policy No. 1, as I omitted to state. For the policy existing at the Equitable, then, the trustees will refund the sum of 514*l.* 9*s.* if surrendered any time before the annual premium due should be paid: so, allowing for this, the sum nominally offered is 566*l.* 13*s.* It is of no utility here to urge any thing as to the variation of the premiums, they being in the one case upon a single assurance, in the other apportioned upon three separately; it is the aggregate of the annual premiums and relative amount of assurance we are directed to, and that strikes as dissonant to reason; it is an instance, in which the mind discriminates truth and error from derived intuitive evidence, and not enveloped in the mists of sophistry.

Further concerning these two values: as referred to the first announced, the discordant medium or discrepancy between them is not easily, if at all, explicable by any order or process of computation on which such kind of deductions should ever be regulated and assumed as the only infallible in result. We may next, as not without its use, since elucidation is the mean intended, proceed to a different view of the subject; for, by placing the same proposition in new and opposite directions, we see clearer what is either apposite or inconsistent. It appears then on the three former-described policies, the gross amount of premiums paid is 663*l.* 11*s.* 8*d.*, or equal to 15, on the first; 13, on the second; 12, on the last; and these, if calculated as distinct annuities at 5 per cent., the first payment of each being made when the policy was opened, and so on at the beginning of every year succeeding, in which the life, it is considered, will be a claim—the equivalent amount is 952*l.* 3*s.* 6*d.* that is, equal in value now to an annuity of 47*l.* 12*s.* 2*d.* certain, to continue for ever, or in perpetuity at the same rate of interest, and in consideration of having received which, it is that the office proffers in-signifi-

cant return the sum of barely 93*l*. Is this then a precedent distinguished by any principle of liberality? is it distinguished by a feature of justice? It would, I imagine, be an excess of candour to confess as much; but nay, more than this unjustly it seems, that at some offices there is a refusal to cancel at all, or rather to make any return whatever of the premiums received; and thus, were it not through the instrumentality of those trafficking in such things, it would be impossible to recover a single shilling of the immense sums so expended, and which, under the peculiar circumstance, would inevitably be sacrificed without any collateral advantage acquired. An observation occurs now, and though implied, yet should be expressly so, to the credit of the Society animadverted upon, it does then at recurring intervals divide with its members a part of the enormous gains of profit made; they do this in reality, and as what is falsely insinuated in many, where the scale of premiums is actually the same as at the Equitable, and yet never add a mite beyond the sum assured; and with this fact so glaring, it bewilders one to think how they should be able to effect any policies with the public at all, excepting by the medium of a corrupt influence, or a courtesy tantamount to servility which their most interested and strenuous supporters may command in the world. If in general this is the arbitrary and unfair practice dictated in all the subscription Societies, how are we to applaud or even give credence to the reiterated professions made with unblushing effrontery, that in all negotiations with the public, both its "interest and convenience are considered," being in all their proceedings actuated by a strict sense of rectitude and of honour?

I have in the present paper intentionally abstained from mixing up any extraneous calculations with it: by resolving algebraic into numerical values, the enunciation is simply apparent and intelligible to the general reader. I must now, in conformity with my original design, revert thereto, and introduce the table itself and auxiliary means of construction suited accordingly.

Table of the Value of an Annuity of 1l. increasing in Order of the Series 1. 2. 3. 4. .. n. on a Life at every Age from Three Years to extreme Duration. Computed from Observations of M. DE PARCIEUX.

Age.	3 per Cent.	3½ per Cent.	4 per Cent.
3	458,393	389,494	333,422
4	463,367	394,275	337,923
5	464,747	396,027	339,846
6	464,227	396,174	340,404
7	462,201	395,042	339,870
8	459,116	393,018	338,564
9	455,451	390,488	336,841
10	450,679	387,006	334,288
11	444,794	382,562	330,899
12	437,806	377,158	326,685
13	430,743	371,679	322,397
14	423,606	366,125	318,02
15	416,399	360,497	313,585
16	409,123	354,797	309,08
17	402,261	349,445	304,853
18	395,327	344,016	300,57
19	388,319	338,511	296,244
20	381,24	332,932	291,813
21	374,555	327,683	287,657
22	367,792	322,354	283,423
23	360,951	316,945	279,108
24	354,031	311,454	274,712
25	347,036	305,881	270,23
26	339,962	300,227	265,662
27	332,813	294,49	261,016
28	325,589	288,672	256,286
29	318,292	282,765	251,471
30	310,922	276,782	246,571
31	303,48	270,718	241,585
32	295,969	264,575	236,513
33	288,389	258,352	231,355
34	280,744	252,052	226,111
35	273,035	245,674	220,782
36	265,266	239,222	215,369
37	257,439	232,695	209,872
38	249,186	225,762	203,988
39	242,07	218,775	198,035
40	233,807	211,7403	192,018
41	225,525	204,663	185,942
42	217,235	197,551	179,813
43	208,944	190,412	173,636
44	200,662	183,254	167,42
45	192,399	176,086	161,171
46	184,591	168,919	154,900

TABLE con-

TABLE continued.

Age.	3 per Cent.	3½ per Cent.	4 per Cent.
47	176,71	162,03	148,81
48	168,863	155,147	142,75
49	161,338	148,532	136,918
50	154,173	141,93	131,076
51	147,012	135,587	125,452
52	140,164	129,493	120,037
53	133,343	123,413	114,616
54	126,573	117,355	109,195
55	120,092	111,542	103,981
56	113,407	105,757	98,774
57	107,037	100,013	93,583
58	100,951	94,51	88,6
59	94,938	89,054	83,641
60	89,01	83,655	78,716
61	83,178	78,324	73,836
62	77,455	73,075	69,013
63	72,028	68,083	64,415
64	66,723	63,187	59,891
65	61,557	58,402	55,456
66	56,697	53,888	51,26
67	52,135	49,642	47,304
68	47,87	45,663	43,588
69	43,9	41,951	40,113
70	40,223	38,503	36,879
71	36,709	35,205	33,775
72	33,497	32,18	30,924
73	30,462	29,316	28,218
74	27,613	26,622	25,666
75	24,96	24,105	23,277
76	22,387	21,662	20,952
77	20,033	19,421	18,814
78	17,907	17,393	16,875
79	15,902	15,475	15,039
80	14,148	13,798	13,431
81	12,537	12,258	11,95
82	11,124	10,859	10,603
83	9,686	9,47	9,263
84	8,249	8,078	7,914
85	6,953	6,821	6,692
86	5,807	5,705	5,607
87	4,814	4,738	4,664
88	3,79	3,735	3,684
89	2,893	2,856	2,821
90	2,126	2,103	2,08
91	1,486	1,472	1,459
92	,9567	,9499	,943
93	,4854	,4831	,481
94	zero.	zero.	zero.

Respecting these numbers and tabulated values, which I have extended to other rates, but cannot well augment here, it is to be observed when at any time needed they are directed to be calculated by M. De Moivre's Hypothesis, from which some neat and appropriate formula has been investigated; but it is obvious the values deduced by it are only approximate to the true ones as found from the series of decrements of life; but this latter value however can be obtained, and, excluding the hypothesis, with greater ease, facility and accuracy of course, after the manner ensuing: first, having from real observations and for any order of increase determined the value of any such annuity upon an advanced life, or beginning from an age near the end of the table, then having for any form chosen so determined the value at the age begun with, we may thence assign in succession the value for every age intercepted, or that preceding, by aid of a subsidiary formula; and the operation being continued until the Table is completed, as also similarly for single, joint or longest of lives, the application may be generalized.

Enumerating therefore the formula of derivation. Of the order of numeral series in the table 1. 2. 3. 4. &c.

Let A signify the value of 1*l.* annuity on life N , at age $n-1$. B , that at age n , but increasing by 1. 2., 3*l.* Then the value of such annuity for life N , is expressed by $(A + B \times \frac{a'}{ax})$.

If the annuity increases in the order 1. 3. 5. 7; if C be its value at age n , then is whole value for $N = 2A + (C-1) \cdot \frac{a'}{ax} = (A + \frac{2Ba'}{ax})$.

If increasing by the triangular series 1. 3. 6. 10; and D be its value at age n , the value for N is $= A + \frac{(B+D)a'}{ax}$.

If it increases as the quadrangular series 1. 4. 9. 16, the value for N , $= A + \frac{(B+2D)a'}{ax}$.

Where in each theorem $\frac{a'}{ax}$ is the expectancy of 1*l.* on life N , a year hence. And, expanding by the combinatorial methods, other forms of series will be developed, and the formula of continuation applied. There are beside certain modifications fitting in practice, and some relative to the deduction of value in the case when an annuity is not for the whole term of duration, but temporary, or in remainder deferred; but these are not here admitting a full description.

This being the first table of the value of a life annuity increasing,

creasing, founded on any registered observations of mortality, that have as yet been published, the computist may lean with due confidence on its accuracy; it contains, I believe, no error of moment, the numbers respectively were derived from a double operation pursued, logarithmically and by calculation, the former to check and verify the latter; and though this is indispensable, yet the process by logarithms is not invariably critically exact in any specific inquiry, as will happen in fact from such tables, having been mostly by interpolation of differences compiled and extended: when then any result occurs so, as dissonant, it is only to be settled by a distinct revision, the work collated anew, or by some independent mode of verification.

Of all the chartered companies established, they avowedly profess to blend with their customary the life annuity business in every species of variety and form: it is unaccountably strange, then, they evince not partially a wish to promote the cause of that science, by which it is, that the stability of their plans is to be determined, and even existence vitally depend; and if we look unto those mushroom societies spontaneously emerging out of obscurity, ignorance or corrupt motives, it is but too visibly beheld that the age of bubble scheme and system has revived again, or is rapidly reviving: it would be a truly laudable task for any one possessing abstractedly the means, to devote some portion of time and talent to the discussion of their principles, views and measures; if however, and I must so qualify the expression, they can sustain this test and scrutiny of examination. I am aware of what is to be done, but, much as inclined, the engagement is such that few cheerfully and with alacrity would undertake and prosecute it through all its minutiae of condition.

Aske Terrace, Hoxton, Feb. 1821.

J. B. BENWELL.

XXIV. *On the Solar and Lunar Periods.* By Mr. JAMES
UTTING, of Lynn Regis.

To Mr. Tilloch.

SIR, — I HAVE sent you for insertion in the Philosophical Magazine (if approved) a few extracts and remarks relative to the solar and lunar periods, as stated by modern astronomers, in order (in part) to meet the wishes of Mr. Yeates: as probably some of your astronomical correspondents are in possession of the means of a more extensive reference, my selections are but few, but I believe them to be the most correct published, agreeable to the present improved state of the science.

I remain, sir, yours truly,

Norfolk-street, Lynn Regis, Feb. 8, 1821.

JAMES UTTING.

M. DE