XXIX.—*On the Determination of Heights by the Boiling Point of Water.* By JAMES D. FORBES, Esq., F.R.S., Sec.R.S.Ed., Inst. Reg. Sc. Paris. Corresp., and Professor of Natural Philosophy in the University of Edinburgh.

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It was observed by FAHRENHEIT, that the boiling point of water depends on the height of the barometer, the pressure of the air hindering the conversion of water into steam by a resistance which must be overcome by an increase of heat. DELUC* and DE SAUSSURE† contrived apparatuses for making the observation in the open air, and at great heights, and appear to have contemplated the substitution of the thermometer for the barometer upon occasion. They, as well as Dr HORSLEY,‡ Sir GEORGE SCHUCKBURGH,§ and Mr CAVENDISH,|| seem to have regarded the question as one which concerned the fixity of the point used in graduating thermometers, and its requisite corrections, rather than as applicable to barometric purposes generally. Several of them have given empirical tables for correcting the boiling point within the limits of the usual barometric variations, but one only, M. DELUC, has given a formula for connecting the indications of the barometer with the boiling point of water throughout the range which the barometer has been observed to vary on the earth's surface. This is the *only* formula immediately deduced from direct observations of the boiling point; and having been verified by DE SAUSSURE at a height greater than the limits for which it was constructed, and having elsewhere been declared by him to be so accurate as to supersede farther experiment on the subject, it might have been expected to be generally adopted, or at least known. We find, however, that though it has been occasionally copied into the formal articles of Encyclopædias, as a correction in graduating thermometers, observers who have used the boiling point for the determination of heights, have always preferred the ordinary tables which give the *elasticity of steam* in terms of its temperature, determined from experiments of quite a different kind from the boiling of water.

Dr DALTON, indeed, has given a table from observation under the air-pump of the boiling point,¶ and that table shews a manifest deviation from the elasticities and temperatures of vapour determined by himself, and now generally accepted as the most accurate below 212°. In boiling, the temperature requires

* Modifications de l'Atmosphere, tom. ii.

† Voyages, § 1275, 2011.

‡ Phil. Trans. vol. lxiv.

§ Ibid. vol. lxix.

|| Ibid. vol. lxvii. p. 816.

¶ Meteorological Essays, 2d edit. p. 127.

to be *higher*, under a given pressure, than the temperature of steam which has the same tension. Thus, comparing DALTON's two tables—

Temperature.	Pressure under which Water boils.	Tension of Vapour.	Difference.
212	30.0	30.0	0.00
200	22.8	23.64	+ 0.84
190	18.6	19.00	+ 0.40
180	15.2	15.15	— 0.05

it is exactly at the part of the scale where the difference is most practically important that it is most conspicuous, namely, between 190° and 212°. The method of observation used by Dr DALTON, does not admit of any great accuracy in observing the boiling points, and the numbers he has given are evidently only approximate. Still, from observations made under *naturally* low pressures (the only ones worthy of much confidence in this case), I have found the same nonconformity of the theoretical tension of steam and the atmospheric pressure.

In 1817, Archdeacon WOLLASTON described a thermometer destined particularly for the purpose of determining heights.* But he seems not to have been aware of the progress which the subject had already made in the hands of DELUC and DE SAUSSURE. The latter used a thermometer indicating $\frac{1}{1000}$ of a degree of REAUMUR. WOLLASTON's instrument, though a neat laboratory one, has almost every fault which a travelling instrument can have, excepting only its small dimensions, to which everything is sacrificed. It is apt to break, and still more apt to be deranged, the contrivance for extending the scale being excessively incommodious; finally, it is impossible to use it in windy weather, and its indications are in an arbitrary scale. No was the method of calculating the heights more happy. At first he contented himself with assuming the progression of height to be proportional to the fall of the boiling point, near 212°;† but he afterwards‡ extended his calculation from Dr URE's table of tensions of vapour, expressly stating, that he had used the proportionality of 1° of FAHRENHEIT to 0.589 inches of the barometer, or 530 feet, merely as an approximation for small heights.

A reference in BOUÉ's *Guide du Geologue Voyageur*, directed me to a paper by Mr PRINSEP, in the Journal of the Asiatic Society of Bengal for April 1833. I hoped there to have found a table of boiling temperatures observed at great heights in India. But it only contains a modification of TREDGOLD's Formula of

* Phil. Trans. vol. cxx. p. 183.

† Ibid. p. 192.

‡ Ibid. vol. cx. p. 295.

Fig. 1.

Fig. 2.

Fig. 6.

Fig. 9.

Fig. 8.

Fig. 5.

Fig. 4.

Fig. 7.

Fig. 3.

the Elasticities of Steam adapted to the measurement of heights by the thermometer, and no original observations.

During a late journey in Switzerland (in 1842), I made several observations on the boiling point of water at great heights. Having long since abandoned WOLLASTON'S thermometrical barometer as practically useless, I was led to resume the method in consequence of a very ingenious and compact apparatus for chemical or culinary purposes having been shewn to me the preceding winter, by Mr STEVENSON, instrument maker, under the name of a Russian furnace, and which was, I believe, introduced into the country from Russia by Dr SAMUEL BROWN. It consists of a very thin cylindrical copper-pan for holding water, Fig. 1, Plate XI., with three moveable wire-legs. The bottom is flat, so that the flame of a spirit-lamp plays fully upon it. This lamp or furnace consists of two parts; a flat dish or saucer, Fig. 4, containing a little alcohol, which is set on fire, and then covered by the double dome-shaped vessel, Fig. 5, also of thin copper, with an air-tight plug *a*, by which a certain quantity of spirit of wine is introduced, and the lower part communicating with a bent tube or nozzle *b*, by which alcohol in ebullition is violently projected by the pressure of its own vapour, when heated by the flame in the saucer. The jet of burning spirit thus thrown up like a volcanic explosion through the aperture of the dome, has such force as to resist the blast of a hurricane, and plays right upon the bottom of the cylindric boiler or pan. Two fluid ounces of spirit of wine, will thus boil above a pint of water in still air in four minutes; and I have frequently first melted snow, and then brought it to boil to the amount of a pint, with little more alcohol, but, of course, in a longer time.

The furnace and boiling apparatus, together with a reservoir of alcohol, packs into the copper-pan, and that into a cylindrical leather case 4 inches high, and 6 in diameter. The thermometer, Fig. 2, is carried separately. It is 15 inches long and the degrees measure $\frac{3}{10}$ inch, which is quite sufficient in practice. Parallax is avoided, by having the scale repeated on each side of the tube on two pieces of copper not in the same plane.

Fig. 6 represents the spirit measure, Fig. 7 a reservoir for spirits, Fig. 8 a water measure or cup, Fig. 9 a handle which opens all the plugs, and serves also for lifting the lamp and pan when heated.

I immediately saw the value of the apparatus for determining the boiling point, and directed Mr ADIE to adapt a thermometer to it, graduated from 185° to 214° of FAHRENHEIT'S scale, divided to 10ths of a degree, the divisions admitting an estimation to 100ths. I am well assured, however, that in no circumstances, even the most favourable, is the observation true to less than $\frac{1}{20}$ of a degree. But this quantity corresponds to only 25 feet of elevation, and is therefore accurate enough for most purposes. The minute subdivisions of DELUC'S, DE SAUSSURE'S, and WOLLASTON'S instruments, are quite unavailing, as I have found by using the instrument of the latter with every precaution.

My barometer having been broken in the course of my journeys, I was glad to have recourse to the boiling point as a means of estimating (only roughly as I expected) some remarkable elevations not before measured. In several cases I had the advantage of comparing my thermometric boiling point with a barometer, and lately I resolved to discuss these observations empirically, without reference to any theory or tables, or previous observations.

I first projected the barometric pressures in terms of the corresponding thermometric observations. These were the following :—

DATE.	PLACE.	BOILING POINT.	Barometer reduced to English inches, and to 32°
1842.			
August 4	Tacul	200°·10	23·154
... 6, 7½ A. M.	Tacul	200°·6	23·353
... 13, 8 A. M.	St Bernard	199°·08	22·674
... 16, 8 P. M.	Prarayon	201°·58	23·893
... 17, 9 A. M.	Col Collon	195°·15	20·77
... 29, 11 A. M.	Gressonay	204°·20	25·143
September 5, 7 P. M.	Martigny	210°·12	28·489

I obtained a curve, which resembled a flattish logarithmic, the barometric numbers appearing to be in geometrical progression, whilst the temperatures varied uniformly. This recalled to me an idea which I had entertained some years ago, that the boiling point would be found to vary simply with the height to which I was led from knowing DELUC's formula; but the idea had since escaped me, or been postponed to other occupations. Now, however, I projected the simple elevations of the points of observation (derived from the barometric pressures from the common tables for computing heights uncorrected for the temperature), in terms of the boiling points, as in the Plate XI., Fig. 3, and I was gratified to find, that a straight line passed almost quite through the whole of them, *showing that the temperature of the boiling point varies in a simple arithmetical proportion with the height*, namely, 549·5 feet for every degree of FAHRENHEIT; so that the calculation of height becomes one of simple arithmetic, without the use of logarithms, or of any table whatsoever.

When I had ascertained this fact, I looked back to DELUC's formula, and found my old conjecture entirely confirmed. Its form is

$$a \log p + C = h,$$

h being the height of a thermometer plunged in boiling water under a pressure p ;

a and C constants. But the first side of this equation is the very form which gives elevations in terms of the barometric pressure. Hence the boiling temperature varies as the height. In other words, the pressure varies in a geometrical ratio, when the temperature of boiling water varies uniformly; but the pressure varies geometrically when the heights above the sea vary uniformly: hence the heights vary uniformly with the boiling temperatures.

It is very singular that so elegant and simple a result should have escaped every writer on the subject (so far as I know); even DELUC himself who proposed the logarithmic law, and WOLLASTON, who unawares adopted the true law as a first approximation, and then took a wrong one.*

It is not to be supposed that the coincidence appears close, because the observations are not accurate enough to test it. Of seven observations between 195° and 210° , no one differs 20 feet of elevation from the mean line,—a quantity corresponding to $\frac{3}{20}$ of a degree, an amount which can by no means be considered as being beyond the errors of observations; and the small errors $\pm$ are well distributed throughout. On the contrary, when the tensions of vapour, from DALTON'S Table, are projected beside them, as in the dotted curve of the figure, not only do they lie wholly above the line, but these tensions cannot be represented (when treated as representing barometric heights) as a straight line at all. They have a manifest curvature convex upwards. In short, as is well known, the tensions of steam cannot be represented by a geometrical progression in terms of the temperature; but when water boils in the free air, *the pressures are then exactly in geometrical progression.*

I never saw any ground for believing that the two laws must be the same. Our theory of vapours is not sufficiently perfect to admit of our drawing any such conclusion. Indeed I cannot help thinking that the influence of the pressure of the *air* upon the elasticity of *nascent steam*, is a fact not easily reconciled with DALTON'S theory of the pressure of elastic fluids. It is one thing to ascertain the elasticity of steam of maximum density which water of a given temperature can yield, and it is another to ascertain under what pressure of air water will yield steam of a given temperature. In practice I have observed the temperature of the boiling water, and not of the steam. The construction of the apparatus required this. But by moving the furnace to a side, so as to prevent the flame from disengaging the steam immediately under the thermometer, I have found the indications as steady

* He says,—“ Having occasion last summer of visiting Caernarvon, which would afford an opportunity of trying the instrument on the known height of Snowdon, and *being aware that in 3550 feet the variations of the boiling temperature were not to be considered uniform, as they might in small elevations*, on which alone I had before tried the experiment, I wished to provide myself previously with a table for making the necessary correction, and from Dr URE'S paper was supplied with data for calculation.”—Phil. Trans. 1820, p. 295. The table given from URE'S law of tensions gives a gradually increasing number of feet, corresponding to every degree that the thermometer falls.

as I believe can be got in any other way. The mass of the water and also of the thermometer favours this.

But I had a farther test of the exactness of the arithmetical progression above established, and that as severe as could well be proposed. It was to compare DE SAUSSURE'S observations on Mont Blanc and the pressure there observed with the result of my formula. But first, it was necessary to correct the zero point of his instrument, and to render it comparable to mine. DE SAUSSURE'S boiling point, 80° of REAUMUR, or 212° of FAHRENHEIT, was adjusted at 27 French inches, or 28.777 English.

At that pressure my thermometer (ADIE) shews 210.58 F. DE SAUSSURE'S stood, therefore, $1^{\circ}.42$ F. higher than mine. Now, on the top of Mont Blanc, the barometer stood at 17.133 English inches.

The boiling temperature by DE SAUSSURE was	. . .	187°.234 Fahr.
Reduced to ADIE,		185°.814 ...
But the boiling point of ADIE'S thermometer, with the barometer at 30 inches, is		212°.62
Subtract		185°.81

At Mont Blanc, below boiling point at 30 inches,	26°.71
By GALBRAITH'S Tables, 30.000 inches =	29228 feet.
27.133 —	= 14593

Height uncorrected for temperature, 14635

Now, by the proportion found empirically above,

Height uncorrected for temperature = $26.71 \times 549.5 = 14677$ feet,—a coincidence really surprising.

I have already stated, that DE SAUSSURE found DELUC'S formula to conform accurately to his observation on Mont Blanc. It may therefore be concluded, that DELUC'S formula and mine agree closely. In fact, if we take its conversion into English measures, as given by Dr HORSLEY,*

$$\frac{99}{8990000} \log z - 92.804,$$

which gives the boiling point, in degrees of Fahrenheit, *reckoned from 32°* , z being the height of the barometer in *tenths of an English inch*, we find that this gives

544.7 English feet of ascent for 1° Fahr.

Practically, I consider it sufficient to find the difference of height, in feet, between two stations, to multiply the difference of the boiling points by 550, and then correct as in barometric observations for the temperature of the air.

* Phil. Trans., vol. lxiv., p. 226.

If the barometer at one station is to be compared with the boiling point at another, the simplest way is to find what elevation the barometer expresses, compared to an imaginary station, where the barometer stands at 30 inches, the boiling point at 212° . Then the height of the station, where the thermometer has been observed, above the imaginary station, is found by the preceding rule.

For example: The corrected boiling point on the *Col d'Erin* between Evolena and Zermatt, in the Vallais, on the 19th August 1842, was $191^{\circ}.93$, the external thermometer 34° , the barometer (English) at Geneva was 28.73, and the temperature 72° , required the height.

Then, by GALBRAITH'S table, for 30 inches,	.	.	.	29228 feet.
28.73 ...	.	.	.	28098
				<hr/>
Difference,	.	.	.	1130

Consequently, supposing the *atmospheric temperature* 32° , the barometer stood at 30 inches, at a level 1130 feet below Geneva. The boiling point at the upper station was $20^{\circ}.07$ below 212° . The Col d'Erin was, therefore, $20.07 \times 549.5 = 11028$ feet above that imaginary station, or 9898 feet above Geneva. Corrected for temperature, this gives 10377; and Geneva being 1343 feet above the sea, the height of the Col d'Erin is 11720 feet.

This is purposely given as a complex case; but let us suppose that the boiling point, at the level of the sea, is assumed to be 212° , then the approximate height of the Col d'Erin is $549.5 \times 20^{\circ}.07 = 11028$ feet; and supposing the mean temperature of the column 54° , the height will be 11586 feet above the sea.