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XX. *An Alternating Current Bridge Method of Comparing Two Fixed Inductances at Commercial Frequencies.* By T. PARNELL, M.A., Lecturer in Physics, University of Queensland.

COMMUNICATED BY PROF. T. R. LYLF.

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THE simple Maxwell inductance bridge cannot be used conveniently for the comparison of fixed inductances on account of the tedious double balance necessary, though by reason of its simplicity and symmetry the bridge has much to recommend it.

By making use of a current detector whose deflections depend on the component of the current in quadrature with the E.M.F. applied to the bridge, it is possible to make the condition for no deflection depend either chiefly on the inductances or chiefly on the resistances.

The arrangement of the bridge is shown in Fig. 1. In series

FIG. 1.

with the bridge is placed either a non-inductive resistance, R , or a condenser, K . If R , r and s are all large compared with p , q , g , $P\omega$ and $Q\omega$, it will be shown that, with R in circuit, the balance depends chiefly on the inductances P and Q , and that with K in circuit the balance depends chiefly on the resistances.

With R in circuit, s is adjusted for no deflection; this gives an approximate balance for the inductances. Then, with K in circuit, q is adjusted for no deflection; this gives an approximate balance for the resistances. The process is repeated, and after a few balances values of s and q are obtained that

give a balance with either R or K in circuit, in which case the bridge is completely balanced, and

$$\frac{P}{Q} = \frac{p}{q} = \frac{r}{s}.$$

The first few balances need only be made roughly, and the whole process can be carried out very quickly; when the apparatus is adjusted, an unknown inductance can be balanced against a standard to one part in a few thousand in a few minutes.

As detector a sensitive moving-coil galvanometer and rotating commutator may be used. In this case the commutator is mounted on the axle of the small alternator or rotary converter that supplies the bridge current, and the brushes of the commutator are adjusted till no deflection is produced when the E.M.F. used on the bridge is applied through the commutator to a high non-inductive resistance in series with the shunted galvanometer, thus showing that the commutator is in quadrature with the E.M.F. When the commutator is thus adjusted the deflection of the galvanometer is approximately proportional to the component of the current in quadrature with the E.M.F. applied to the bridge.

A more convenient detector is the Sumpner reflecting electro-dynamometer, manufactured by Paul. This instrument is fully described in a Paper by Sumpner and Phillips.* The suspended coil moves in the field of a laminated iron electromagnet excited by a field winding with a large number of turns; the resistance of this field winding is small compared with its reactance, and therefore the total flux through the field coils is approximately in quadrature with the applied E.M.F. The flux density is sensibly constant throughout the air-gap and proportional to the total flux. When, therefore, the moving coil is connected across the bridge, and an E.M.F. in phase with the E.M.F. used for the bridge is applied to the field coils, the mean torque on the moving coil is approximately proportional to the component of the current in quadrature with the bridge E.M.F.

If the mutual inductances between the various conductors, and also their capacities and the inductances of R , r and s , are negligible, the final balance is independent of wave-form and frequency. The preliminary balances depend on wave-form and frequency. An approximate knowledge only of the con-

* "Proc." Phys. Soc., Vol. XXII, p. 395. "Phil. Mag.," August, 1910, p. 309.

ditions for these preliminary balances is required; an approximate calculation of the current through the detector, therefore, is sufficient, and it may be assumed with a sufficient degree of accuracy that the mean torque on the moving coil is proportional to the component of the current in quadrature with the bridge E.M.F.

FIG. 2.

Consider the bridge as in Fig. 2

For continuous currents $x = \frac{qr - ps}{\Delta} \cdot E$,

where $\Delta = bg(p+q+r+s) + b(p+r)(q+s) + g(p+q)(r+s) + pq(r+s) + rs(p+q)$.

For the bridge, Fig. 3, with the harmonic E.M.F., $e = E \sin \omega t$,

FIG. 3.

if the inductance of g is small compared with P and Q ,

p becomes the operator $p + i\omega P$

q " " $q + i\omega Q$

b " " $R - \frac{i}{\omega K}$

and if R, r, s are large compared with $p, q, g, P\omega$ and $Q\omega$, Δ becomes

$$\begin{aligned} (p+i\omega P)\left(r+R-\frac{i}{\omega K}\right)s + (q+i\omega Q)\left(s+R-\frac{i}{\omega K}\right)r \\ + \left(R-\frac{i}{\omega K}\right)rs \text{ approximately} \\ = M+iN, \end{aligned}$$

where $M = p(rs+Rs) + q(rs+Rr) + Rrs + \frac{1}{K}(sP+rQ)$

$$N = \omega P(rs+Rs) + \omega Q(rs+Rr) - \frac{1}{\omega K}rs.$$

$$\therefore x = \frac{(qr-ps) + i(\omega Qr - \omega Ps)}{M+iN} \cdot E.$$

If $A = \frac{q}{p} - \frac{s}{r}$, i.e. the "resistance error," and $B = \frac{Q}{P} - \frac{s}{r}$, i.e. the "inductance error,"

$$\begin{aligned} x &= \frac{prA + i\omega PrB}{M+iN} \cdot E \\ &= \frac{prMA + \omega PrNB + i(\omega PrMB - prNA)}{M^2 + N^2} \cdot E. \end{aligned}$$

The mean torque on the moving coil is proportional to the component of x in quadrature with E , i.e. to

$$\frac{\omega PrMB - prNA}{M^2 + N^2}.$$

$$\therefore \text{Mean torque on moving coil} = XB - YA$$

where

$$\begin{aligned} \frac{X}{Y} &= \frac{\omega PM}{pN} \\ &= \frac{\omega P \left\{ p(rs+Rs) + q(rs+Rr) + Rrs + \frac{1}{K}(sP+rQ) \right\}}{p \left\{ \omega P(rs+Rs) + \omega Q(rs+Rr) - \frac{1}{\omega K}rs \right\}}. \end{aligned}$$

Consider now the two cases for preliminary balances :—

Case I. (inductive)—

$$\begin{aligned} \frac{1}{K} &= 0 \\ \frac{X}{Y} &= \frac{\omega P R r s}{\omega p \{ P(rs+Rs) + Q(rs+Rr) \}} \text{ approx.} \\ &= \frac{R r s}{p r s + p R s + \frac{Q}{P}(p r s + p R r)} \end{aligned}$$

$$\therefore \frac{X}{Y} \text{ is positive and fairly large.}$$

Case II. (resistance)—

$$R=0$$

$$\begin{aligned} \frac{X}{Y} &= \frac{\omega P \left\{ (p+q)rs + \frac{1}{K}(sP+rQ) \right\}}{p \left\{ rs\omega(P+Q) - rs\frac{1}{\omega K} \right\}} \\ &= - \frac{\omega^2 P \left\{ (p+q)rs + \frac{1}{K}(sP+rQ) \right\}}{\frac{1}{K} p r s} \quad \text{approx.} \end{aligned}$$

$\therefore \frac{X}{Y}$ is negative, and small for ordinary values of P , Q and ω .

The above assumes an harmonic E.M.F., but since the final balance is independent of frequency and wave-form, the effect of the higher harmonics in the case of the E.M.F. obtained from an ordinary machine may be neglected, for it is clear from the above expressions that the effect of the higher harmonics in the preliminary balances will be much less than that of the fundamental.

A more rigorous treatment leads to the same result as the simpler theory given above.

For no deflection the mean torque is zero—

$$\text{i.e.} \quad XB - YA = 0 ;$$

$$\text{or} \quad X \left(\frac{Q}{P} - \frac{s}{r} \right) - Y \left(\frac{q}{p} - \frac{s}{r} \right) = 0.$$

If this preliminary balance is obtained by varying s with R alone in circuit (Case I.), X and Y are of the same sign, and X is fairly large compared with Y ; hence the balance depends chiefly on the inductances. If in this balance q is too small for a resistance balance A is negative, and therefore B is negative, and the value of s is too great. Similarly if the value of q is too great, the value of s obtained will be too small.

If a balance be now obtained by varying q (or p) with K alone in circuit (Case II.), X and Y are of opposite sign and X is fairly small compared with Y ; hence the balance depends chiefly on the resistances. If in the first balance q is too small, a value of s too great for a true inductive balance is obtained—i.e., in this balance B is negative, and therefore A is positive, and the value of q is too great. The next inductive balance will, therefore, give too small a value of s , which means

that the next resistance balance will give too small a value of q , and so on.

Two series of values of s and q respectively are thus obtained whose terms are alternately too great and too small, and these terms rapidly converge to final values corresponding to a balance of both inductances and resistances—

$$i.e. \quad \frac{P}{Q} = \frac{r}{s} = \frac{p}{q}.$$

The following experimental results serve to illustrate the use of the method and to show the agreement of the above theory with experiment.

In these measurements P was a standard inductance and Q the coil to be measured. An inductive balance was first obtained by varying s , then a resistance balance by adding resistance to the arm containing Q , or to the arm containing P , by means of a dial box and graduated non-inductive rheostat, which could be switched into either arm.

Column I. gives the series of values of s for inductive balances; Column II. the corresponding values of the added resistances.

One example of the use of the rotating commutator is given: the other measurements were made with the electro-dynamometer as detector.

Coil A.—Resistance, 15.6 ohms; $P=100$ M.H., $R=2,000$ ohms, $K=\frac{1}{2}$ M.F., $r=2,000$ ohms, $p=34.8$ ohms

Using electro-dynamometer.		Using rotating commutator.	
I. s .	II. Resistance added to arm Q .	I. s .	II. Resistance added to arm Q .
3,480.0	0	3,300.0	0
3,040.0	45.0	3,060.0	43.0
3,120.0	37.0	3,108.0	37.0
3,099.0	38.7	3,101.0	38.22
3,102.6	38.39	3,102.0	38.04
3,102.3	38.425	3,101.7	38.06
3,102.4	38.42	...	...
...		...	
$Q=155.12$ M.H.		$Q=155.08$ M.H.	

Coil B.—Resistance, 0.73 ohm ; $P=10$ M.H., $R=2\,000$ ohms, $K=\frac{1}{2}$ M.F., $r=2,000$ ohms, $p=7.4$ ohms.

I. s.	II. Resistance added to arm Q.
	0
1,000.0	3.0
800.0	2.2
860.0	2.45
846.0	2.36
853.0	2.405
850.0	2.39
850.1	
$Q=4.250$ M.H.	

Coil C.—Resistance, 0.3 ohm ; $P=10$ M.H., $R=2\,000$ ohms, $K=\frac{1}{2}$ M.F., $r=2,000$ ohms, $p=7.4$ ohms.

I. s.	II. Resistance added to arm Q.
	0
129.0	0.13
119.0	0.09
121.4	0.105
120.0	0.101
120.6	0.102
120.5	
$Q=0.602$ M.H.	

With the larger inductances the final balance could easily be made to 1 in 10,000, but this degree of accuracy cannot be expected in the actual results, as the non-inductive resistance boxes were specified as correct to 1 in 1,000 only and no small corrections were made.

Measurements of a number of coils with various values of the voltage and of R , K and r gave results consistent within 1 in 1,000. As a further test of the accuracy of the method, two coils were placed a considerable distance apart, and their in-

ductances (including the leads) measured in situ, singly and in series. The results of two such measurements are given below.

Inductance of A	155.12 M.H.
Inductance of A'	158.02 M.H.
Sum of inductances	313.14 M.H.
Inductance of A and A' in series	313.24 M.H.

Inductance of B	4.250 M.H.
Inductance of B'	3.942 M.H.
Sum of inductances	8.192 M.H.
Inductance of B and B' in series	8.190 M.H.

Reversing the connections of A' or B' did not appreciably affect the final balances.

Owing to the presence of R or K , it is necessary to use fairly high voltages in order to keep up the sensitiveness. The value of R need not be known with much accuracy, so that any reasonably non-inductive resistance may be used, and as s and s' are large compared with p and q , the voltage is limited only by the permissible current through the inductance coils. In the measurements made, voltages from 10 to 85 were used, and the higher voltages were found the more satisfactory.

The mutual induction between R and the rest of the bridge is liable to produce considerable error; it is necessary, therefore, to keep R some little distance away.

Leakage to earth and electrostatic capacity effects gave some trouble. The detector was kept permanently in circuit, and the alternating-current supply connected to the two terminals of the bridge through a pair of tapping keys; the apparatus was manipulated until putting down either key alone gave only a negligible deflection. In obtaining a balance the two keys were put down simultaneously. The "false zero" obtained by keeping the detector in circuit fluctuates to some extent, but little difficulty was experienced from this cause, as it was always sufficiently steady to enable the final balance to be made accurately.

When the rotating commutator was used it was found that the best results were obtained by taking the bridge current from a small transformer, with secondary well insulated and shielded from primary connected to the machine. The galvanometer was connected to the brushes bearing on the split part of the commutator, and the bridge to the brushes on the rings. When the core of the transformer, the frame of the machine and one of the rings of the commutator were

earthed, the stray effects due to leakage, capacity, &c., were reduced to a negligible amount.

In setting the brushes of the commutator the current from the transformer was passed through the commutator and the suitably shunted galvanometer in series with R , and the brushes adjusted for no deflection. This allowed for any change of phase occurring when the bridge current was taken from the transformer. It was found that great accuracy in the position of the brushes was not necessary. If the setting was not sufficiently exact, the defect was noticed at once, as the values of s and q did not alternate in the manner described above.

The use of the rotating commutator was found much more troublesome than that of the electro-dynamometer, and the results obtained were somewhat less consistent.

When the electro-dynamometer was used as detector, the currents for the bridge and for the field were taken from a small transformer having two secondaries well insulated and shielded from one another. It was found that with this arrangement electrostatic and capacity effects could be reduced to a negligible amount by earthing the core of the transformer and one terminal of the moving coil. Voltages from 40 to 75 volts were employed on the field coils.

In conclusion I wish to express my thanks to Prof. Lyle for his kindness in reading through this Paper and offering valuable criticisms. It was at his suggestion that the simpler proof given here was substituted for a more elaborate and cumbersome one employed originally.

NOTE.—Sumpner and Phillips mention the use of the electro-dynamometer for the direct comparison of inductances, but they state that the resistance balance is unnecessary. This does not agree at all with the results obtained in this Paper.

ABSTRACT.

The Paper describes a method of avoiding the troublesome double adjustment required in Maxwell's method of comparing inductances. A current detector, whose deflections depend on the component of the current in quadrature with the E.M.F., is employed, which makes it possible to arrange that the condition for no deflection depends chiefly either on the inductances or resistances. In series with the bridge is placed either a non-inductive resistance or a capacity. In the first case the balance depends chiefly on the inductances, and in the second case on the resistances. A few alternate repetitions of the two adjustments suffice to balance the bridge, both for resistances and inductances. As detector a sensitive moving coil galvanometer

in conjunction with a commutator, or a Sumpner electro-dynamometer, may be employed; the latter proved more satisfactory.

DISCUSSION.

Dr. D. OWEN regarded the Paper as a valuable contribution to the subject of alternate-current bridge measurements. To secure zero current through the detector of a bridge to which an alternating voltage is applied it is necessary to satisfy two conditions. It has been claimed that by the use of the alternate-current moving-coil galvanometer inductances or capacities might be accurately determined without the necessity of a double adjustment. The claim is a mistaken one. The mere attainment of zero deflection, implying zero torque on the moving coil, by itself cannot give the information sought, for neither of the conditions for zero current in the detector is necessarily satisfied. The author of the present Paper shows a way out of the difficulty—namely, by applying alternately voltages to the bridge which are approximately—but not necessarily exactly—in quadrature with each other, the voltage applied to the field coil remaining unchanged. The attainment of a balance in *both* of these cases justifies the inference that the galvanometer current due to the voltage impressed on the bridge is zero, and that the two fundamental conditions of balance are simultaneously reached. A Paper by Weibel in the Bureau of Standards "Bulletin" (February, 1917) deals very fully with this same problem. Weibel points out how the tests may be made also by varying the phase of the voltage applied to the field coil, voltages in quadrature being secured by the aid of a phase-shifting device. The present author applies his method solely to the Maxwell bridge for comparing two self-inductions. There seems, however, no good reason for this exclusive treatment. The mode of procedure advocated is applicable to all the bridge methods, and those methods in which the two conditions of balance are independently attainable will still have the advantage in practice over those, such as the Maxwell bridge of the Paper, in which the opposite is the case.

Mr. T. SMITH pointed out that in double adjustments of this nature, if the wrong adjustment were made first, the successive errors got larger instead of smaller. It would have been helpful if the author had extended his mathematics somewhat, and given some criterion (*e.g.*, some function of the various quantities involved being either positive or negative) as to which of the adjustments—inductance or resistance—should be made first.

Dr. BRYAN asked if the method could be used for small inductances of a few micro-henries?

Prof. BOYS asked if inductances with iron cores could be measured, or only air cored coils, in which the sine law was nearly fulfilled?

Dr. SUMPNER said, in reply, that the author had not gone in great detail into the range of usefulness of the method. He might point out, however, that the sine law was not essential to work with the electro-dynamometer. The author did not appear to have experienced any difficulty about the order of the adjustments; though in some cases this might doubtless cause considerable trouble. He admitted the validity of the author's criticism of the Paper by himself and Phillips; and agreed that the double adjustment was essential. At that time, however, he was principally concerned with describing the instrument, and was not using small inductances in which the resistance effect was important.

Mr. A. CAMPBELL communicated the following: Mr. Parnell's system is interesting, and may be of value in other methods where two adjustments are necessary to obtain a balance. In connection with his commutator method it may be mentioned that L. T. Robinson applied a commutator in the galvanometer circuit in a somewhat similar way for testing transformers by inductance methods ("Trans." American Institute of Electrical Engineers, p. 1040, Vol. XXVIII, June 30, 1909).