On the Law of the Electromagnet and the Law of the Dynamo

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XXIX. *On the Law of the Electromagnet and the Law of the Dynamo.* By SILVANUS P. THOMPSON, D.Sc., B.A.*

IT is remarkable, considering the vast amount of research that has been made, that authorities in physics are in so little agreement as to what the law of the electromagnet is. Upon the one topic that is all-essential to the electric engineer the professed physicist is either silent, or gives mathematical rules that are hopelessly wide of the observed facts, and based in most cases on no consistent theory. The expressions given by Weber† for the relation between the magnetizing force and the induced magnetism are an exception; but they are unmanageable in the extreme, and have led to no useful results. The expression given by Lamont‡ is also based on a certain consistent theory, but it appears to be entirely unknown both to physicists and to electric engineers, and is to be found only buried in the depths of Lamont's own treatise. The two mathematical formulæ which are most often given in treatises on physics as representing the relation between the magnetizing current and the induced magnetism of the electromagnet are the following:—

Lenz and Jacobi's Formula.—According to the experiments of these early investigators (1839), the magnetism of the electromagnet is simply proportional to the strength of the current and to the number of turns of wire in the coil. This may be written as

$$m = kSi,$$

where i is the strength of the current, S the number of turns in the coil, k a constant depending on shape and quality of iron, and m the strength of the pole. Joule (1839) showed this rule to be incorrect, and that as the iron became saturated, m ceased to be proportional to Si .

Müller's Formula.—Müller gave an expression of the form:—

$$m = b \tan^{-1} \frac{Si}{a},$$

* Read November 14, 1885.

† Weber, *Elektrodynamische Maasbestimmungen*, p. 572. See also Maxwell, 'Electricity and Magnetism' (2nd edition), vol. ii. p. 78.

‡ Lamont, *Magnetismus*, p. 41.

where b and a are constants, depending on the form, quantity, and quality of the core and on the form and disposition of the coils. Very similar formulæ have been used by Von Waltenhofen, Dub, Cazin, and Breguet. They are purely empirical, fail to represent the facts with accuracy, and have moreover the defect of not lending themselves conveniently to use in the equations of electromagnetic motors, dynamo-electric machines, and other kinds of electric machinery.

More recently another formula has been coming into use, and is known—for reasons which will presently appear—as Frölich's formula: it is written

$$m = \frac{i}{a + bi},$$

where a and b are again constants, b being the reciprocal of the maximum value of m . This formula, like the preceding, is a perfectly empirical expression not based in itself upon any physical theory, and it accords with the observed facts of the electromagnet better than any of the preceding formulæ. Frölich, who in 1878 was using merely Lenz and Jacobi's formula*, appears to have adopted this expression in 1880†, in consequence of the experiments which he had made with dynamo-machines. A similar formula had been used twenty-five years before by Robinson‡ to express the lifting-power of an electromagnet, and similar formulæ have been used by Oberbeck§, Fromme||, Clausius¶, Ayrton and Perry**, and Rücker††.

Now it must be obvious that since almost the whole of electric technology deals with the electromagnet and its applications, it is of utmost importance to know what the true law of the electromagnet is. If the technologist is left to the mercy of the physicist, he has to choose between formulæ

* Frölich, *Die Lehre von der Elektrizität und dem Magnetismus*, p. 237.

† Frölich, *Berl. Berichte*, Nov. 18, 1880, p. 973; also *Elektrotechnische Zeitschrift*, 1881, April, p. 139.

‡ Robinson, *Trans. Roy. Irish Academy*, vol. xxii. p. 1, 1855.

§ Oberbeck, *Pogg. Ann.* cxxxv. pp. 74-98, 1868.

|| Fromme, *Pogg. Ann.* clv. p. 305, 1878.

¶ Clausius, *Wied. Ann.* xx. p. 307, 1883.

** Ayrton and Perry, *Journ. Soc. Electr. Eng. and Electr.* xii. p. 317, 1883.

†† Rücker, *Phil. Mag.* ser. 5, xix. p. 463 (June 1885).

that are based on physical conceptions and which are utterly unmanageable for his purpose, and formulæ that are purely empirical expressions and have no rational basis.

But the study of the modern dynamo-electric machine has brought us to a point that makes some decision a necessity. The entire action of the dynamo is dependent on the magnetizing action of the electric current, and the law of the dynamo is, in the very nature of things, not to be discovered, save by the discovery of the law of the electromagnet. It is indeed extraordinary that such able physicists as Mascart and Angot, Mayer and Auerbach, Schwendler, and Herwig sought in vain for the true law to connect the electromotive force of the dynamo with its speed, the resistance of its circuit, and the constants of its construction. Hopkinson came nearest to the mark when in 1879 he described the properties of the dynamo in terms of those of a certain curve, which we now call the characteristic curve of the machine; but he did not state the law of the dynamo algebraically, and apparently he accepted as true Weber's formula for the electromagnet.

But the law of the dynamo is now known, and expressions have been deduced which are found to agree with the utmost accuracy with the observed facts. The discovery is almost entirely due to Dr. Frölich*, whose papers, published at intervals from 1880 to 1885, are in their way already classical. The results first obtained by Frölich related exclusively to that form of dynamo in which the armature-coils and the field-magnet coils are both included in the main circuit—the series-dynamo—though they have been further extended by the present writer, and by others, including Frölich himself, to other forms of machine. A very brief resumé of Frölich's research will explain his method of arriving at the law.

His theory is based upon (1) Faraday's law of induction, (2) Ohm's law, (3) a curve, called by him the current-curve, expressing certain results of experiments made on the series-wound dynamo.

Following Faraday's principle, the induced electromotive

* Frölich's chief papers are as follows:—*Berl. Berichte*, 1880, p. 978; *Elektrotechnische Zeitschrift*, vol. ii. p. 134 (1881); *ib.* vol. iii. pp. 69 and 113 (1882); *ib.* vol. iv. pp. 60, 67, and 71 (1883); *ib.* vol. vi. pp. 128, 139, 227 (1885).

force E will be proportional to the speed of the machine n and to a quantity M which Frölich calls the "effective magnetism," and which is itself proportional to the effective area of the armature-coils and to the intensity of the magnetic field. Writing this as an equation, we have

$$E = nM,$$

and, by Ohm's law,

$$E = iR,$$

which gives us for the current i ,

$$i = \frac{n}{R}M,$$

or

$$\frac{i}{M} = \frac{n}{R}.$$

Since M is itself a function of i , and not primarily of n or R , it is clear that i is itself a function of n/R , and we may write

$$\phi(i) = \frac{n}{R}.$$

This function Frölich set himself to investigate by pure experiment without making any hypothesis whatever. He determined the values of i at various speeds and with various resistances, and plotted out the results as a curve, values of i being taken as ordinates and values of n/R as abscissæ. This curve (see figure) he termed the "current-curve." It shows itself to be very nearly a straight line, which, however, does not pass through the origin.

The portion which departs from the line is difficult of observation because it is only obtained when either n is very small or R very large, that is to say when the dynamo is scarcely able to excite its magnets. Neglecting this unstable state, and dealing only with that part of the curve which relates to the machine as it is when in action, it is clear that the relation between the two variables may be expressed as

$$bi = \frac{n}{R} - a,$$

where b and a are constants ; giving us as the equation of the series-dynamo :—

$$i = \frac{1}{b} \left(\frac{n}{R} - a \right).$$

Substituting for n/R its equivalent i/M , Frölich then obtained from the law of the dynamo the following expression for the “effective magnetism” :—

$$M = \frac{i}{a + bi},$$

an expression which he himself appears, in his original paper, to have considered as a mere interpolation-formula. It is, however, this expression which is generally referred to as Frölich’s formula for the electromagnet.

It was in 1883 that the author of this paper, in working at the problem of the dynamo, realized the significance of this result of Frölich’s. He had been using as the fundamental law

$$E = 4nAH$$

to express the average value of the electromotive force developed by rotating in a uniform magnetic field of intensity H , an armature whose total effective area was A ; and requiring a formula to connect H with the exciting current, he was considering the various equations of Müller, Weber, and others when he became aware that Frölich’s expression furnished what was needed ; and, finding it necessary for the case of compound-wound machines to express the excitation in terms of ampère-turns rather than of ampères, he rewrote the formula in the following way :—

$$H = GSi \frac{k}{1 + \sigma Si},$$

where, as before, Si is the number of ampère-turns, G a geometrical coefficient, k the coefficient of magnetic permeability in the initial stage, and σ a small saturation-coefficient dependent on the quantity and form as well as on the quality of the iron of the core.

Writing similarly Zi_s for the ampère-turns of a shunt winding, the expressions for H became

$$H = GZi_s \frac{k}{1 + \sigma Zi_s}$$

for shunt-wound machines, and

$$H = G(Si \pm Zi_s) \frac{k}{1 + \sigma(Si \pm Zi_s)}$$

for compound-wound machines.

Inserting these values in the fundamental equation at once gave the following expressions :—

Series dynamo.

$$E = \frac{1}{\sigma} \left\{ 4nAGk - \frac{\Sigma R}{S} \right\}$$

$$i = \frac{1}{\sigma} \left\{ \frac{4nAGk}{\Sigma R} - \frac{1}{S} \right\}$$

Shunt dynamo.

$$E = \frac{1}{\sigma} \left\{ 4nAGk - \frac{r_a r_s + Rr_s + Rr_a}{ZR} \right\}$$

Compound dynamo (at critical speed for regulation).

$$e = \frac{1}{\sigma} \left\{ \frac{r_s(r_a + r_m)}{S(r_s + r_a)} - \frac{r_s}{Z} \right\} \text{ (short shunt),}$$

or

$$e = \frac{1}{\sigma} \left\{ \frac{r_s(r_a + r_m)}{S(r_s + r_a + r_m)} - \frac{r_s}{Z + S} \right\} \text{ (long shunt),}$$

where r_a , r_m , r_s , and R are the resistances of armature, main-circuit coil, shunt-coil, and external circuit respectively, and e the difference of potentials at the terminals of the machine. These results were announced by the author at the Montreal meeting of the British Association, and were published by him in his volume on *Dynamo-Electric Machinery*, as was also the important proposition that the factor $\frac{1}{\sigma}$ (which may be called *the determinant* of the working power of the dynamo, since it appears in every one of the equations, both of current and electromotive force) is that number of ampère-turns which will reduce the effective permeability of the

electromagnet, or rather of the magnetic circuit, to half its initial value. It will be seen that all these synthetical results were new, except that for the current of the series dynamo, which is identical with the expression that had been deduced by Dr. Frölich from the current-curve which represented his experimental results.

Since the publication of the author's treatise, Frölich has further elaborated the equations of the compound-wound machines; and a more generalized form has been deduced by Prof. Rücker in the masterly paper which he communicated to the Physical Society in March last, in which, however, he attributes to Frölich (I know not on what grounds) the general equation for the electromotive force of all kinds of dynamos. Frölich has also verified the adequacy of these formulæ by comparing results calculated from them with actual experiments on compound-wound machines. One example will suffice.

<i>n.</i>	<i>R.</i>	<i>e</i> obs.	<i>e</i> calc.	<i>i</i> obs.	<i>i</i> calc.
850	0·841	127	127	151·0	151·0
853	1·22	133	133	109·0	109·0
855	2·34	139	140	59·9	59·4
850	4·52	138	139	30·3	30·5
850	9·25	136	138	14·7	14·7
850	22·1	137	136	6·2	6·2
845	222·0	133	134	0·6	0·6

Such a series of results proves the law of the dynamo to be fully established. But if the law of the dynamo, so deduced from Frölich's empirical expression for the electromagnet, be true, *then*, to an equal degree of precision, *must the law of the electromagnet be itself true.*

But Frölich's expression is not the expression of any physical law: it is a mere interpolation-formula, destitute of rational significance. What then is the true law of induced magnetism? And how comes it that an expression devoid of physical significance so nearly expresses the physical law?

That question the author believes to have been already

solved by a forgotten investigation of Lamont, published almost without note or comment in his *Handbuch von dem Magnetismus* in 1867. On p. 41 of that work he gives the following equation:—

$$m = \frac{aMx}{M + ax},$$

as a convenient first approximation to another equation based on a new theory of induced magnetism. In the above equation x stands for the magnetizing force, m the induced magnetism, and M the maximum value of the latter. Writing Gk for a , Si for x , and σ for $\frac{a}{M}$, we at once get

$$m = \frac{GkSi}{1 + \sigma Si},$$

which is the author's way of writing Frölich's equation.

The physical theory which led Lamont up to this result is of great interest, the more so since it has been revived during the current year in a more elaborate form by Bosanquet*, who has apparently been led thereto by an independent line of thought, and who has gone much further than Lamont in bringing in the additional consideration of the forces resisting the orientation of the molecules. Lamont's theory is, briefly, that the permeability of the iron diminishes as the permeation increases, being at every stage of the magnetization proportional to the deficit of saturation. He assumes that for every bar there is a certain maximum of magnetization, to which it could only attain under the influence of an infinitely great magnetizing force; and that the permeability of the bar is at every stage of the magnetization proportional to the difference between the actual magnetization and the possible magnetization. This is, in other words, as if in every bar there were room for only a certain limited number of magnetic lines of force, and that, when any lesser number have been induced in it, the susceptibility of the bar to the reception of additional lines is proportional to the room yet left for them in the bar. Lamont's theory is expressed as follows: Let the magnetism present at any stage be called m , and let the maximum magnetism be called M . Then the

* Bosanquet, *Phil. Mag.* ser. 5, xix. p. 85 (Feb. 1885).

amount which the bar can still take up is $M-m$; and it is to this that the permeability dm/dx is proportional. Here x may be understood, as itself proportional to the number of ampère-turns of the exciting current; and we may write

$$\frac{dm}{dx} = k (M-m),$$

where k is a constant depending on the units employed, and on the initial value of the permeability of the magnetic circuit when $m=0$. Integrating we obtain

$$M-m = A\epsilon^{-kx},$$

where A is a constant of integration. But when $x=0$, $m=0$ also; whence $A=M$, giving

$$m' = M(1 - \epsilon^{-kx}),$$

which is Lamont's formula.

Lamont also develops this expression in ascending powers of kx ,

$$m = M k x \left(1 - \frac{kx}{1.2} + \frac{k^2 x^2}{1.2.3} - \dots \right),$$

and he then makes the remark that it may be approximately represented by the simpler, but empirical expression

$$m = \frac{a M x}{M + a x},$$

which is identical in form with the expression known as Frölich's. Now, writing k for a/M ,

$$m = \frac{M k x}{1 + k x},$$

and expanding this in ascending powers, we get

$$m = M k x (1 - kx + k^2 x^2 - \dots).$$

Neglecting the fourth and higher terms, it will be seen that the formulæ are very nearly equal for all small values of kx , and are identical for the value

$$kx = \frac{3}{5},$$

or when the actual magnetism has about .456 of the value it would have under an infinite magnetizing force. For larger values of the magnetizing force the values of m calculated by

the empirical formula are slightly greater than those calculated by the exponential formula.

None of these formulæ render any account of a phenomenon noticed in many cases where an electromagnet is magnetized with gradually ascending magnetic forces, namely, a concavity in the *early* part of the curve of magnetization, the permeability apparently becoming greater after a certain degree of magnetization has been attained. The researches of Chwolson and of Siemens seem to show that this apparent increase in the permeability (which is not observed with descending magnetic forces) is due to non-homogeneity, and to the resistance of some of the molecules to magnetization. Bosanquet's theory may be taken in connection with the recent observations of Rowland, Warburg, Ewing, and Hopkinson on this subject, which, however, is of no great importance here since the apparent maximum of permeability is attained in most cases at a much lower degree of magnetization than that at which the dynamo is actually worked.

The very close agreement between the observed values of current and potential of the dynamo and those calculated by the equations based upon Frölich's formula, proves that nowhere within the working range do the actual values of the magnetization differ sensibly from those calculated by the Frölich formula. That is to say this formula may be taken as *something more than a first approximation* to the true law of the electromagnet for all degrees of saturation of the magnets employed in practice, and may be taken as a first approximation for all degrees of saturation beyond that usually attained.

It is eminently desirable, in the face of these facts, that the imperfect formula of Lenz and Jacobi, as well as that of Müller, should henceforth disappear from the text-books of physics, and that instead thereof the true law of the electromagnet should be stated, either in the exponential form or in the simpler form that is found to be equally true within the range of saturation attained in practice.