A Graphic Method of dealing with Combinations of Co-axial Refracting Surfaces

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XXXI. *A Graphic Method of dealing with Combinations of Co-axial Refracting Surfaces.* By H. STANLEY ALLEN, M.A., B.Sc. *Senior Lecturer in Physics at King's College, London* *.

[ABSTRACT.]

IN this paper a graphic method is given that may be applied to the case of two thin lenses not in contact, or to the case of two co-axial refracting surfaces (as in the case of the thick lens). Further the method may be extended to the combination of any two co-axial systems. It affords a way of finding, by a simple geometric construction, the focal length of the combination, and also the positions of the focal planes and principal planes (planes of unit magnification) of Gauss.

Although in actual experiments we are restricted by the formulæ of the first order to rays making small angles with the axis, it is legitimate for purposes of construction to consider rays making finite angles with the axis. By the choice of two particular rays passing through a definite point in the plane of the geometric centre of two refracting systems, the cardinal points of the combination may easily be determined.

In this abstract the method is applied to the general case of the combination of two refracting systems in any media †. We assume that the positions of the focal points and principal planes of each system are given. We can then deduce the existence and positions of the nodal points, defined by the property that an in-ray through one nodal point is parallel to an out-ray through the other nodal point.

Through F_1' , the second focal point of the first system (fig. 1), draw a line $F_1'E_1$ at right angles to the axis, equal to the second focal length of that system. Draw F_2E_2 similarly through the first focal point of the second system, equal to the first focal length of that system. Let $F_1'E_2$ and F_2E_1 intersect in C. Draw CB at right angles to the

* Read November 27, 1908.

† Drude, *The Theory of Optics*, p. 28.

axis. The point B may be called the "geometric centre" or the "optic centre" of the system. It divides the distance between the components in the ratio of their focal lengths. Hence the image of CB formed by the first system is of the

Fig. 1.

same size as the image of CB formed by the second system. It follows that these images are planes of unit magnification with regard to the combination, in other words they are the principal planes*.

The construction proceeds as follows. Let CE_1 , produced if necessary, meet the first principal plane, H_2 , of the second system in G_2 , and the first nodal plane, N_2 , of that system in L_2 . Let CE_2 , produced if necessary, meet the second principal plane, H_1 , of the first system in G'_1 , and the first principal plane of the second system in P_2 . Then G'_1G_2 is parallel to the axis. Through P_2' , a point in the second principal plane of the second system at the same distance from the axis as P_2 , draw a line parallel to E_2N_2 meeting the axis in F' and G'_1G_2 in K' . Then F' is the second focal point of the combination, and K' , the image of C formed by the second system, lies in the second principal plane H' of the combination. The second focal length of the combination is given by $H'F'$ or by N_2L_2 .

* "If we imagine a circular aperture or pupil to be placed at the geometric centre then the image of it formed in the first principal plane by L_1 (the first lens) will be exactly equal to the other image of it formed in the second principal plane by L_2 (the second lens). The two principal planes are in this way shown to be planes of unit magnification relatively to one another."—H. Dennis Taylor, "A System of Applied Optics."

I am indebted to my former colleague, Mr. J. Nicol, for drawing my attention to the importance of the optic centre in the general case.

To verify these results trace the course of the rays CE_2 , CF_2 through the system from left to right.

The in-ray corresponding to $F_1'C$ passes through G_1 and is parallel to the axis. The out-ray passes through P_2' and is parallel to E_2N_2 , for it must pass through the image of E_2 , and this image is formed at infinity since E_2 lies in the first focal plane of the second system. This out-ray cuts the axis in F' , the second focal point of the combination.

The ray CF_2 emerges through G_2' in a direction parallel to the axis. The intersection of the two out-rays gives the position of K' the image of C , in the second principal plane of the combination. Thus H' , F' are determined and $H'F'$ is the focal length. But by similar triangles

$$\frac{F'H'}{H'K'} = \frac{F_2N_2}{F_2E_2} = \frac{F_2N_2}{F_2H_2}.$$

Therefore

$$F'H' = \frac{F_2N_2}{F_2H_2} \times H_2G_2 = N_2L_2.$$

The first focal point, the first principal plane, and the first focal length can be determined in a similar way.

In the more common case in which both systems are in the same medium, as for two lens systems in air, the construction is still simpler, since the nodal points and principal points coincide. In this case $F_2N_2 = F_2H_2 = F_2E_2$ so that E_2N_2 makes an angle of 45° with the axis*.

In fig. 2 we have the application of the method to the case of two co-axial refracting surfaces a finite distance apart, as in the thick lens. The principal planes of the first refracting surface coincide at A_1 , the nodal points coincide at N_1 the centre of curvature.

The case of two thin lenses separated by a finite interval is shown in fig. 3. It is to be noticed that P_1K , P_2K' make

* We may note that if a stop is put at B , the geometric centre of two lens systems in air, the images of this point formed by the two systems are orthoscopic points, for in this case the images coincide with the nodal points of the combination. (Drude, 'The Theory of Optics,' pp. 64-65).

angles of 45° with the axis, so that the focal length, HF , of the combination, is equal to HK or to A_1G_1 .

Fig. 2.

Interesting illustrations of the method are found in eye-pieces formed of two lenses, such as those of Ramsden or Huyghens.

It appears to the writer that the method here given may be of value for investigating the way in which the properties of a combination are affected by making given alterations in

Fig. 3.

the character or separation of the components. It may also be of use in enabling non-mathematical students to realise the existence of the cardinal points for a complicated system of refracting surfaces.

DISCUSSION.

Dr ECCLES remarked that the Author's treatment of refracting surfaces was limited to spherical surfaces and to the direct incidence of

pencils of infinitesimal aperture. If the treatment were extended to the discussion of oblique pencils and the construction of caustics it would be very valuable. The Author's method, as developed to the present, consisted of applying the well-known graphical construction for a harmonic mean, to the successive lenses or spherical surfaces met with in the coaxial system of lenses or surfaces under consideration. Now the harmonic construction just mentioned is a particular case of the general graphical connection between two collinear projective (or homographic) ranges of points; and therefore the speaker preferred, in his own teaching of geometrical optics, to use general projective constructions rather than this harmonic construction. As a fact, the general projective method would be felt to be the easier of the two, by anyone familiar with the fundamental principles of projective geometry, or, as it is sometimes called, "geometry of position." From the point of view of projective geometry the object-points and the image-points on the axis of a refracting (or reflecting surface), since they have a true one-one correspondence (as shown by their algebraic homographic connection), constitute two collinear projective ranges of points. Two such ranges can be graphically correlated, it is known, by at most three projections. The rays used in the first and last of these projections may be interpreted optically as rays of light through a source of light and through its image. The intermediate projection gives the central point on which the Author lays stress. If this point be taken at infinity the planes of unit magnification become defined. Since any number of successive projections at successive surfaces leave the ultimate range of image-points homographic with the original range of object-points, it is proved that any lens system whatever of the kind specified by the Author can be summarised graphically by three projections. Moreover, no graphical construction can be obtained which is not comprehended in the general projective method just sketched. In treatises on projective geometry the properties of these projective ranges and pencils are demonstrated, and they need merely be translated into optical language to give many beautiful and powerful graphical optical theorems. There is reason to think that caustics can be approached along these lines. The speaker expressed the hope that the Author would devote still further effort to these much desired simplifications of geometrical optics.

Dr A. RUSSELL congratulated the Author, and indicated a simple graphical method of determining the focal length of two thin lenses in contact when the focal length of each was known.

Prof. C. H. LEES thought the graphical method described by the Author was an improvement upon algebraic methods.