To summarize the points involved: first, we have four definite rules for teaching beginners how to attack the elementary problems; second, problems involving simultaneous equations yield to the same mode of attack; third, it is possible by these problems and charts to teach something of analysis, and to develop toward all problems a certain attitude of mind which will be useful after the algebra has been forgotten. This is the valuable part of the subject whose presence in a curriculum teachers are frequently called upon to justify.

SOME APPLICATIONS OF THE VARIOUS FORMS OF ZERO AND UNITY.

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We have the symbols 0 and 1 representing zero and unity respectively. In subtraction we also know that when a quantity is subtracted from its equal, the remainder is zero. This latter expression for zero, however, very often serves as a useful device in algebraic operations when the mere symbol would be of no service.

Unity may also have different forms of representation, such as x^0 , $\sin^2 a + \cos^2 a$, $\log_e e$, $(-1)(-1)$, i^4 , besides many others. When any quantity is divided by itself, the quotient is unity, whether the operation be performed or indicated. And this indicated form of unity is likewise a helpful device in effecting algebraic transformations. Since a quantity may be multiplied or divided by unity without altering its value, the multiplier or divisor may be written in any one of the various forms instead of by the symbol 1.

Some examples will serve to illustrate our meaning in the use of these indicated forms:

1. In solving $ax^2 + bx + c = 0$ for x , we may use the form $b^2 - b^2$ for zero. Multiplying the equation by $4a$, we get $4a^2x^2 + 4abx + 4ac = 0$. Evidently if b^2 is added to the terms $4a^2x^2 + 4abx$ there is obtained the square of $(2ax + b)$. Hence, if b^2 is both added and subtracted, that is, if we write zero as $b^2 - b^2$, we may add it at a convenient place in the equation without destroying the equation.

The writer appreciates that the ability to find a third term to combine with two given terms in order to form a trinomial which is a perfect square is involved in this problem. But the ability to complete the square is *very* essential and should be stressed.

2. In order to factor the expression $x^4 + a^2x^2 + a^4$, it will be necessary to write zero in the form $a^2x^2 - a^2x^2$. This is evident since $2a^2x^2$ in place of a^2x^2 , changes $x^4 + a^2x^2 + a^4$ into the square of $x^2 + a^2$. We then have $(x^2 + a^2)^2 - a^2x^2$, which is easily factored.

3. Let it be required to change $x^4 + 2x^3 - x^2 - 2x - 3 = 0$ into the form of a quadratic. We may write it $(x^4 + 2x^3) - (x^2 + 2x) - 3 = 0$. If now zero be written in the form $x^2 - x^2$, the equation takes the form $(x^2 + x)^2 - 2(x^2 + x) - 3 = 0$ which is evidently in the form of a quadratic, since one parenthesis is the square of the other. In order to change it still further and remain a quadratic, put $x^2 + x = y$. It then becomes $y^2 - 2y - 3 = 0$.

4. Similarly, change $2x^2 - 4x + 3\sqrt{x^2 - 2x + 6} = 15$ into the form of a quadratic. It is evident that the part $2x^2 - 4x$ must be built up so as to be identical with the radicand $x^2 - 2x + 6$ save for a possible coefficient. Taking out the common factor 2 we get $2(x^2 - 2x)$. Zero in the form $6 - 6$ will effect the necessary transformation for the trinomial and will make a change in the constant term. Thus we will get $2(x^2 - 2x + 6) + 3\sqrt{x^2 - 2x + 6} - 12 - 15 = 0$, or $2y^2 + 3y - 27 = 0$ by making $x^2 - 2x + 6 = y^2$.

5. Take this example from analytical geometry, viz., $x^2 + y^2 - 6x - 10y + 9 = 0$ which is the equation of a circle, the problem being to find the coördinates of its center and the length of the radius. By completing the square for those terms involving x and y separately, we see that zero must have the two forms $9 - 9$ and $25 - 25$, respectively. Using these we get $x^2 - 6x + 9 - 9 + y^2 - 10y + 25 - 25 + 9 = 0$ or $(x - 3)^2 + (y - 5)^2 = 25$. The form $9 - 9$ is not necessary if one observes that $x^2 - 6x + 9$ is one of the two perfect squares needed.

6. If r is a root of $ax^2 + bx + c = 0$, which means that $ar^2 + br + c$ is a form for zero, we can show that $x - r$ is a factor of $ax^2 + bx + c = 0$.

Taking the identity $ax^2 + bx + c = ax^2 + bx + c$ and subtracting zero from the right member but in the special form above we get $ax^2 + bx + c = ax^2 + bx + c - ar^2 - br - c$ or $ax^2 + bx + c = a(x^2 - r^2) + b(x - r)$.

7. Take this problem from the calculus, in which it is required to put $x^2 + 6x + 13$ in the form of the sum of two quantities, the expression in terms of x being a perfect square. Zero must be written in the form $9 - 9$, in which case we get $x^2 + 6x + 9 - 9 + 13$ or $(x + 3)^2 + 4$.

8. Put $8+4x-4x^2$ in the form of the difference of two quantities, the subtractive one being a perfect square. In this case, zero must have the form $1-1$, when we get $9-(2x-1)^2$.

9. Put x^2-6x+5 in the form of the difference of two squares. Zero must be written $9-9$, thus giving $(x-3)^2-4$.

Thus we might multiply examples to show that zero in its various forms is a useful device and of frequent application. In our judgment, the value of this should be impressed upon the pupil and he should be encouraged to be on the alert for opportunities to employ it. Surely it will mean the exercise of his ingenuity and the satisfaction in using it will impart a relish for things mathematical.

There are many problems in which unity may be introduced, written as the quotient of two equal quantities, in order to effect transformations in solving or factoring. The following examples will illustrate:

10. In the expression $3x^2+7x+5$, take out the common factor 3. It is not apparent that there is such a common factor, but if unity is written in the form $3/3$ and applied to $7x$ and 5 , we get $3x^2+3\cdot 7x/3+5\cdot 3/3$ or $3(x^2+7x/3+5/3)$.

11. Let it be required to put the final form of example 10 in the form of the difference of two squares in order to factor it. In this, zero must be written $49/36-49/36$. Applying this we get, $3[x^2+7x/3+49/36-49/36+12/12\cdot 5/3]$, unity being written $12/12$. This condenses to $3[(x+7/6)^2+11/36]$. Again unity may take the form $(-1)(-1)$, and for one of these factors i^2 may be substituted. This gives

$3[(x+7/6)^2-i^2 11/36]$ whose factors are $3[x+7/6+i\sqrt{11/6}][x+7/6-i\sqrt{11/6}]$.

12. Express ax^2+bx+c as the difference of two squares. Write unity as a/a and apply it to bx and c . Taking out the common factor a and writing zero as $b^2/4a^2-b^2/4a^2$, we get $a[x^2+bx/a+b^2/4a^2-b^2/4a^2+c/a]$ or $a[(x+b/2a)^2-b^2-4ac/4a^2]$ whose factors are

$a(x+b/2a-(\sqrt{b^2-4ac})/2a)(x+b/2a+(\sqrt{b^2-4ac})/2a)$.

13. Take the interesting problem to show that in the division of fractions, multiplication is substituted for division by inverting the terms of the divisor.

$$a/b$$

Take — as the fraction. In this, unity must take the form

$$c/d$$

d/c
 of $\frac{d}{c}$. When the pupil has reached division of fractions, he
 d/c
 will have had practice in cancellation and will have had the
 definition of the reciprocal of a number, both of which ideas are
 involved in this problem. Using the above form of unity, we get
 $a/b \cdot d/c$
 $\frac{a}{b} \times \frac{d}{c}$ or $a/b \times d/c$ by cancellation.
 $c/d \cdot d/c$

14. In order to change $\sqrt{7/5}$ into a mixed surd, unity must
 have the form $5/5$ or $7/7$. Thus we have $\sqrt{7 \cdot 5/5 \cdot 5}$ or
 $\sqrt{7 \cdot 7/5 \cdot 7}$ which become $1/5 \cdot \sqrt{35}$ or $7/\sqrt{35}$.

15. Rationalize the denominator of $(5+\sqrt{3})/(5-\sqrt{3})$. A
 knowledge of conjugate quantities is essential to see that unity
 must be written $(5+\sqrt{3})/(5+\sqrt{3})$.

16. Transform the binomial $(8+12x)^{\frac{2}{3}}$ into a form with
 unity for the first term. In order to take out the common
 factor 8, unity must be written $2/2$, $4/4$, or $8/8$. Applying
 the last form we get $(8+8/8 \cdot 12x)^{\frac{2}{3}}$ or $4(1+3x/2)^{\frac{2}{3}}$.

17. Let it be required to express the coefficient of the middle
 term of $(x+y)^{28}$ in the form of factorial quantities. This co-
 efficient is $28 \cdot 27 \cdot \dots \cdot 15 \cdot \frac{14}{14}$, whence it is evident
 that unity must be written $\frac{14}{14}$. Applying this to the
 coefficient we get $\frac{28}{14} \cdot \frac{14}{14}$.

18. Show that $\sqrt{(1-\cos x)/(1+\cos x)} = (1-\cos x)/\sin x$.
 If the pupil is ready with his trigonometric formulae, he will see
 at once that unity must have the form $(1-\cos x)/(1-\cos x)$,
 and when this is applied to the radicand, we get
 $\sqrt{(1-\cos x)^2/(1-\cos^2 x)}$ or $(1-\cos x)/\sin^2 x$.

19. Change the following fractions to similar fractions,
 $a/(x+y)$, $b/(x-y)$, $c/(x^2-y^2)$. A knowledge of the lowest com-
 mon denominator would suggest that unity in the first fraction
 must have the form $(x-y)/(x-y)$ and in the second fraction it
 must be $(x+y)/(x+y)$. In order to change the sign of a fraction,
 unity must be written $-1/-1$.

20. As a final problem, let the coefficients of
 $ax^4+bx^3+cx^2+dx+e=0$ be integers. In order that this may
 be put in the form of a quadratic, we must have
 $8a^2d=4abc-b^3$.

Taking out the common factor a , we have
 $a[x^4+(b/a)x^3+(c/a)x^2+(d/a)x+e/a]=0$, and building up

a trinomial perfect square, using $x^4 + (b/a)x^3$ as part of the trinomial, we get

$a[x^4 + (b/a)x^3 + (b^2/4a^2)x^2 - b^2x^2/4a^2 + cx^2/a + (d/a)x + e/a]$
 $= 0$. Writing unity in the forms, $4a/4a$ and

$(4ac - b^2)/4a^2 \cdot 4a^2/(4ac - b^2)$, and multiplying the terms cx^2/a and ax/a by these respectively, we get,

$a[x^4 + (b/a)x^3 + b^2x^2/4a^2 + 4acx^2/4a^2 - b^2x^2/4a^2 +$
 $4ac - b^2/4a^2 \cdot 4a^2/4ac - b^2 \cdot (d/a)x + e/a] = 0$ or

$a[(x^2 + bx/2a)^2 + (4ac - b^2)/4a^2(x^2 + 4adx/4ac - b^2) + e/a] = 0$.

Hence this will take the quadratic form only when

$4ad/(4ac - b^2) = b/2a$ or if $8a^2d = 4abc - b^3$.

These problems suffice to exhibit the wide range to which the various forms of unity and zero may be applied, and the constant effort on the part of the pupil to discover opportunities for their application will mean much to him in his mathematical career.

ATOMIC THEORY IN BASEBALL TERMS.

By DR. IRVING LANGMUIR.

According to our present views, all forms of matter are built up of atoms, but we no longer regard these atoms as indivisible nor even as simple structures. If a lump of ordinary matter the size of a baseball could be magnified to the size of the earth, the atoms in it would then have become about the size of baseballs. In other words, an atom is about as big compared to a baseball as the baseball is when compared to the earth. The atoms are constructed of particles of positive and negative electricity arranged in a very open structure. All the positive electricity is concentrated into a very small particle, called the nucleus, located at the center of the atom. The negative electricity exists in the form of electrons which arrange themselves in space about the nucleus. The size of the electrons and nucleus is small compared with that of the atom itself. Thus if we imagine an atom magnified until it has a diameter of one mile, the electrons would be about five feet in diameter while the nucleus at the center would be only the size of a walnut.

The electrons in different kinds of atoms are alike but there are as many different kinds of nuclei as there are chemical elements, that is, about 92 in all. These differ from one another only in the amount of positive electricity they contain. Thus for the simplest element, hydrogen, the nucleus has a unit positive charge which is able to neutralize the charge of a single electron. A hydrogen atom then consists merely of the nucleus and a single electron. The next element, helium, has a nucleus with a double positive charge and the atom thus contains two electrons. In a similar way we find that the atoms of carbon have six electrons, while oxygen has eight, aluminum, thirteen, sulfur, sixteen, iron, twenty-six, copper, twenty-nine, silver, forty-seven, gold, seventy-nine, lead, eighty-two, and radium, eighty-eight electrons.

These electrons do not revolve around the nucleus the way the earth revolves around the sun, but they are arranged in three dimensions in a series of layers or concentric shells surrounding the nucleus. The electrons are probably not stationary but each revolves in its own orbit