VI. ABSTRACT of EXPERIMENTS *made to determine the TRUE RESISTANCE of the AIR to the SURFACES of BODIES, of various figures, and moved through it with different degrees of velocity.* By CHARLES HUTTON, LL.D. *Professor of Mathematics in the Royal Military Academy, Woolwich, and F. R. S. LOND. & EDIN.*

[Read by Mr ROBISON, Jan. 1. 1787.]

1. **T**HE experiments from which the following are extracted, make part of a course, instituted at the Royal Military Academy, for determining the resistance of the air to a surface of any form whatever, either plane or curved, moved through it with any degree of velocity. I was induced to undertake these experiments, both for the improvement of my students in the Academy, and with a view to apply the conclusions derived from them towards perfecting the theory and practice of military projectiles, as well as other branches of natural philosophy, in which the pressure or resistance of fluids is concerned: Circumstances, concerning the laws of which, authors on the theory have widely differed; some making the pressure or resistance equal to the weight of a column, whose altitude is equal to the whole height due to the velocity, while others make the altitude very different, either the half or the double of that. This altitude, however, it is evident, will be various, according to the nature of the fluid, whether elastic or non-elastic, &c. or according to its different degrees of compression. I have, therefore, confined these experiments to the pressure and resistance of the air only, being that which affects
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the accuracy of the branch, for the improvement of which I am more particularly solicitous at this time; and therefore the laws here deduced are not meant to be extended to other fluids of a different nature.

2. THE machine with which these experiments were performed, was made after the pattern of, and by the same workman as that which is described by the late excellent Mr ROBINS, in the first volume of his works, as published by Dr WILSON, and of which a view is inserted in that volume, at least of the principal parts of it. Suffice it, therefore, in this place, just to observe, that it consists of a small vertical axis, with a long horizontal arm connected with it. A body of any form is fixed on the extremity of the arm; then a fine, but strong filken thread, or cord, is wound about the axis, with a given small weight at the end, which is passed over a vertical pulley, and left to descend by its weight, and so turning the axis, gives motion to the arm and body at the end of it. Hence it is evident, that a slow motion of the axis, or of the actuating weight, will give a very quick motion to the resisting body at the end of the arm; this latter being to the former indeed, as the length of the arm, measured to the centre of the body, is to the radius of the axis, which, in these experiments, was as $51\frac{1}{7}$ to 1.

3. THE actuating weight would descend continually with an accelerated velocity, were it not for the friction of the axis, and the resistance of the air to the arm and the body placed at the end of it. But this resistance always increasing with the velocity, and indeed as the square of it, it must needs happen, that, by the resistance constantly gaining on the velocity, this will soon arrive at its maximum, and after that proceed with a uniform motion, the resistance neither gaining on the velocity, nor the velocity on the resistance, but each mutually balancing the other. As soon as this happens, then the actuating

tuating weight is the measure of the resistance of the air on the body and the arm, and of the friction of the axis.

4. NEXT, to find what part of the retardation is owing to the friction on the axis, and the resistance of the air against the arm, both the actuating weight and the resisting body were taken off, and, instead of the latter, a very thin bit of lead of the same weight was put on; then various smaller actuating weights were employed, till at last such a one was found as gave to the machine the very same degree of uniform motion as it had before, when the resisting body and the larger actuating weight were employed. Then, the degree of velocity being the same in both cases, the smaller actuating weight will be the measure of the friction on the axis, and the resistance of the air to the arm; both of which, however, were reduced to as small a quantity as possible, the former by means of friction-wheels, and the latter by being made thin and feather-edged.

5. SUBTRACTING now the less weight from the greater, the remainder is the measure of the resistance of the air against the body alone; that is, when reduced for the different lengths of lever, namely, by diminishing the remaining weight in the ratio of the length of the arm to the radius of the axis, measured to the middle of the thread. Thus, then, we obtain a weight which is the measure of the resistance of the air against a given surface, moving with a given velocity; that is, a weight which is equal to the pressure of the air against the surface, or which, if it were laid upon and uniformly diffused over the surface when it is a plane, would press the surface just as much as the air does.

6. THEN, lastly, finding what altitude a column of air must have, which is of the weight of the aforesaid remaining weight reduced, and whose base is the plane of the resisting surface; it will be the altitude of the column of the fluid whose weight or pressure is equal to the resistance, and which pressure would consequently generate the same velocity in the fluid.

7. THE

7. THE body used to affix to the end of the arm, in the following abstract, was a hemisphere of pasteboard, the hollow part being covered with a flat circle of the same, that either the round or the flat side might be made to go foremost against the air. The diameter of the hemisphere was $6\frac{3}{8}$ inches; and consequently the area of its great circle, or flat side, was 32 square inches or $\frac{2}{7}$ of a square foot, and it weighed 4 oz. 3 dr. avoirdupois. The hemisphere being fixed on the end of the arm, with either side foremost, by a medium of several times, and different ways of measuring, it was found, that the radius of the axis, including half the thickness of the thread, was 1.043 inches, and the length of the arm, measured to the centre of the hemisphere, was 53.34 inches; so that the two radii, namely of the path of the body and of the axis, are to each other as 53.34 to 1.043, or as 51.14 to 1: And therefore every experimented actuating weight must be divided by 51.14 or $51\frac{1}{2}$, to reduce it to the equivalent weight acting at the centre of the hemisphere.

8. THE times of revolutions of the arm were counted by a peculiar pendulum clock, beating seconds, which was made for the purpose. The method was thus: The clock being placed close by the machine, and the hemisphere and actuating weight fixed in their places, an assistant held the hemisphere in a particular situation by his hand, while a second assistant audibly counted the beats of the clock, beginning at 50 seconds, and counting on from 1 to 10, which consequently would end at 60 or 0; and the instant he pronounced 10, the first assistant let the hemisphere go. The consequence was, it began at first to move very slowly, and gradually increase for a short time, and then to move uniformly. The first assistant, keeping his station, called out at every time the body passed him, in its revolution, and the other assistant called out the corresponding number of seconds and half seconds beat by the clock, which I instantly wrote down with a pencil on a paper held in my hand,

hand, previously ruled for that purpose. And thus we could with ease mark the precise time of every revolution. The number of revolutions was usually continued to about 35; and as the motion, with the hemisphere commonly became uniform after two or three revolutions, and with the lead only, after about 20 revolutions, I subtracted the time of the first 25 revolutions from that of 35, and the remainder was the mean time of 10 revolutions; and, consequently, dividing by 10 gave me the mean time of one revolution very correctly; and thence, from the space of one circle or revolution, which is 27.93 feet, the velocity of the hemisphere *per* second of time.

9. IN this manner, then, by varying the actuating weight, by 1 dr. or 2 dr. &c. at a time, I obtained a long series of corresponding times and velocities, both with the round and flat side of the hemisphere foremost, and with the equivalent lead only. After which I subtracted the numbers of this latter from the corresponding ones of the two former, and the remainders, when divided by 51.14, gave the true measure of the pressure of the air at the centre of the hemisphere.

IN the following table are selected only the velocities in whole numbers of feet, namely, of 3 feet *per* second, of 4 feet, of 5 feet, and so on to that of 20 feet *per* second of time, with the corresponding actuating weights in all the three cases, namely the flat side foremost, the round side foremost, and without the hemisphere, with the lead only; namely, such actuating weights as were really experimented, and before dividing them by 51.14, to reduce them to the centre of the body.

ABSTRACT of VELOCITIES and ACTUATING WEIGHTS.

Velocity, <i>per</i> sec.	Weights with			Diff. or true resistances.		Ratios.
	Flat side.	Round side.	Lead only.	Flat.	Round.	
feet.	oz.	oz.	oz.	oz.	oz.	
3	3.8	2.2	1.2	2.6	1.0	2.60
4	6.2	3.4	1.4	4.8	2.0	2.40
5	9.2	4.9	1.7	7.5	3.2	2.35
6	12.8	6.7	2.0	10.8	4.7	2.30
7	17.0	8.7	2.3	14.7	6.4	2.30
8	21.9	11.0	2.8	19.1	8.2	2.33
9	27.6	13.5	3.3	24.3	10.2	2.38
10	34.0	16.2	3.8	30.2	12.4	2.44
11	41.0	19.2	4.4	36.6	14.8	2.47
12	48.7	22.6	5.1	43.6	17.5	2.49
13	57.1	26.4	5.8	51.3	20.6	2.49
14	66.2	30.6	6.5	59.7	24.1	2.48
15	76.0	35.1	7.2	68.8	27.9	2.47
16	86.6	40.0	7.9	78.7	32.1	2.46
17	98.2	45.3	8.7	89.5	36.6	2.45
18	111.0	51.0	9.5	101.5	41.5	2.45
19	125.0	57.2	10.3	114.7	46.9	2.45
20	140.0	64.0	11.0	129.0	53.0	2.44
1	2	3	4	5	6	7

HERE the first column contains the velocity *per* second; the second column contains the experimented actuating weight, with the flat side foremost; the third column that for the round side foremost; and the fourth column that for the lead only: Then the fifth column contains the difference between the second and fourth, or actuating weights for the flat side and lead; and the sixth column the difference between the third and fourth, or actuating weights, for the round side and lead only; so that the fifth and sixth columns, when divided by $51\frac{1}{2}$, will be the true measure of the resistance of the air to each side of the

the hemisphere, moving with the corresponding velocity on the same line of the first column; and in the last column are contained the ratios of these two resistances, or how often each resistance of the round side is contained in that on the flat side of the hemisphere.

10. FROM a slight contemplation of the last three columns of this table, we may easily draw several important consequences. As, *first*, From the fifth and sixth columns, it appears, that the resistance to either surface, with different velocities, is always as the square of the velocity, as near as such experiments can be expected to show.

THUS, in the fifth column, taking the resistances corresponding to the velocities of 4 feet and 8 feet, which are as 1 to 2, and their squares as 1 to 4; the resistances 4.8 to 19.1 are as 1 to 4 very nearly; and the resistances in the sixth column, namely, 2 to 8 $\frac{1}{2}$, are also nearly in the same ratio. And so of others.

11. *2dly*, FROM the last column, it appears, that the resistance to the flat side is to that on the round side, on an average, nearly as 2.45 to 1, or 2 $\frac{1}{2}$ to 1 nearly, if a medium be taken among all the numbers in the last column. But, by the theory of the resistance of fluids, we are led to expect, that this ratio would have been only that of 2 to 1, instead of 2 $\frac{1}{2}$ to 1, as by the experiment. Now, what this difference is owing to, may be at present difficult to determine with precision. The greater part of it may probably arise from the air differing in its nature from the perfect fluid which the theory contemplates; but some small part of it may arise from the different figure of the hinder parts of the hemisphere, though I hardly suspect that this may cause any sensible difference. I intend, however, soon to try whether it be sensible to experiments; in which I intend to employ a cylinder, to compare with the flat side foremost of the hemisphere, and a whole sphere, each of the same diameter, to compare with the round side foremost of the hemisphere. I

propose also, at the same time, to try the resistance of some other figures.

12. *3dly*, FROM any of the numbers in the sixth column, it appears, that the altitude of a column of air, whose pressure is equal to the resistance on the round side of the hemisphere, is half the altitude due to the velocity of the figure; that is, half the altitude from which a body must freely fall by gravity to acquire that velocity; and, in this instance, agreeing with the theory. Thus, if we take the velocity of 10 feet *per second*, whose resistance in the sixth column is 12.4, we shall have as $32^2 : 10^2 :: 16 : \frac{100}{64} = 1.56$ feet, which is the altitude due to the velocity 10, and the half altitude is .78; but the weight is 12.4 ounces, which being divided by 51.14, to reduce it from the axis to the centre of the body, gives .2411 oz. for the true resistance to the convex side. Now, a cubic foot of air weighs $1\frac{1}{4}$ oz.; therefore, as $1\frac{1}{4} : .2411 :: 1 : .1929$, which is the bulk of the column of air whose weight is equal to the resistance, which being divided by $\frac{2}{3}$ of a foot, the area of the base, we have .86 feet for the altitude of that column, and which, therefore, is nearly equal to the half altitude above found for the velocity, exceeding it only by about the 13th or 14th part.

13. *4thly*, BUT, from the fifth column, it appears, that the altitude of the column of air, whose pressure is equal to the resistance on the flat side of the hemisphere, is to the altitude due to the velocity of the body, as $2\frac{1}{4}$ to 2, instead of being equal, as required by the theory.