

irrespectively of the direction of the stream lines in its neighbourhood ; a result easily deduced from the elementary formula for fluid pressure in hydrokinetics.

I have long ago shown that an elongated diamagnetic body in a uniform magnetic field tends, as tends an elongated ferromagnetic body, to place its length along the lines of force. Hence a long solid, pivoted on a fixed axis through its middle in a uniform stream of liquid, tends to place its length perpendicularly across the direction of motion ; a known result (Thomson & Tait's "Natural Philosophy," § 335). Again, two globes held in a uniform stream with the lines joining their centres, require force to prevent them from mutually approaching one another. In the magnetic analogue, two spheres of diamagnetic or ferromagnetic inductive capacity repel one another when held in a line at right angles to the lines of force. A hydrokinetic result similar to this for the case of two equal globes, is to be found in Thomson and Tait's "Natural Philosophy," § 332.

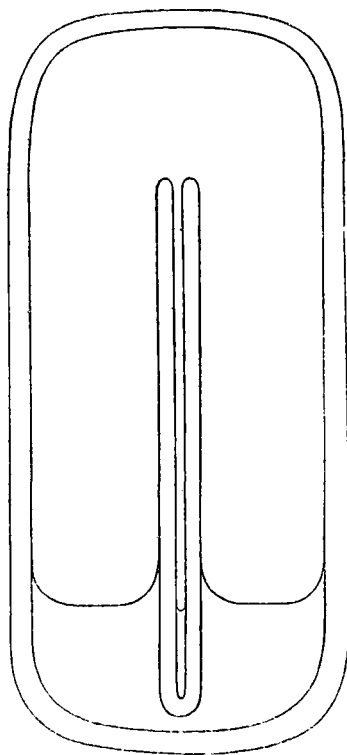
IV. (Proposition.) If the second body considered in § III., that is to say, a body either having no apertures, or, if perforated, having no circulation through the apertures, be acted on by one system of forces applied so as always to balance the resultant of the fluid pressure, calculated for it according to II. and III. for whatever position it may come to at any time, and if it be influenced, besides, by any other system of applied forces, superimposed on the former, it will move just as it would move, under the influence of the latter system of forces alone, were the fluid at rest, except in so far as compelled to move by the body's own motion through it. A particular case of this proposition was first published many years ago, by Professor James Thomson, on account of which he gave the name of "vortex of free mobility" to the cyclic irrotational motion symmetrical round a straight axis.

#### 4. On the Equilibrium of Vapour at a Curved Surface of Liquid. By Sir William Thomson.

In a closed vessel containing only a liquid and its vapour, all at one temperature, the liquid rests, with its free surface raised or depressed in capillary tubes and in the neighbourhood of the solid boundary, in permanent equilibrium according to the same law of

relation between curvature and pressure as in vessels open to the air. The permanence of this equilibrium implies physical equilibrium between the liquid and the vapour in contact with it at all parts of its surface. But the pressure of the vapour at different levels differs according to hydrostatic law. Hence the pressure of saturated vapour in contact with a liquid differs according to the curvature of the bounding surface, being less when the liquid is concave, and greater when it is convex. And detached portions of the liquid in separate vessels all enclosed in one containing vessel, cannot remain permanently with their free surfaces in any other relative positions than those they would occupy if there were hydrostatic communication of pressure between the portions of liquid in the several vessels. There must be evaporation from those surfaces which are too high, and condensation into the liquid at those surfaces which are too low—a process which goes on until hydrostatic equilibrium, as if with free communication of pressure from vessel to vessel, is attained. Thus, for example, if there are two large open vessels of water, one considerably above the other in level, and if the temperature of the surrounding matter is kept rigorously constant, the liquid in the higher vessel will gradually evaporate until it is all gone and condensed into the lower vessel. Or if, as illustrated by the annexed diagram, a capillary tube, with a small quantity of liquid occupying it from its bottom up to a certain level, be placed in the neighbourhood of a quantity of the same liquid with a wide free surface, vapour will gradually become condensed into the liquid in the capillary tube until the level of the liquid in it is the same as it would be were the lower end of the tube in hydrostatic communication with the large mass of liquid. Whether air be present above the free surface of the liquid in the several vessels or not, the condition of ultimate equilibrium is the same; but the processes of evaporation and condensation through which equilibrium is approached will be very much retarded by the presence of air. The experiments of Graham, and the kinetic theory of Clausius and Maxwell, scarcely yet afford us sufficient data for estimating the rapidity with which the vapour proceeding from one of the liquids will diffuse itself through the air and reach the surface of another liquid at a lower level. With air at anything approaching to ordinary atmospheric

density to resist the process, it is probable it would be too slow to show any results unless in very long continued experiments. But if the air be removed as perfectly as can be done by well-known practical methods, it is probable that the process will be very rapid: it would, indeed, be instantaneous, were it not for the cold of evaporation in one vessel and the heat of condensation in the other. Practically, then, the rapidity of the process towards hydrostatic equilibrium through vapour between detached liquids, depends on the rate of the conduction of heat between the several surfaces through intervening solids and liquids. Without having



made either the experiment, or any calculations on the rate of conduction of heat in the circumstances, I feel convinced that in a very short time water would visibly rise in the capillary tube indicated in the diagram, and that, provided care is taken to maintain equality of temperature all over the surface of the hermetically sealed vessel, the liquid in the capillary tube would soon take very nearly the same level as it would have were its lower end open; sinking to this level if the capillary tube were in the beginning filled too full, or rising to it if (as indicated in the diagram) there is not enough of liquid in it at first to fulfil the condition of equilibrium.

The following formulæ show precisely the relations between curvatures, differences of level, and differences of pressure, with which we are concerned.

Let  $\rho$  be the density of the liquid, and  $\sigma$  that of the vapour; and let  $T$  be the cohesive tension of the free surface, per unit of breadth, in terms of weight of unit mass, as unit of force. Let  $h$  denote the height of any point,  $P$ , of the free surface above a certain plane of reference, which I shall call for brevity the plane level of the free surface. This will be sensibly the actual level of the free surface in regions, if there are any, with no part of the edge (or bounding line of the free surface where liquid ends and solid begins) at a less distance than several centimetres. Lastly, let  $r$  and  $r'$  be the principal radii of curvature of the surface at  $P$ . By Laplace's well-known law, we have, as the equation of equilibrium,

$$(\rho - \sigma)h = T\left(\frac{1}{r} + \frac{1}{r'}\right) \quad . \quad . \quad . \quad (1).$$

Now, in the space occupied by vapour, the pressure is less at the higher than at the lower of two points whose difference of levels is  $h$ , by a difference equal to  $\sigma h$ . And there is permanent equilibrium between vapour and liquid at all points of the free surface. Hence the pressure of vapour in equilibrium is less at a concave than at a plane surface of liquid, and less at a plane surface than at a convex surface, by differences amounting to  $\frac{T\sigma}{\rho - \sigma}$  per unit difference of curvature. That is to say, if  $\varpi$  denote the pressure of vapour in equilibrium at a plane surface of liquid, and  $p$  the pressure of vapour of the same liquid at the same temperature presenting a curved surface to the vapour, we have

$$p = \varpi - \frac{T\sigma}{\rho - \sigma} \left(\frac{1}{r} + \frac{1}{r'}\right) \quad . \quad . \quad . \quad (2),$$

$\frac{1}{r}$  and  $\frac{1}{r'}$  being the curvatures in the principal sections of the surface bounding liquid and vapour, reckoned positive when concave towards the vapour.

In strictness, the value of  $\sigma$  to be used in these equations, (1) and (2), ought to be the mean density of a vertical column of vapour, extending through the height  $h$  from the plane of reference.

But in all cases to which we can practically apply the formulæ, according to present knowledge of the properties of matter, the difference of densities in this column is very small, and may be neglected. Hence, if  $H$  denote the height of an imaginary homogeneous fluid above the plane of reference, which, if of the same density as the vapour at that plane, would produce by its weight the actual pressure  $\varpi$ , we have

$$\sigma = \frac{\varpi}{H}.$$

Hence by (1) and (2)

$$p = \varpi \left(1 - \frac{h}{H}\right) \quad . \quad . \quad . \quad (3).$$

For vapour of water at ordinary atmospheric temperatures,  $H$  is about 1,300,000 centimetres. Hence, in a capillary tube which would keep water up to a height of 13 metres above the plane level, the curved surface of the water is in equilibrium with the vapour in contact with it, when the pressure of the vapour is less by about  $\frac{1}{10000}$ th of its own amount than the pressure of vapour in equilibrium at a plane surface of water at the same temperature.

For water the value of  $T$  at ordinary temperatures is about  $\cdot 08$  of a gramme weight per centimetre; and  $\rho$ , being the mean of a cubic centimetre, in grammes, is unity. The value of  $\sigma$  for vapour of water, at any atmospheric temperature, is so small that we may neglect it altogether in equation (1). In a capillary tube thoroughly wet with water, the free surface is sensibly hemispherical, and therefore  $r$  and  $r'$  are each equal to the radius of the inner surface of the liquid film lining the tube above the free liquid surface; we have, therefore,

$$h = \cdot 08 \times \frac{2}{r}.$$

Hence, if  $h = 1300$  centimetres,  $r = \cdot 00012$  centimetres. There can be no doubt but that Laplace's theory is applicable without serious modification even to a case in which the curvature is so great (or radius of curvature so small) as this. But in the present state of our knowledge we are not entitled to push it much further. The molecular forces assumed in Laplace's theory to be "insensible at sensible distances," are certainly but little, if at all, sensible at distances equal to or exceeding the wave lengths of ordinary light. This is directly proved by the most cursory observation of soap

bubbles. But the appearances presented by the black spot which abruptly ends the series of colours at places where the bubble is thinnest before it breaks, make it quite certain that the action of those forces becomes sensible at distances not much less than a half wave length, or  $\frac{1}{40000}$  of a centimetre. There is, indeed, much and multifarious evidence that in ordinary solids and liquids, not merely the distances of sensible inter-molecular action, but the linear dimensions of the molecules themselves, and the average distance from centre to nearest centre,\* are but very moderately small in comparison with the wave lengths of light. Some approach to a definite estimate of the dimensions of molecules is deducible from Clausius' theory of the average spaces travelled without collision by molecules of gases, and Maxwell's theory and experiments regarding the viscosity of gases. Having perfect confidence in the substantial reality of the views which these grand investigations have opened to us, I find it scarcely possible to admit that there can be as many as  $10^{27}$  molecules in a cubic centimetre of liquid carbonic acid or of water. This makes the average distance from centre to nearest centre in the liquids exceed a thousand-millionth of a centimetre!

We cannot, then, admit that the *formulae* which I have given above are applicable to express the law of equilibrium between the moisture retained by vegetable substances, such as cotton cloth or oatmeal, or wheat-flour biscuits, at temperatures far above the dew point of the surrounding atmosphere. But although the energy of the attraction of some of these substances for vapour of water (when, for example, oatmeal, previously dried at a high temperature, has been used, as in the original experiment of Sir J. Leslie, to produce the freezing of water under the receiver of an air-pump), is so great that it might almost claim recognition from chemists as due to a "chemical affinity," and resulting in a "chemical combination," I believe that the absorption of vapour into fibrous and cellular organic structures is a property of matter continuous with the absorption of vapour into a capillary tube demonstrated above.

\* By "average distance from centre to nearest centre," I mean the side of the cube in a cubic arrangement of a number of points equal to the number of real molecules in any space.