ENGINE BALANCING.

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1. *Introductory.*—The subject of engine vibration and the balancing of reciprocating engines and other machinery has been dealt with on many previous occasions in various papers and treatises, and it is customary to regard the subject as one involving mathematical treatment of an abstruse character. In the present paper the author has endeavoured, as far as possible, to treat the subject by direct methods and to avoid the use of symbolic notation.

The purpose of the paper is the study of the elementary causes of vibration, and the demonstration of methods by which the elimination of vibration may be effected; the examples taken for the purpose of illustration are such as to substantially cover the ground that ordinarily comes within the purview of the automobile engineer.

Vibration in an automobile vehicle of any kind may be regarded as a disease which may be constitutional or otherwise, and which it is the object of the designer and constructor to cope with and prevent.

2. *Classification.*—At the outset, it may be stated that there are two fundamentally distinct problems connected with the subject of vibration and balancing, between which a sharp line of definition exists. There are vibrations proper to any given engine in which the component parts are assumed as rigid within themselves, that is to say, vibrations which are quite independent of any distortion or springing of the material of which the engine is composed. On the other hand, there are vibrations, or sources of vibration, which are due entirely to the elastic yielding or springing of the engine components, principally the springing

of the crank shaft and engine frame, and vibrations of this type may exist in an engine which, considered as rigid, should be vibration free. In the present paper, it is assumed in Part I. that the engine components are in themselves rigid, and the question of engine vibration is dealt with on this assumption. Vibrations due to elastic yielding are dealt with in the sections constituting Part II.

PART I.

3. *Line of Treatment.*—If the problem with which we are faced were not confined and limited by the actual conditions of engine construction, that is to say, if we had required to consider the vibration of an entirely undefined engine or piece of mechanism in its broadest possible aspect, we should take our stand initially on the usual mathematician's or physicist's basis of *six degrees of freedom*, the meaning of this being that the vibrations may be constituted of motions of translation in any or all of the three co-ordinate dimensions of space or of rotations about any or all of the three co-ordinated axes. Fortunately, however, the problem has not to be treated in its broadest form; the conditions are considerably narrowed by the actual nature and character of the machines with which we are concerned, and consequently we can abandon the generalised basis with advantage, and adopt a specialised treatment as more appropriate to the conditions of the problem.

4. *Narrowed Basis of Treatment.*—When we have before us the design of a reciprocating motive power engine, or, more broadly, any fluid pressure engine employing the usual piston and connecting rod or equivalent, we find that the complete symmetry of the problem is disturbed, firstly, by the existence of a definite axis of rotation, that is, the functional axis of the engine—the axis direction of the crank shaft, and, secondly, by the existence of one or more axes of piston motion. When we require to consider the balancing problem, and study questions of forces and moments, we find that the constructional features of the engine aforesaid give us a circumscribed problem in which we know just where to look for our unbalanced forces—the reciprocating parts are confined to directions at right angles to the crank axes—and in which we look for moments in the form of torque variations when we deal with the crank shaft axis, and of rocking moments when we deal with axes at right angles thereto.

5. *Two Lines of Thought.*—There are two different ways of thinking of any problem in balancing or vibration, which, though not fundamentally distinct, are at least sufficiently different to merit a few words of discussion. We may, for instance, in the case of an ordinary reciprocating engine, consider the bed of the engine as fixed (as if, for example, bolted to a foundation of overwhelming size), and consider the forces and moments that will result from the functional movements of its parts, or alternatively, we may consider the engine as suspended perfectly freely in space, and study the resultant movements of the engine bed, or of that portion of the engine which is in practice employed as its mounting.*

In this latter method of considering the problem, it is necessary to assume a torque applied to the engine equal to its mean torque in the performance of work, in order to prevent its rotating bodily in space. Both the foregoing lines of thought are of value, and are employed in the present paper in dealing with the different phases of the problem.

6. *The Single-Cylinder Engine.*—As an initial study, it is convenient to take the case of a single-cylinder reciprocating engine, and to introduce the various defects of balance to which such an engine is subject, one by one. Briefly, we are proposing to deal with the problem as it presents itself to the motor-cycle builder of the present day. Let us take the elements of a single-cylinder engine and imagine, in the first instance, that the connecting rod is infinitely long, and let us further assume that there is no flywheel, and that the engine is "belted" round without compression. These conditions are naturally impossible of fulfilment, but before we reach the point of discussing the engine as a practical machine, we shall have made the necessary additions to the study as above presented.

Let us suppose that the piston have a mass m , and the remainder of the engine have a mass M . Let us firstly assume that the mass M is fixed, then we have the mass m moving with a strictly harmonic motion through a distance s , by which we will denote the stroke; then at any known number of revolutions R we can calculate precisely the forces that will be set up, acting on the foundation bolts by which the mass M is held, and we know

* In the case of a rotary engine of the Gnome type, for example, the crank shaft.

that the motion in question is harmonic and gives rise to forces represented by a straight line diagram as in Fig. 1, in which the maximum value of the force F (in pounds) is given by the expression

$$F = 2\pi^2 msR^2/32 \cdot 2.$$

If, alternatively, in the above example we suppose the engine frame, that is the mass M , to be free in space, the resultant motion may be derived from the principle that the centre of gravity of a self-contained system cannot undergo change of motion without a force applied from without (this being a paraphrase of the

FIG. 1.

third law of motion); thus, if the centre of gravity of the combined mass $M+m$ undergoes no motion, then it is self-evident that the movement of M will be (under the conditions supposed) a harmonic motion whose amplitude in relation to that of m will be inversely as the respective masses, that is to say, s as before being the stroke, the piston will have a motion to $Ms/(M+m)$, and the motion of the engine bed will have an amplitude $ms/(M+m)$.

Under neither of the conceptions above given will it be directly possible to give an account of vibration as transmitted to the chassis of a car, owing to the fact that, under the one conception

the bed of the engine is assumed to be incapable of movement, and therefore cannot receive any energy, and under the other condition, the bed of the engine is supposed to be free and unrestrained, and so can transmit no energy.* The meaning of this is that in actuality neither of the extreme conditions can exist, and the transmission of vibration from a given engine is a matter apart from what may be termed the vibration constants of the engine itself.

Let us now suppose that the simplified conception of an engine be modified to the extent of giving it a connecting rod of ordinary

FIG. 2.

proportions. Then, as every engine draughtsman knows, the motion of the piston is no longer a simple harmonic motion, but is an unsymmetrical motion, and the accelerations are greater at the in-centre than at the out-centre, the acceleration curve, and therefore the diagram of force, acting upon the engine foundation (assuming that the engine is fixed) being as represented in Fig. 2, where a dotted line shows the curve (straight line) representing the forces as due to the rod of infinite length previously assumed. Now we know that every periodic curve may be re-

* It is unnecessary to point out that the transmission of vibration in the ordinary sense of the term means the transmission of energy.

solved by Fourier's Theorem into a number of simple harmonic components, and it is a fact of no inconsiderable importance in connection with the present subject that the Fourier series may, in the present example, be, for all practical purposes, represented by two members, namely, the main "fundamental" component represented by the *sine* curve in Fig. 3 at *a*, and a secondary motion of twice the frequency, sometimes termed the octave component, also a *sine* curve shown in Fig. 3 at *b*. It will be seen in referring to Fig. 3, that by superposing the ordinates of the two *sine* curves *a* and *b*, a curve *c* is obtained optically coincident with the actual piston motion plotted (curve *d*).*

The full Fourier series of the piston movement is actually an

FIG. 3.

infinite series, but of such rapid convergence that only the two first elements are of practical importance to the automobile engineer,† though there are cases in which the third member of the series may have appreciable effect.

7. *Torque Vibrations*.—Attention will now be directed to a cause of vibration which, under certain conditions, may render itself most apparent to users of automobile vehicles. It will be noted, in considering the problem as far as it has been developed, that the piston of our one-cylinder engine has two positions per stroke when its kinetic energy is zero, and two positions when its kinetic energy is maximum. This betokens that

* The two curves *c* and *d* are, as shown, based on different datum lines; if referred to the same datum their difference would be indistinguishable.

† See Appendix I., also Discussion, p. 261.

for a quadrant (roughly speaking) after crossing the in or out centre, the crank shaft is doing work on the piston, and for the succeeding quadrant to the next centre the piston is giving up its energy and doing work on the crank shaft; thus, there is an alternation of torque twice per crank shaft revolution, or in Fig. 1 the applied torque (that is to say, the torque required to belt round the engine) will be clockwise in the quadrants *a* and *c*, and counter clockwise in the quadrants *b* and *d*. This is not all, however; the torque applied to the crank of the engine and acting on the piston through the connecting rod involves an equal and opposite torque applied to the engine bed or frame from without, and thus the crank shaft torque may be regarded as acting directly on the frame of the engine and undergoing two reversals per revolution. Such an intermittent torque is evidently a potential cause of vibration.

Now let us assume that the engine be furnished with a flywheel. The whole of the foregoing facts remain unaltered provided that we regard the torque applied to the crank shaft as that existing between the crank itself and the flywheel; it may be noted, however, that the belting torque, that is, the torque by which the engine is being artificially driven, no longer need correspond to the internal torque of the engine. For the present it is sufficient to call attention to this point; its bearing on the vibration question will be developed later.

We may assume, as the next step, that the condition of compression be added. As we know, the compression in a single-cylinder engine may take place at every revolution as in a two-stroke cycle engine, or it may take place every alternate revolution as in a four-stroke cycle engine. On the question of vibration, the effect of compression is felt again in the torque variation; thus, during the compression stroke work is expended on the working fluid, and during the expansion stroke the work so expended is restored to the flywheel, so that during the half-revolution represented by the compression there is a torque acting on the frame of the engine in the direction of rotation, and during the expansion period there is a torque acting on the frame of the engine in the direction contrary to that of rotation. The torque intermittences due to compression (and explosion) and those due to piston inertia are in practice superposed, and the algebraic sum of the two, which is their resultant, is that which acts on the engine bed. Over certain portions of the stroke and at certain speeds these torques may, to some extent, neutralise one another.

The torques due to compression and those due to piston inertia follow different laws; whereas the former are approximately constant in relation to engine speed, the latter vary as the square of the number of revolutions per minute. The question of the interaction of these two intermittent torques will be considered more fully when the multi-cylinder engine is under discussion.

When we add the question of the explosion pressures, we are, in effect, augmenting to the extent of some four or five times, the torque during the expansion stroke, that is, an increase of the torque acting on the crank and flywheel in the direction of rotation and on the frame of the engine in the contrary direction; here we have the recoil or kick so well known in the case of single-cylinder engines as the cause of a most disagreeable vibration. Its effect is also felt in multi-cylinder engines, in what is frequently referred to as the "pick up" vibration.

8. *Partial Balancing of the Single-Cylinder Engine—a Compromise.*—Before we can regard our one-cylinder engine as complete, we must assign weight or mass to the crank itself and the connecting rod. So far as the mass of the crank itself is concerned, it may be completely balanced, as is usual, by rotating counterweights on the opposite side of the shaft, and so treated it ceases to interest us from the present point of view. The connecting rod is a matter of greater subtlety. As a first approximation, we may, as is customary, consider the mass of the rod as divided between the large and small ends, according to their respective weights, the large end mass being treated as part of the crank itself and balanced by an addendum to the rotating counterweight on the crank shaft, and the small end mass being treated as in effect part of the piston weight.

If the shank of the connecting rod were of considerable mass, it is probable that the above approximate method of treatment would require reconsideration, but, generally speaking, and certainly for the purpose of the automobile engineer, the above method of treatment is all that is required.

From the foregoing it will be clear that in a single-cylinder engine, apart from unavoidable torque variations, which are in themselves of a serious character, the question of vibration resolves itself into a question of the mass of the reciprocating parts. Taking an ordinary 85 mm. by 88 mm. bicycle motor running at 3,000 revs. per minute, the forces due to the main harmonic motion amount to about 800 lb., and those due to

the secondary amount to about 180 lb., making a total maximum at the in-centre position of 980 lb. It is customary in single-cylinder engines to endeavour to mitigate the gravity of the position by partially balancing the equivalent reciprocating mass of the piston by rotating weights, but it is quite obvious that such a device is, at the best, a rough and ready make-shift. The practice, however, although it does not enable the engineer to eliminate vibration from the cause in question, gives him considerable control over its magnitude and direction; the method is, therefore, of more utility than otherwise might be supposed.

If we take the extreme case when the whole of the reciprocating parts are treated and balanced as if they were rotating parts, that is to say, as if their mass were concentrated on the crank pin, it is of interest to recapitulate the well-known fact that the resultant unbalanced forces may be represented by a reciprocation at right angles to the direction of piston motion of equal magnitude to the original unbalanced forces, at least so far as the fundamental harmonic movement is concerned. Thus, if the structure on which the engine is mounted is better capable of taking vibration or of resisting vibration stresses in one direction than in another, the method in question, even as a means of control, may be of extreme utility.

If, as is frequent, half the reciprocating weight be balanced by a rotating mass on the crank shaft, the well-known result is that the unbalanced residue is the equivalent of a mass equal to the added balance weight rotating at the radius of the crank pin in the direction opposite to the motion of the engine, the maximum disturbing force thus being half its original value. For other proportions of rotary balance weight, the resultant motions are reverse rotations in elliptical orbits of various degrees of eccentricity. In some cases the best degree of balance can be estimated or computed by a study of the conditions of the mounting of the engine or the conditions of usage,* in other cases, as where it is a question of personal comfort, experiment with a variety of different degrees of "over-balance" is the only safe guide.†

9. *The Two-Cylinder Engine.*—We will now pass to a brief

* In railway work, for example, forces tending to produce a vertical pounding action are objectionable. Horizontal forces tending to vary the draw bar pull are also objectionable, but in most cases less so.

† The term "over-balance" is here used to denote the extent of rotary balance added beyond that necessary to balance the actual rotating parts.

study of the additional factors introduced in the case of the two-cylinder engine. There are many arrangements of two-cylinder engine, but, firstly, we will consider the ordinary vertical two-cylinder, that is to say, that in which the cylinders are arranged side by side. In this case it may be noted, first, that the main reciprocating forces may act in the same or in contrary direction according to whether the crank pins are in line or opposed, that is, whether they are together or separated by an angle of 180 degrees. In the former case, so far as reciprocating forces are concerned, the problem resolves itself again into that of the single-cylinder engine, but in the latter case a new factor in the vibration question makes its appearance; the main forces due to the fundamental piston motion (not to its octave) are balanced, but as they act at points separated by the distance between the cylinder axes they give rise to a variable torque or moment about a transverse horizontal axis, and it is customary to term moments of this kind rocking moments, as distinguished from the torque moments about the crank axes itself. It is almost unnecessary to point out that in either case the octave vibrations synchronise, so that, whether the two-cylinder engine has its cranks together or at 180 degrees, the forces that give rise to an octave vibration will be present with full severity.

When we consider the question of torque variation, it is clearly desirable in any multi-cylinder engine to arrange the explosion recoil torques of the different cylinders at equal periods of time; in the two-cylinder engine we have been considering, this will be consistent with neutralising the main inertia force only so long as we are dealing with the two-stroke cycle; that is to say, a two-stroke cycle engine with cranks at 180 degrees will have the impulses occurring at equal intervals of time; when we are concerned with the four-stroke cycle the equal spacing of the impulses can only be obtained by arranging the crank pins in line, and thus the designer has to choose between two conflicting requirements, the equal distribution of impulses and the balancing of the fundamental components of the piston motions. This difficulty was encountered some twenty-five or thirty years ago in the construction of two-cylinder gas engines; in the early engines of this type built by Messrs. Crossley Bros. the piston motions synchronised, that is to say, the balancing of the inertia forces was sacrificed to obtain equal distribution of the working cycles, but after a very few years this type was superseded by a type

of engine in which the cranks were arranged at 180 degrees, the question of inertia balancing being found to be the more important factor of the two. To some extent history repeated itself in the development of the motor car engine, until of recent years the two-cylinder vertical engine has been virtually abandoned.

When we pass to the question of the octave component of the piston motion and the intermittent torque due to piston inertia, we find that neither of these can be dealt with satisfactorily on the two-cylinder engine in either form without abandoning both of the other factors. It will be necessary to arrange the cranks at right angles to eliminate these as disturbing forces, and bring them down to the status of disturbing couples. Such a crank distribution is generally considered inadmissible.*

There is a form of two-cylinder engine which certainly never had the vogue to which it is entitled—the type in which the cylinders are opposed and the cranks are at 180 degrees. An examination of this type of engine shows, firstly, that the impulses are equally distributed, secondly, that the fundamental piston motions are balanced, thirdly, that the octave piston motions are balanced, fourthly, that the rocking moments due to cylinder axis spacing may be made as small as we please; by a little ingenuity and design the cylinders can be arranged co-axial. The only unbalanced features worthy of consideration are the intermittence of torque due to piston inertia (compare Section 7), and the intermittence in torque due to the explosion. Without adopting some specially devised means of balancing or going to a multiplicity of cylinders or some form of rotating engine, it is impossible to obtain a more perfect proposition from the balancing standpoint. The author attributes the fact that this type of engine has not come into prominence almost entirely to popular prejudice. The popular idea has been for many years that an engine cannot be right unless it is vertical. It would show quite as high a degree of popular intelligence if an engine were not deemed “the thing” unless *painted blue*.

10. “*Looking Glass*” Symmetry.—In the foregoing Section the question of the rocking moment due to the axial separation of the cylinders with their reciprocating parts was touched upon. At the present juncture it is desirable to call attention to the method by which the rocking moment is eliminated in the types of multi-cylinder engine commonly employed for automobile purposes, that is, the method of symmetrical distribution. The essential point is

* Compare Section 13.

that the arrangement of the crank throws should be one of "looking glass" symmetry. If we examine the case of the one-cylinder engine in which no rocking moment exists, or the case of the two-cylinder opposed engine in which no rocking moment exists (i.e., with the two cylinders *exactly* opposite), or that of the four- or six-cylinder engine commonly employed in high-class automobiles, it will be found that there is a plane of symmetry at right angles to the crank axis, that is to say, a plane which may be said to represent a reflector, in which the one half of the engine is a perfect reflection of the other half; thus, in the case of the one- or two-cylinder engine the said plane passes through the cylinder centre line, and in the four- and six-cylinder engine the plane is situated half way between the middle pair of cylinders. Speaking broadly, it is not strictly essential that a plane of geometrical symmetry should exist; the symmetry may be of a dynamic kind. It is of interest, however, to point out that in the case of all actual automobile engines in which the rocking moment is eliminated, the distribution of cylinders is, as a fact, on a strictly symmetrical basis. It requires no proof to show that an engine possessing symmetry of the kind under discussion is free from rocking moment, since, whatever couple one half of the engine may set up at any instant, the couple in the opposite half of the engine, being a reflection of this, must be equal and opposite. The three-cylinder engine may be cited as an example of a type of engine at one time to be met with in which the rocking moment had not been eliminated.

11. *The Torque Recoil*.—Passing from the consideration of rocking moment, that is to say, of couples about axes at right angles to the crank axis to those about the crank axis itself, we have to deal with considerations of an entirely different character. The ideal engine from the present point of view would be one in which, as in an electro motor, the driving torque is constant, but no reciprocating engine can be said to have this quality even approximately; the best approximation is obtained by employing a great multiplicity of cylinders. For many years, and in fact until comparatively modern times, the question of torque variation was exclusively dealt with from the point of view of the crank shaft; that is to say, in the steam engine for example, efforts were continually made by designers to get as constant a torque as possible, but always with the end in view of minimising speed and torque variations on the crank shaft and drive. It was well

understood that the greater the torque variation the heavier the flywheel necessary to ensure constant speed, but little, if any, attention was devoted to the question of the *torque recoil*. When the gas engine came into prominence, and the flywheel at once became a more essential and functionally more important part, the fact was almost entirely overlooked that, although a sufficiently massive flywheel would give the necessary steadiness of running from the point of view of speed variation, it has no influence whatever as affecting the rotational recoil. The author believes that the question was first dealt with prominently in his own patent specification, 15045 of 1895, but even since that date the question has been comparatively little understood, and many claims have been made to dispose of the rotational recoil, or, as it is sometimes termed, the "kick," of the explosion, by means that can have no effect whatever, as, for example, the employment of two opposed pistons in one cylinder.

12. *Torque Recoil as a Cause of Vibration.*—Let us consider the case of a single-cylinder engine, which we will imagine to be provided with some kind of torsion balance, by which the recoil torque about its crankshaft axis can be measured or recorded. We will examine a period comprising one explosion and expansion, and to fix our ideas we will take it that the (brake) effective mean pressure in the cylinder amounts to 100 lb. per square inch; we will assume, as must be approximately the case, that a quarter of this is expended in overcoming the work done externally by the motor during the expansion stroke, thus leaving 75 lb. per square inch as applying an acceleration torque to the flywheel; this will represent roughly 6 ft.-lb per cubic inch of cylinder volume energy added to the flywheel; if we know the mass and diameter of the latter, we can calculate exactly what increase of velocity this means, and what additional angular momentum the flywheel has received. Now angular momentum received by the flywheel means torque applied to it, and if we regard the whole engine as a self-contained system, then since the bed and cylinder are not set in opposite rotation, a torque must have been applied to the bed of the engine sufficient in the time of the stroke to impart the calculated angular momentum to the system, and this torque is, in other words, the torque required to resist the rotational recoil of the explosion. It evidently matters nothing whether the cylinder is fitted with a multiplicity of pistons or whether there is an elaboration of connecting rods or link work between the said pistons and the crank shaft. There

is a fundamental principle at stake, and as long as energy is stored in the flywheel at each explosion by a torque applied directly or indirectly from the motor cylinder, the equal and opposite torque must be borne by the attachments of the engine, whether it be a foundation or the chassis of a vehicle. There is only one method of evading this intermittent torque known to the author, and it will be discussed later in the paper.

We know that if any given torque act for a time t , the angular momentum it will communicate is directly proportional to the time t ; thus, in cases where the bed of the engine is not fully secured, such as when mounted on the chassis of a motor car, the amplitude of the kick of the engine will be greater as the length of time over which the recoil torque acts is greater. Now the torque is virtually independent of the speed of rotation (within the limits of ordinary usage), hence the amplitude of the kick is, in some measure, inversely as the velocity of rotation. It is well known to users of motor vehicles that the roughness of a single- or a multi-cylinder engine is greater at low speeds than at high speeds; in fact this "roughness" is frequently referred to as the "pick up" vibration, since it is felt more acutely when the speed of the engine is low and the throttle is fully open, as when picking up speed from rest or from a lower gear.

13. *Balancing illustrated by model.*—It has already been pointed out that one of the advantages of the multi-cylinder engine is the reduction of the irregularity of the recoil torque; the greater the number of cylinders, the more nearly the driving effort is constant, and counterpart to that, the more nearly the recoil torque is constant, but in order to obtain the best results from the multiplicity of cylinders, it is self-evident that the distribution of the impulses should be made as nearly uniform as possible. This point has been touched on in considering the two-cylinder engine.

The author some ten or twelve years ago devised a simple means of illustrating in four-stroke cycle engines the relations between impulse frequency, the fundamental piston frequency and the octave piston frequency, in order to illustrate in a diagrammatic manner the virtues and failings, from the point of view of vibration, of engines of different cylinder number and type. This device is represented in Figs. 4 to 8.* Three

* These figures were illustrated by means of a model during the reading of the paper.

drums are provided whose diameters are in the relation of the respective frequencies in question, namely, in the ratio

FIG. 4.—Four cylinders.

of 1 to 2 and 4, and strings threaded with a number of beads to correspond with the number of cylinders are provided.

FIG. 5.—Six cylinders.

The drums will be referred to as the cycle drum, the main drum, and the octave drum. Taking firstly, as an illustration, Fig. 4, a

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string threaded with four beads representing a four-cylinder engine, in order that the impulses should be equally spaced, the beads

FIG. 6A.—Two cylinders, cranks together.

are arranged at equal distances, that is to say, quadrants on the periphery of the *cycle* drum. If, now, the string is passed round

FIG. 6B.—Two cylinders, cranks at 180 degrees.

the *main* drum, it will take two turns, and the beads will be found two on one side and two on the other diametrically opposite;

this demonstrates that the main vibration is balanced by two cylinders being always opposed to the other two. Now let the

FIG. 7.—Two cylinders opposed, cranks at 180 degrees.

string be passed round the *octave* drum; it will be found that in the four turns the beads all come on one side of the drum, thus

FIG. 8.—Two cylinders, cranks at 90 degrees.

denoting what we already know quite well, that in the four-cylinder engine the secondary or octave vibration is totally unbalanced, the

four cylinders acting synchronously. Let us now take the case of the six-cylinder engine (Fig. 5). On spacing this equally on the *cycle* drum at the corners of an imaginary hexagon, we find that on the *main* drum the beads take their place in pairs on the corners of an imaginary equilateral triangle, and on the *octave* drum, the beads are again found to be in pairs at the corners of an equilateral triangle, showing that the six-cylinder engine is balanced for both fundamental and octave vibration. Let us now try a smaller number of cylinders. We will take the two-cylinder engine in which the impulses are equally divided (Fig. 6A), and we now find that when the string is placed on the *main* drum, two beads come together, and similarly on the *octave* drum, showing that both fundamental and octave piston vibration are unbalanced. If we now so distribute the beads that the fundamental vibration is balanced (Fig. 6B), by arranging these opposite on the *main* drum, on passing the string round the *cycle* drum we see that the two beads are 90 degrees apart, and that the impulses are not properly spaced; on testing the string on the octave drum we find again that want of balance exists. If we take the special two-cylinder engine with opposed cylinders (Fig. 7), we require to remember that as the cylinders are opposite the reciprocation balance will be in order when the beads come together, and not when they are on opposite sides; to remind us of this, the author has arranged a black and white bead on the string representing this type of engine. Here, arranging the impulses as equally spaced, that is, separated at equal distances on the *cycle* drum, we find, on placing these on the *main* drum, that the black and white beads come together, and similarly on the octave drum the black and white beads come together, showing the engine to be balanced in both cases.

It is quite true that the model does not tell us anything we did not previously know, or at least that we could not ascertain in other ways, but it appears to the author none the worse on that account. It is, in effect, a pictorial analogue, and, as such, is of service.

14. *Recoil Balancing—Reverse Rotation.*—The fundamental character of rotational recoil has already been demonstrated, and also the fact that no combination of pistons and connecting rods, however elaborate, can affect the recoil kick of a given engine. In the ordinary multi-cylinder engine it has been shown that the trouble is mitigated by substituting a number of distributed

torque applications of small magnitude for the intermittent and heavier torque of the equivalent single-cylinder engine. There is a method, however, by which the torque recoil may be eliminated. The author believes the method in question to have been first described in his patent application in 1895; it consists in the employment of two flywheels having opposite rotation. Where two

FIG. 9.

flywheels of equal moment of inertia are arranged or geared together to rotate in opposite directions, the *storage of energy does not involve a change of angular momentum of the system*; thus if, for example, starting from rest, two equal flywheels be given equal velocities of rotation, the one clockwise and the other counter clockwise, as in Fig. 9, a storage of energy has taken place just as effectively as if the two flywheels had been running

in the same direction, but the angular momentum of the one is positive and that of the other negative, and the total angular momentum is zero, just as it stood when the flywheels were at rest; hence no torque has been applied to the system from without, and the engine possessing two such flywheels can function without any rotational recoil kick being transmitted by its holding-down bolts. The same result can be demonstrated by a careful analysis and resolution of the connecting rod, piston and crank shaft forces, but the author considers the fundamental method of dealing with the problem here adopted to be preferable. It is easy to make a

FIG. 10.

slip in resolving forces and moments in a complicated mass of link work, and it is an altogether wrong way of tackling the problem.

The method of reverse rotation also provides the solution to another problem connected with balancing; thus, an unbalanced reciprocation can be, in effect, balanced by two rotating weights. Referring to Fig. 10, if two spindles, 1 and 1', be arranged to rotate in opposite directions and carry respectively balance weights 2 and 2', whose centres of gravity in rotation take up the positions respectively $aa', bb', cc', dd', ee', ff'$, it will be seen that the combined centre of gravity of the balance weights follows a straight line path with strictly harmonic motion, the

combined centres of gravity being represented by the letters *A*, *B*, *C*, *D*, *E*, *F*, as the spindles revolve.

15. *Reverse Rotation—The Lanchester (1895) Engine.*—The principles discussed in the two preceding sections, 13 and 14, are conveniently illustrated by the old Lanchester type of engine, which may be said to date from patents Nos. 15045 of 1895 and 13960 of 1896, and which was manufactured over a period of years extending from 1896 to 1903. Referring to Fig. 11, two parallel single-throw crank shafts were provided and fitted with flywheels of equal moments of inertia, geared together

FIG. 11.

by helical gearing. Two opposed cylinders were arranged symmetrically on opposite sides of the plane containing the crank axes, and the connecting rod system consisted of a symmetrical parallelogram of, functionally speaking, four rods, actually six rods being employed. Both crank shafts were fitted with balance weights whose function was not only to balance the rotating portions of the cranks themselves but also to balance, on the principle already discussed, the reciprocating parts of the engine.

By an inspection of Fig. 11, it will be seen that the distribution of pistons and connecting rods at all times formed a symmetrical figure about a point *A* half way between the centres of the

two crank pins, and so the whole motion of the reciprocating parts could be treated as a mass concentrated at the point in question with strictly harmonic motion; it could, therefore, be balanced with mathematical accuracy by oppositely rotating weights on the crank shafts. The engine was not troubled with any rocking moment owing to the fact that the whole of its reciprocating parts had "looking glass" symmetry about the transverse vertical plane. The rotational recoil impulse was balanced completely by the reversely rotating flywheels of equal moment of inertia; the fact that these lie in two different planes is of no importance. Though the distribution of the impulses does not take place at equal angular intervals, this is without effect on the balance of the engine owing to the absence of rotational recoil—since there is no unbalanced kick it is of no consequence when the kick takes place. From a scientific point of view, probably this engine is the only completely balanced type of reciprocating engine that has ever been built.

In connection with this engine, it is of interest to call attention to a kind of vibration that may be felt, or at least may make itself appreciable, apart from any vibration of the engine proper. Let us take an engine of the type under discussion, and, admitting that it is perfectly free from vibration, let us put it to work to drive a car. At each impulse there will be a variation in the flywheel velocity; this, in an ordinary engine, would produce a recoil on the frame, which in the present engine, owing to the reverse rotation, does not take place. It produces, however, an increased acceleration on the vehicle, that is to say, every time the flywheels are accelerated by the expansion of the gases, the vehicle has to follow suit, and so, owing to the inertia of the vehicle, the transmitted torque, that is to say, the torque **between the engine flywheel and transmission mechanism**, is a variable torque, and this in turn reacts on the engine and its mounting.

We have here an extremely interesting property of the engine with reverse rotation, *the torque carried by its mounting is precisely the torque impressed by the driven mechanism*; in other words, if the work done is intermittent, the torque on the engine mounting is intermittent to a like degree, whereas if the *driven* torque is constant, the engine mounting is not subject to fluctuation.

It is an interesting point in connection with the motor with

reverse rotation that the same means taken to eliminate rotational recoil or kick also eliminates the torque variations and torque vibrations which have been shown to be due to piston inertia (section 7).

It is, in the author's opinion, very important to be clear on the distinction between the transmission of vibration to the engine mounting as due to variations in the work done and the transmission of vibration as due to the working cycle of the engine itself. Thus, it may be laid down that in cases where the work done is of an intermittent and vibratory nature, the employment of a motor with reverse rotation may result in transmitting vibrations to the engine mounting to a detrimental degree, and such an engine is therefore bad, whereas in an ordinarily constructed engine with a single flywheel all such vibrations and torque variations are intercepted by the flywheel and are not felt by the engine mounting; it will therefore be seen that, broadly speaking, where questions of vibration have to be dealt with, the conditions have to be studied individually, and what may be right in one case may be equally wrong in another.

16. *Reverse Rotation—The "Valveless" (Lucas) Engine.*—The manufacture of the Lanchester engine, made in accordance with the patents aforesaid, and described in the foregoing sections, has been discontinued for many years; in spite of this there are, however, several hundreds still on the road. More recently an engine having the same main feature, that is to say, the two crank shafts with oppositely rotating flywheels, has been put on the market under the name of the "Valveless," Fig. 12; this engine, made under patents of Mr. R. Lucas, works on the two-stroke cycle. This is a matter quite apart from its balancing devices, which are in their essence identical with those described above. In the Lucas engine, the crank shafts are acted on by two cylinders side by side with axes parallel; the design is inferior to the Lanchester type in so far that the secondary or octave vibration is not balanced; it is superior from the point of view of simplicity and ease of manufacture, two of the factors which eventually led to the abandonment of the author's earlier type.

17. *The Four-Cylinder Engine—Octave Vibration.*—The four-cylinder engine, which may be taken broadly as the popular motor car engine of to-day, has been dealt with to some extent in previous sections, but on account of the commercial importance of this particular type, it is necessary to devote to it more than passing attention. We have seen that

the main defective balance is due to the synchronising of the octave components of the motion of all four pistons, that is to say, the displacements of the pistons as due to connecting rod angularity all take place in the same sense, so that the forces in a four-cylinder engine tending to set up octave period vibration are four times as great as would be the case in a single-cylinder engine of equal bore and stroke. In Fig. 13 we have represented

FIG. 12.

diagrammatically the movement of the gudgeon pin, corresponding positions of which are given at the "in" and "out" centres and at 90 degrees thereto. If the connecting rods were of infinite length, all four pistons would reach mid-stroke position at the same instant, and at all other points pistons Nos. 1 and 4 would by their motion accurately compensate for Nos. 2 and 3. In an engine with a rod length of about two and a quarter times the stroke (Fig. 13), there is a considerable departure from this in the position of the pistons when they *should be* at mid-stroke. Thus, when the crank pin is at 90 degrees from the in-centre, all four pistons are somewhat below their mid-stroke position; in the case of an ordinary 20 or 25 h.p. engine (R. A. C. rating) the

error of position amounts to about a quarter of an inch. Let us express the length of the connecting rod in terms of the crank throw = unity, and take the ratio as 4.5; then, in Fig. 13, if a be the position error of the gudgeon pin at mid-stroke, as due to connecting rod angularity, we know that a is approximately equal to the *square of the crank throw divided by twice the connecting rod length*, or in the present case one divided by nine. But the quantity

FIG. 13.

a is the total amplitude of the octave vibration of the piston, hence the octave amplitude is $1/18$ th of the stroke. Now, since the force is proportional to the square of the periodic speed, the maximum disturbing force due to the octave components of the piston motion will be $4/18$ ths of that due to the main component of each piston; multiplying this again by four to represent the whole engine, we have the unbalanced force due to the octave component in a

four-cylinder engine nearly as great (in the relation of 16 to 18)* as the main unbalanced piston force of a single-cylinder engine of equal bore and stroke. This comparison is given graphically in Fig. 14, which represents the unbalanced forces in a four-cylinder engine in comparison with the unbalanced forces in a single-cylinder engine of equal bore and stroke, assuming the base chambers of both engines to be bolted rigidly to the foundations.

FIG. 14.

As an illustration, we may take the case of a 130 mm. stroke engine of about 20 R. A. C. rating running at 2,000 revs. per minute with a total reciprocating mass of 12 lb.; the maximum value of the unbalanced alternating vibratory forces under the conditions stated is approximately 800 lb.

18. *The Four-Cylinder Engine* (continued).—In addition to its defect in the matter of the secondary vibration as above, the four-cylinder engine is affected by another of the forms of want of balance to which the single-cylinder engine is

* If the rod/throw ratio had been taken = 4 we should have had equality.

liable, that is to say, the torque alternations due to piston inertia. In the four-cylinder engine the whole of the pistons are at rest at the same instant of time, and the whole have their maximum velocity at approximately the same point. Thus, as in the one-cylinder engine, there is a continual give and take of energy between the flywheel and the pistons, which results in an intermittent torque about the crank shaft axis just as described in section 7. The magnitude of this torque in the four-cylinder engine is that of the sum of its combined elements, that is to say, it is four times as great as in the case of a single-cylinder engine of equal stroke and piston weight. The four-cylinder engine also (as in the single-cylinder) has a torque alternation due to the work done in compression and restored in expansion. This torque variation may be treated as part of the functional torque variation due to the total cycle of operations, and from this point of view the torque variations due to the compression and explosion pressures cannot be separated.

We have already seen that torques due to pressure and those due to piston inertia are actually superposed, and since in the four-cylinder engine there is a compression in one cylinder or another on every stroke of the engine, they are of the same frequency, and there are conditions of running under which these variations in some degree cancel out.

Let us suppose that powerful springs are fitted, tending to bring the pistons to their mid-stroke position; then it is evident that there is some particular running speed of the engine which corresponds to the period of the springs and piston masses, and at this critical speed no torque variation will be required. The particular speed in question is that at which the kinetic energy of the piston in its mid-stroke position is just equal to the energy stored in its spring when fully compressed, so that at every point in the stroke the total of the energy in the springs and the kinetic energy in the pistons will be constant. At this speed, since the energy of the system does not fluctuate, there will be no give and take of energy between the pistons and the flywheel, and so the torque fluctuations that otherwise are necessary will not occur.

Now, in fact, there is no such spring system as that indicated, but if we consider the combined effect of the four indicator diagrams, assuming the engine to run against compression only, it will be seen by referring to Fig. 15 that the combined indicator diagrams would be such as to simulate with a fair degree of approximation a straight line spring diagram, and so in an actual

engine the form of vibration now under consideration will be far less apparent when running on compression than when closely throttled down, and so the spring effect of the compression is destroyed.

As evidence that this is not a mere academic digression, the fact may be cited, known to every motorist of experience, that there are conditions of running under which a vibration of disagreeable magnitude manifests itself the instant the throttle is closed. The speed at which this effect manifests itself depends

FIG. 15.

upon the specific weight of the pistons and the degree of compression.

19. *The Lanchester Anti-Vibrator*.—The author has recently introduced a device with a view to the elimination of some of the defect of balance of the four-cylinder engine.* The device in question is capable of dealing with both forms of vibration discussed in the preceding section. The function of the new device or "Anti-Vibrator" is to supply a counter-vailing movement of mass of the same periodicity, and of

* Patents 26038 and 26777, 1911.

Balancing Erection.
FIG. 16.

sufficient magnitude to neutralise the unbalanced octave component of the piston motion. To this end, two reverse rotating balance weights are employed, or two pairs of such weights each of which supplies a vertical component sufficient to quench one-half of the unbalanced piston component, the horizontal component of motion of the rotating masses being arranged to neutralise one another after the style of the balance weights fitted to the crank shaft of the old 12 h.p. Lanchester motor. This apparatus is shown in two of its forms in Figs. 16, 17 and 18, Fig. 16 showing the apparatus as fitted to the 25 h.p. Lanchester car, and Figs. 17 and 18 as applied more generally to four-cylinder cars where the space in the crank base permits.

An interesting point relating to this new application of the principle of reverse rotation to balancing is found in the fact that though, from a purely functional standpoint, the reverse rotating masses are merely the equivalent of a single reciprocating mass, from the dynamic or engineering point of view this is not the case. Attempts have been previously made to balance the octave vibration by a reciprocating mass or masses of appropriate frequency, but the remedy has proved worse than the disease. In a reciprocating mass, the whole energy has to be taken out and put back again twice per revolution, and if such a mass be driven through the intermediary of gearing and, say, crank and connecting rod, the gearing and mechanism requires to be designed to transmit very heavy stresses, and the lost motion or backlash results in intolerable noise and wear. In the author's Anti-vibrator, on the contrary, the energy content for uniform motor speed is constant, and the gearing is mere motion transmission, and the existence of some backlash in the gears is quite harmless.

This Anti-vibrator can be employed simultaneously for the correction of the octave vibration and for the elimination of vibration due to piston inertia. A note in Appendix III. deals with its application in this respect. In practice, however, owing to the compression pressure effect already discussed, it has not hitherto been found of material advantage to extend its application in that direction.

20. The Radial Engine—The Vee Type.—We have so far dealt exclusively with engines in which the cylinder axes are in the same plane, such as in the ordinary vertical motor car type. There are other types of engine (the importance of which at the moment is mainly due to their appli-

Front end of Engine.

Section thro' A.B.

Section thro' C.D.

FIG. 17.

FIG. 18.

cation to aeronautics) in which the cylinders are arranged radially round a crank shaft at different points of the circumference. In some cases, as in the 80—100 h.p. Anzani, or the R. E. P. engine, the distribution of the cylinders is symmetrical, their axes being spaced at equal angles round the crank axis, in other cases only a portion of the circumference is utilised, or again in other cases, as in the ordinary Vee type of engine, the arrangement resembles more the “twinning” of two multi-cylinder engines whose cylinder axis planes are arranged at an angle. The latter type is best considered from the point of view of balance as two distinct engines, and the unbalanced components of the two superposed to give the resultant.

In the case of the eight-cylinder Vee engine (two fours), the octave disturbance is only balanced if the angle between the cylinder planes be made 180 degrees. Such an arrangement is

FIG. 19.

considered bad owing to the fact that the working strokes cannot be properly distributed, the explosion always taking place in pairs of cylinders simultaneously. The usual arrangement for the eight-cylinder engine is for the two fours to be set at an angle of 90 degrees, so that four impulses take place per revolution at 90 degrees interval. In such an arrangement the octave vibration is not balanced, the resultant vibration being represented by the horizontal diagonal of a square in which the component vibrations of the two sets of cylinders are represented by two of the inclined sides, Fig. 19. In such a case the vibration can be corrected by the Anti-vibrator, the disposition of the rotating balance weights being that corresponding to a four-cylinder engine in a horizontal position, Fig. 20.

In the case of a “double six” built on the Vee system, the

angle between the cylinder axes should be either 60 degrees or 180 degrees; by this means the impulses, six per revolution, can be distributed evenly at 60 degree intervals; the balance of such an engine requires no comment, inasmuch as each component engine is in itself balanced in respect of its fundamental and its octave vibration, and is not subject to inertia torque variation.

21. *The Radial Engine—Symmetrical Types.*—The simplest

FIG. 20.

case of the radial cylinder engine with symmetrical distribution is the two-cylinder engine with opposed cylinders, discussed in section 9; in an engine of this type there is one impulse per revolution, and thus, if it be taken as a unit of a four-cylinder radial engine, it will take the form given in Fig. 21, in which there is an impulse at every half revolution. As an eight-cylinder, it will take the form of two such four-cylinder engines "twinned" at right angles to each other, giving

an impulse at each quarter of a revolution. This type of engine, with any of the numbers of cylinders above given, or any higher multiple of four, is balanced in respect of both fundamental and octave vibration.

It will be noted that in all the types of radial engine discussed in the present section the cylinders run in pairs, that is to say, on each cylinder axis plane there will be found two cylinders, or a multiple of two, side by side. The origin of this peculiarity is found in the fact that we have assumed the four-stroke cycle as the basis of discussion. Where the two-stroke cycle is adopted, in most cases each pair of cylinders may be replaced by a single cylinder, but in some cases, as in the ordinary motor car type of engine, the difficulty of maintaining "looking glass" symmetry may arise and result in the appearance of unbalanced rocking moments. In the radial engines so far considered, the crank shafts have more than one throw; there is, however, a simpler

FIG. 21.

type of radial engine in which a single throw crank is employed. In this type, if the four-stroke cycle is used, equal spacing of the impulses can only be obtained by employing an *odd number of cylinders*; where the two-stroke cycle is employed, an even number of cylinders may be adopted. In the author's opinion, the most satisfactory view to take of the balance of such a motor is that where the cylinders are sufficiently numerous the equal radial distribution of the pistons disposes with a sufficient degree of approximation of the secondary or octave inequality. A mere inspection of the problem shows clearly that this must be the case. Beyond this the energy content of the pistons as a whole is approximately constant, so that an engine of the radial type is not liable to torque irregularity due to piston inertia. The main piston motion

can be dealt with with sufficient accuracy by regarding the whole weight of the pistons and connecting rods as being concentrated as a rotary mass on the crank pin and balancing by an equivalent rotary counterweight on the crank shaft in the usual way.

22. *The Rotating Engine.*—Closely akin to the radial type of engine dealt with in the last paragraph of the preceding section is the rotating engine, of which the Gnome is, perhaps, the best known example. From a kinematic stand-

FIG. 22.

point, this type of rotating engine is identical with the corresponding radial engine, the difference being that in one case the crank shaft rotates within the cylinders, and in the other case the cylinders and the whole engine rotate around a fixed crank shaft. Although the two engines are identical as a matter of kinematics, the difference being merely a question of relative motion, as a matter of dynamics there is considerable difference between the two. The fixing of the crank shaft renders

the balance weights unnecessary, and so a considerable saving of weight is at once effected. The dynamic system becomes inverted, and if treated by other than mathematical methods requires to be considered afresh. The connecting rods with their pistons attached now swing round the stationary crank pin, and although the path of the gudgeon pin is circular (if we ignore irregularities introduced by want of symmetry in the big end

FIG. 23.

articulation) an angular irregularity of motion exists that prevents us regarding the balance as necessarily exact.

Referring to Fig. 22, we will assume that we are dealing with a four-cylinder rotating engine with fixed crank shaft; in this case, as in every case where the number of cylinders in this type of engine is even, it is easy to show by a simple geometrical demonstration that the so-called reciprocating parts are in balance; thus, referring to Fig. 23, which represents in skeleton the engine in

question, we have the four piston masses 1, 2, 3 and 4 situated on the connecting rod circle $p p p$, whose centre is the crank pin o . The lines 1, 3 and 2, 4 represent the axes of the rotating cylinders, and are at right angles. If at different points of the stroke perpendiculars are let fall from the centre of the crank pin o on to the lines 1, 3 and 2, 4, they will be bisected at the points q_1 and q_2 , the locus of which points will be situated on a circle passing

FIG. 24.

through the centre of the crank pin o and the centre of the crank shaft S , and of which the line $o S$ is a diameter. From geometrical considerations, it follows that the point q_1 bisects the distance between the points 1 and 3, and so the point q_1 will be the common centre of gravity of pistons 1 and 3. Likewise the point q_2 will be the centre of gravity of the two pistons 2 and 4. From geometrical considerations we know that the point q_1 is

diametrically opposite to the point q_2 , consequently the common centre of gravity of the four pistons 1, 2, 3 and 4 is situated at the point Q , and is immobile. Hence a rotating engine of the type in question with four or in fact any even number of cylinders is accurately balanced so far as its main reciprocating parts are concerned.

23. *The Rotating Type—Number of Cylinders Odd.*—The author has not been able to find a complete geometrical solution that is applicable to the rotating type of engine in which the number of cylinders is *odd*, and in fact the balance in such a case is not theoretically perfect. The imperfect nature of the balance of a rotating engine with an odd number of cylinders may be demonstrated geometrically by considering an extreme case. Firstly, let us examine an extreme case where the number of cylinders is *even*, and let the connecting rod length be made equal to the crank throw, a case represented in Fig. 24. Here we have the crank pin o , the crank centre S , and the centre axes 1, 3 and 2, 4, and it will be observed that the connecting rod circle $p p p$ passes through the point S . It is needless to remark at the outset that this extreme case must be taken merely as representing a theoretical study, and in nowise a workable engine. Let us now assume the cylinder system to rotate; it will be seen that for uniform rotation of the cylinder centre lines about the point S the piston masses represented by 1, 2, 3 and 4 will in turn make a complete circuit of the path p , each circuit occupying half a revolution of the cylinder system; during the orbit of any two piston masses, say, 1 and 2, as shown in Fig. 24, the remaining piston masses remain fixed at the point S , and at the instant the mass 2 reaches the point S , the mass 4 situated on the corresponding cylinder axis will start on its orbit, so that there is no effective break in the continuity of the motion. If we do not pay attention to the numerals borne by the piston masses, we can regard the conditions as fulfilled by two masses continuously traversing the circular orbit p at symmetrical opposite points and two masses permanently situated at the point S . It is evident in such a system that the common centre of gravity of the two travelling masses is constant in position, and is situated at the centre of the crank pin o , and as the two other piston masses are stationary, the main centre of gravity of the four is situated at the point Q , and the balance is theoretically perfect.

This method of demonstration, however, is only applicable to the special case given in Fig. 24. If, instead of using this line

of argument, we employ the same reasoning as that employed in Fig. 23, we see that the centre of gravity of piston masses 1 and 3 is situated at the point q_1 , and the centre of gravity of piston masses 2 and 4 is situated at q_2 , and the common centre of

FIG. 25.

gravity of all four piston masses will be at the point Q , the geometrical construction being precisely as before.

The demonstration employed in Figs. 23 and 24 is applicable without break of continuity to all intermediate proportions of engine, and from the point of view of the four or even number of cylinder engine, the proof of the special case might be

considered redundant. If, however, we take the special case and apply it to an odd number of cylinders, we have the immediate proof that the latter is inconsistent with perfect balance. In Fig. 25 we have the case of a three-cylinder engine, in which the letters *o* and *S* stand as before, and in which the axes of the cylinder system are represented by the lines *S1*, *S2* and *Sk*. If, now, we imagine the cylinder system to be rotated clockwise,

FIG. 26A.

the piston mass 2 is rapidly approaching the point *S*, where the piston mass 3 is already situated. As the piston mass 1 reaches the point *a*, the line *S2* becomes tangential to the circle *p p p*, and the piston mass 2 reaches the point *S*. During the period that the piston mass 1 is traversing the path *p p p* from *a* to *b*, that is to say, through an angle of 60 degrees, piston masses 2 and 3 remain stationary at the point *S*. When the piston mass 1 reaches the point *b* the line *Sk* becomes tangential to the circle

sented in Fig. 25, from which it is quite evident that the three-cylinder rotating engine is not a perfectly balanced proposition. Here the path of the centre of gravity of the three piston masses is given by two intersecting arcs of circles *e e e*. In Fig. 26 b, a similar diagram is given for a seven-cylinder engine. Here the line *c* represents the path of the centre of gravity of the moving pistons when three are in motion, and *d* the path when four are in

FIG. 27.

motion; *e e* is the orbit described when paths *c* and *d* are combined and the masses of the stationary pistons are taken into account.

In any actual engine, the proportions do not even plausibly resemble those depicted in Figs. 25 and 26, and where the crank throw is quite small in comparison to the length of connecting rod, it is not difficult to show that the irregularity of motion

becomes a quantity of far less magnitude. It is quite evident, however, that the irregularity is not confined to the special case illustrated, and must exist theoretically in rotating engines having an odd number of cylinders as actually built at the present day.

Figs. 25 and 26A refer, strictly speaking, to a three-cylinder and to a seven-cylinder engine only, but the demonstration applies also to every odd number of cylinders; thus, let Fig. 27 represent any odd number of cylinders, in which three of the cylinder axes only are shown, axis No. 1 and the pair m and n nearest opposite. Then from the time of piston mass 1 reaching point S till the start of piston mass m , the cylinder system will require to rotate through an angle β equal to half the angle between an adjacent pair of cylinders. Consequently, a want of balance must exist as in the case of the three- and seven-cylinder examples.

24. *The "Offset" or Desaxé Crankshaft Engine.*—A question of some interest in the matter of balancing is raised by *offsetting* the crank, which has become the practice of many firms during the last few years. The main object of this practice is to obtain a more direct thrust from the connecting rod and diminish the piston friction on the explosion or expansion stroke; incidentally, in many cases advantage is taken of the offset of the crank to secure greater clearances for cam shaft and valve mechanism. It is of interest to study to what extent this offset introduces new factors into the balancing of the engine.

In Fig. 28 let the crank shaft centre be represented by the point S , and the cylinder axis by the line OY , the distance SO being the offset measurement. Taking the two opposite positions with the crank at right angles, we have the connecting rod indicated by the heavy lines A_1p and A_2p_1 . The distance separating the points A_1 and A_2 may be reached by the construction shown, in which the point C represents the piston pin position of an engine without offset, and the lines CA_1 and CA_2 arcs struck from the crank pin positions p and p_1 , or, as shown in the figure as an approximation, tangents to these arcs drawn from the point C . In the position of the stroke shown, it is evident that the angle A_1CA_2 is at its maximum, and since the angle A_1CA_2 is equal to pCp_1 , it is clear that the motion of the points A_1 and A_2 may, to a first degree of approximation, be derived by superposing a harmonic motion of amplitude A_1A_2 on the normal motion of the point of C in an engine in which there is no offset. Thus, if we take a phantom crank pin orbit whose centre is the point O , and out of phase with the main crank by an angle p_1OD and

FIG. 28.

coupled up with a phantom connecting rod, as shown in Fig. 28, we shall have nearly an identity of motion in a phantom piston driven from the crank pin D and crank centre O (without offset), and in the real piston driven by the crank pin p on the crank centre S with offset OS . We may therefore regard the effect of the offset on the piston motion as that of an alteration of phase through an angle p_1OD derived from the construction given in Fig. 28. Thus it follows that the engine with offset is not materially different, so far as its main balance is concerned, from that without offset, the forces and moments being, however, retarded by difference of phase represented by the angle p_1OD .

An alternative and more exact treatment may be given as follows:—

Case 1. Rod length taken as infinite.—Let offset angle be represented by α , Figs. 29 and 30, and let m m_1 represent piston pin travel, and let o be point of midstroke. Take the radial component of stroke n n_1 to represent the stroke path of a phantom engine with the same crank throw but with no offset. Take q_1 , q_2 corresponding positions (on basis of rod length = infinity) by normals to the connecting rod direction, Fig. 30; then the motions and velocities of the actual and phantom pistons will at every instant be in constant ratio and the motion of the piston in the offset engine is therefore, as in the phantom engine, harmonic. Also, since its phase corresponds to the path n n_1 , which in the actual engine is inclined at angle α to the vertical, the phase of the main piston disturbance will be displaced through an angle α from that of the engine without offset.

Case 2. Rod of finite length.—We now require to investigate the octave vibration. This, as we have already seen, is due to the vertical variations in the length of the connecting rod owing to its angularity.

In the offset engine, Fig. 29, eliminating the vertical component of motion, one end of the connecting rod is describing a harmonic motion along a horizontal* path p p_1 whilst the other end is describing a path whose horizontal component is of amplitude k k_1 , which is harmonic to a first degree of approximation, and 90 degrees different in phase from the motion p p_1 . This is represented graphically in Fig. 31.

Let r = crank radius = o n (Fig. 29).

„ b = half distance k , k_1 .

„ α = offset angle.

* As drawn.

FIG. 29.

In Fig. 31, let abscissae represent angular motion in circular measure.

Then $\beta = b j / (r + b)$.

And from the properties of the sine curve j is unity (=1 radian) hence

$$\beta = b/r + b$$

but

$$b/r = \tan \alpha$$

but b is small in relation to r

$\therefore$ (approximately)

$$\beta = \tan \alpha,$$

and α coming within the definition of a small angle

$$\tan \alpha = \alpha \text{ and}$$

$$\beta = \alpha.$$

FIG. 30.

Thus, the octave vibration is out of phase with the main vibration to the extent of the offset angle. Inspection shows that this is in direction such as to cancel the offset angle, so that the octave

LANCHESTER.

Q

vibration in an engine with offset is the same in direction and phase as in an engine without offset.

That is to say, in a vertical cylinder engine with crank offset—

1. The octave component of the piston motion is vertical.
2. It is in phase with the vertical component of the crank.
3. In the axial line of the cylinder.
4. Out of phase with the main component of the piston motion to the extent of the offset angle.

25. *The Balancing of Components in Practice.*—Quite apart from the question of balancing in design and the theoretical aspect

FIG. 31.

of the subject, it is necessary to devote some consideration to the actual carrying into effect of the intentions of the designer in the shops. It is useless to devote time and attention to the elimination of vibration on paper unless an appropriate degree of care be exercised in the course of manufacture to ensure the necessary degree of accuracy in the weights of reciprocating parts, the proper balance of rotating parts, etc. On the other hand, the author has of recent years had personal experience of balancing appliances which in practical use have proved an entire delusion, and we frequently hear talk of a degree of accuracy in weighing reciprocating parts, etc. that is quite beyond the requirements of the

case. It is useless, for example, in the case of an ordinary four-cylinder engine, in which the unbalanced forces may run into half a ton or more, with a reversal of sign twice per revolution, to expect to obtain any advantage whatever by weighing pistons to within a quarter of an ounce or by whittling down connecting rod stampings to a similar degree of accuracy; such fastidious exactitude shows an absence of all sense of proportion.

Until a few years ago, it was considered sufficient for the purpose of the automobile engineer to balance statically such items as the flywheel and the crank shaft. The shape of the former, namely, its short axial dimension, renders the possibility of serious dynamic want of balance extremely remote. In the days when cranks were machined all over, the dynamic balance could be well taken care of by the dimensions assigned on the drawing, but during the last few years it has become almost general to employ stamped cranks which are not machined over their whole surface, consequently at the present time there is something to be said in favour of balancing the crank shaft dynamically.

In the author's own experience, it is certain that where dynamic balancing is adopted, the machines and the method employed should be controlled with considerable care, and made subject to special inspection.

One case coming within the author's experience not only illustrates the danger of putting a machine or appliance that requires special knowledge into the hands of an ordinary shop foreman or mechanic, but also incidentally shows the futility of supererogatory balancing where the ordinary four-cylinder engine is concerned. The circumstances were as follows:—A bad case of synchronous vibration in connection with a four-cylinder car of a new type being under investigation, the author decided at the outset to make sure of his ground by personally testing the balance of all components; the flywheel had previously been balanced both statically and dynamically on a special machine of well-known type, it carried the proper view marks and showed evidence of both balancing processes. It was duly removed from the engine and personally tested for static balance by the author, when a defect of over 4 oz. in the rim was discovered. This was ultimately traced to the fact that the dynamic balancing machine was defective in principle, and although the static balancing had been quite properly carried out, the subsequent use of the dynamic balancing appliance had resulted in the static balancing being undone and

the wheel being passed officially with the error mentioned. Incidentally, it may be stated that the defect in this machine appeared to be that the flywheel was mounted on a cup and peg support, which, owing to its angle of friction, allowed a certain ambiguity as to the axis of rotation. After some investigation the machine was condemned, and its use discontinued. As an amusing sidelight on the subject the fact may be recorded that when the flywheel was put back with 4 oz. removed from the rim on one side to secure static balance, the vibration of the engine was not distinguishably better or worse! This is not the author's only experience in the same direction; the constitutional error of balance in a four-cylinder engine is so great as, under some circumstances, to entirely mask anything less than a missing piston or lack of balance of that magnitude.

In connection with the crank shaft, whilst for the stamped crank dynamic balancing might be of considerable service, difficulties have in practice arisen. When, for example, on the Norton balancing machine, or other machine of similar type, a four or six throw crank is running at speed, the distortion owing to the unsymmetrical distribution of the throws is so great a disturbing factor as to render the process uncertain and quite uncommercial. Attempts have been made to get over the difficulty by attaching balance weights to the crank throws in order to prevent their centrifugal force causing flexion, but the whole business becomes far too elaborate to be of real service.

26. *Methods of Static Balancing.*—There are three alternative forms of support that may be adopted when mounting parts for static balancing. The wheel or part may be mounted in ordinary lathe centres, a mandrel being used if necessary; the centres require to be jarred by hammering in order to eliminate the static friction and render this method of support sufficiently sensitive to indicate small errors of balance. Another method is to roll the part (mounted on a mandrel in the case of a flywheel or similar part) on two parallel horizontal straight edges mounted on a cast iron bed. Both mandrel and straight edges should be properly hardened and ground, and the cast iron bed permanently installed and accurately levelled. A third method, which is not easily applicable excepting in the case of a flywheel, is to employ a mandrel furnished with perfectly axial knife edges at its extremities.

The question of weighing reciprocating components, etc. calls

for no particular comment. It is customary in the best class of practice, as represented by the six-cylinder engine, to bring all pistons within a definite weight variation limit, which is usually fixed at about half an ounce or one ounce for pistons and the like, and to weigh the ends of the connecting rods separately, the end which is not being weighed being supported on a loose mandrel or knife edge. Recently, a special weighing machine has been introduced to weigh both ends of the rod simultaneously; an appliance of this kind, however, has yet to justify its existence.

26A. *Methods of Dynamic Balancing.*—Any running part that is statically balanced may or may not be dynamically balanced; but any piece that is in dynamic balance is of necessity also in balance statically. In the event of a statically balanced piece being out of dynamic balance, it requires two weights of equal and opposite moment to be applied at some definite axial distance apart, and any given want of dynamic balance can always be disposed of by two such weights and two only. It is obvious that in place of definitely adding weights, weight may be subtracted by drilling or otherwise. It appears well to make the above formal statement, since the author has frequently seen wheels or other components after going through the so-called process of dynamic balancing with two or three entirely different sets of drillings, and it is not always easy to convince a foreman that one pair of drillings is all that is necessary if the work is properly carried out.

There are two principles on which the design of a dynamic balancing machine may be based; the one is to run the piece about its geometric axis and measure or indicate the resulting forces or couple, weights being added until such couple is eliminated. The second method is to mount the piece in a springy or yielding support, spinning it at a high speed till it settles down to run on its principal axis, and it is then marked by any usual means to show defect of balance. It is the latter method that is most usually adopted. In this method it is advisable to test and mark the piece, first running in one direction and then reversed, to eliminate the effect of lag.

The author does not consider that the question of dynamic balancing has as yet been solved in a thoroughly satisfactory manner; the appliances in use, as already stated, do not, generally speaking, give reliable results in the hands of the ordinary skilled mechanic, and, as at present constructed, they are better placed

in the laboratory than in the machine or fitting shop; also the indications of a dynamic balancing machine may be entirely masked by centrifugal distortion.

An item of balancing which is of dynamic character, though not dynamic balancing in the sense of the present discussion, is that necessary to determine the moment of inertia of flywheels used in engines in which reverse rotation is employed. Where such flywheels can be made of identical design the matter is simplified, but where this is not possible, some ready method of measuring the moment of inertia must be adopted. In connection with the author's experimental cars and the early 12 h.p. Lanchester car, the flywheels were tested by mounting them on a mandrel to which a pendulum was attached, and the moment of inertia was calculated from the time period of the pendulum. This apparatus was only used for initially checking the flywheel design, and after the first pair of wheels had been standardised by the pendulum method, the drawings were corrected to suit.

PART II.

27. *Vibration due to Want of Rigidity.*—We now pass to the consideration of vibration of a kind entirely different from that dealt with in the preceding sections, and one involving a departure from our initial hypothesis. We have to deal with vibrations due to want of rigidity of the structural components of the engine. In the case of an engine having "looking glass" symmetry, such as the ordinary four-cylinder or six-cylinder engine, if we think of the engine as flexible about its middle point, as though the frame and crank shaft were articulated at that point, we can see that the two halves of the engine would rock in a symmetrical manner just as two separate two- or three-cylinder components would do if built as separate engines. We can, therefore, see that any want of rigidity in the crank case and structure of the engine would result in a vibratory bending of the engine itself under the influence of the reciprocating masses; this is one of the forms of vibration now contemplated.

In such a case as the above, and in practically all cases in which distortion of the engine components is concerned, it is quite easy to give the necessary strength to resist the vibratory motion and to prevent it from directly causing trouble; any reasonably good design will suffice for this purpose. The real difficulty arises when the vibration period due to this elasticity coincides with the

running speed or with any periodic disturbance associated with the functioning of the engine. Thus, any natural vibration period in the engine structure may pick up the main piston period, or the octave piston period, or even one of the higher harmonics of the piston motion, or in some cases the impulse period may be the exciting cause, so that synchronisation may take place at several different running speeds of the engine.

In certain respects, vibration of the character now under discussion resembles the vibration set up in the chassis of the car by direct dynamic vibration of the engine of the type already discussed, and in the case of an engine fitted to a chassis it is frequently difficult to ascertain whether the synchronous vibration found to extend over a certain range of speed is due to resonance in the chassis or in the motor itself. A question of this kind, should it arise, frequently has to be settled by the removal of the motor to the test bench, when any critical period which relates to the resonance within the engine will remain without material alteration, whereas any due to resonance between the engine and chassis will be found to have disappeared, and possibly some new resonance periods will be found related to the natural period of the test bench or its appurtenances.

The period of a resonance within the engine frequently undergoes *slight* modification when the engine is transferred from a chassis on the test bench, owing to the difference in the rigidity of the mounting affecting the apparent stiffness of the engine itself. In this way it is sometimes found that a threshing point may be moved some 20 or 30 revolutions higher or lower in speed, or it may, in the one case, be found to have a range of some 30 or 40 revolutions only over which it is acute, or it may be found to come on gradually and extend over a range of, say, 100 revs. per minute; e.g., an engine which on a chassis might show a vibration period ranging from 1,400 to 1,450 revs. may, when removed to the test bench, be found to have a threshing point ranging from 1,440 to 1,540 revs.; any variation of this character may at any time be expected. In spite of slight variations of this kind, it is usually possible to clearly distinguish threshing points due to internal engine resonance from chassis or test bench resonance; the former are found to be a characteristic of the engine, and soon become generally known as such by the testing staff.

The author has already pointed out that a number of different threshing points may be given by a single source of resonance

in the engine, the exciting cause being a different time factor of the engine cycle; at the same time cases of multiple resonance in the engine structure undoubtedly occur, that is to say, it is quite possible for an engine to suffer both from a bending resonance and a crank torsional resonance; in fact, there is nothing actually to prevent a greater number of sources of resonance being present in one engine. Where a single source of resonance alone is present, the threshing points, that is to say, the speed at which they make themselves manifest, are usually found to bear a simple arithmetical relationship to each other, and in some cases the effect of the various factors—the main piston vibration, octave vibration, and explosion frequency—may be detected coming successively into evidence as the speed is increased; in a six-cylinder engine it must be remembered that the explosion frequency is three per revolution, and so it is not uncommon in six-cylinder engines to find threshing points related in the relation of 2 and 3 or 3 and 4. Where there is multiple resonance, the periods of the two sources may be related to one another by accident, but, ordinarily speaking, there will be no set numerical relation between them, but each source of resonance may have its own series of threshing points.

28. *Determination of Causes of Resonance.*—When a new design of engine is under investigation, and it is established that undesirable resonance exists in the engine structure, it is important to ascertain as rapidly as possible its nature, with a view to elimination. The facts discussed in the foregoing section form the main basis of any investigation in this direction. Frequently additional light may be thrown on the subject by tests of the engine under full load and running light, or even belted round at speed, and it is usually not difficult to track down the cause of the trouble. At the present day most of the possible causes have been located, and we are in a position to know what to look for.

Where possible, the most satisfactory solution to the difficulty lies in so stiffening the parts as to carry the period of resonance outside the range of the running speeds, that is to say, to so stiffen the structure of the engine as to make the resonant period above the frequency of any known disturbing cause at the maximum proposed running speed. This method may be said to be applicable to all questions of bending, and to the torsion of the body of the engine itself, but it is not conveniently applicable in the case of the crank shaft.

29. *Sources of Resonance Discussed.*—In the structure of the engine, a well designed crank chamber and base is usually sufficient to give the engine the necessary rigidity in flexion; where a block cylinder is used, the rigidity of the engine in this respect can be made quite above suspicion. In certain cases the question of bolting the cylinder groups one to another is worth consideration.

The torsional stiffness of the crank case, owing to its tubular form, should be, and usually is, sufficient to prevent torsional vibration of the engine as a whole, but it is well to note that if the crank case be considered a tubular member, the joint between the crank case and the base must be made a thoroughly sound job. There is frequently a source of weakness here; the attachment bolts are often too light and too widely spaced to do more than act as a means of retaining oil. It should be remembered by the designer that the joint in question is in shear when the crank case as a whole is in torsion, and either sufficient bolts should be provided to give a sound frictional grip capable of taking shear, or a sufficient number of the bolts should be made a dowel fit. The torsional stiffness of the engine also would be considerably aided by the bolting together of the cylinder groups.

30. *Crank Shaft Torsion.*—By far the greatest trouble in multi-cylinder engines is on the score of torsional vibration, and in particular, crank shaft torsion. At the present time, when this source of weakness has been so clearly established, the possibility of trouble from such a cause seems more than obvious, but it required a considerable amount of labour to definitely locate the disease from which the early six-cylinder engine suffered; it is unquestionably a fact that the popularity of the six-cylinder car was held back for many years by the vibration troubles which, for a time, were not properly understood. More than one firm attempted to market a six-cylinder car, but had, for a time, to admit failure.

It is difficult to deal with the question of crank shaft torsion solely by giving greater stiffness to the offending member, and the difficulty becomes greater as the stroke-bore ratio is increased. For engines of short stroke up to square proportions, that is to say, when the stroke becomes equal to the bore, it is not difficult to keep out of serious trouble by designing the crank for the maximum stiffness, but for long stroke engines, such as at the present time are in fashion, the crank shaft, if so designed, becomes excessive in weight and costly in bearing friction.

31. *Crank Shaft Torsion* (continued).—Before going further, it is desirable to study carefully the exact nature of crank torsional vibration, and the manner in which it makes itself felt on the bed of the engine. As a rudimentary model, we may take the case of a length of shaft on which two flywheels are keyed. Such a system, which we may assume mounted in bearings, would have a vibration period proper to it, that is to say, that if the shaft were put in torsion and the wheels released, they would vibrate with a certain definite periodicity, the two wheels moving in opposite directions until the vibration is damped out by bearing friction and other causes.

In the six-cylinder engine we have a shaft to which is keyed one flywheel—we have no second flywheel, but we have in place of it a number of crank throws, with parts having considerable mass attached thereto; such a combination is evidently capable of behaving as a second flywheel, and the whole system has a capacity for vibration just as in the rudimentary illustration given.

A slight difficulty occurs in this comparison; the two-flywheel system mounted on the common shaft in bearings would be capable of vibration and torsion to any extent without communicating its vibration to its mounting; from a rotational point of view it is a self-contained system whose vibrations will not be communicated to anything without. In the case of the crank shaft, however, where the pistons form one of the vibrating masses, the vibrations are communicated to the frame of the engine by virtue of the restraint exercised by the cylinders and the connecting rod reactions during the vibratory motion, and so the crank shaft vibrations make themselves powerfully felt on the framework of the engine and its mounting.

32. *Crank Shaft Vibration* (continued).—The main exciting causes of torsional crank shaft vibration are to be found in the torque variations due, firstly, to inertia, and, secondly, to cylinder pressure; the first of these causes is that discussed in Section 7 in our analytical study of the one-cylinder engine, the torque reversal that takes place twice per revolution owing to the give and take of energy between the flywheel and the piston mass. This would manifest itself when the revolution speed of the engine is half the natural complete torsional period of the crank shaft, that is to say, the time of one revolution of the crank shaft is equal to two complete vibration periods of the crank in torsion. The second exciting cause in a six-

cylinder engine (and it is here that torsional vibrations give the most trouble) occurs three times per revolution, that is to say, there are three explosion impulses per revolution, and thus the synchronous vibration will take place when one revolution of the engine corresponds to three complete periods of the crank in torsion. We may consequently expect, in cases of torsional vibration, to have two marked threshing points having speed relation of two to three; the lower of these points will be that consequent on cylinder pressure, the higher of these points will be that due to piston inertia. It is a common experience of designers who have been troubled with six-cylinder engine vibration to have had two marked periods of the character mentioned.

33. *Crank Shaft Vibration—Critical Speeds giving Resonance.*—The matter, however, is far from being as simple as set out in the previous section. There is no doubt that the cylinder farthest from the flywheel, acting as it does on the tail of the crank shaft where its amplitude of motion is greatest, is the dominant factor in stimulating vibration, but inasmuch as the inertia torque variations of the other pistons also take effect, and these are separated by intervals of a third of a revolution, the total influence is one of some complexity. However, if a number of curves be superposed as representing the piston inertia effect the correct amount out of phase, and if they be given values proportionally to their flywheel value, that is, proportional to the probable amplitude of the vibration, it will be found that the periodicity of the end cylinder predominates.

Beyond the above, it will be found on more closely examining the problem that there is a torsional variation due to the octave component of the piston motion which we may expect to give a synchronous revolution speed half that of the main inertia torsion, so that if, for example, the threshing point due to the explosion torque be located at 1,000 revs. per minute, there will be a bad threshing due to the main piston inertia at 1,500 revolutions, and the minor threshing point, due to the octave component piston inertia, at 750. Groups of this character are commonly met with.

In addition to well defined threshing points, or points of resonance, such as discussed, it is often noted in running the speed of an engine up (or letting it down) on the test stand, that there are, as it were, changes of voice at certain speeds, that is to say, changes in the character of the throb or noise; in most

cases the author is inclined to attribute this to resonance of an incipient character where the natural damping of bearing oil films, etc. is sufficient to prevent anything in the nature of a defined vibration.

34. *Crank Shaft Vibration—Its Elimination.*—The author has found the solution to the torsional vibration trouble in a vibration damper attached to the tail or free end of the

FIG. 32.

crank shaft.* This damper, in the form fitted by the Daimler Company, comprises a small flywheel mounted to rotate freely on bearings, and in driving connection with the crank shaft through a multi-disk clutch arranged in an oil bath, ordinary viscous cylinder oil being used as lubrication. The damper, so fitted, does not impede rotational motion in the smallest degree—

* Patent No. 21139, Sep. 12, 1910.

it is carried round with the crank shaft without offering any resistance—but it forms an immediate and considerable resistance to anything in the nature of angular or torsional vibration. The construction of the damper will be seen from reference to Fig. 32. It is so proportioned as to render anything in the nature of a crank vibration “dead beat,” that is to say, if the crank were twisted through a small angle and let go, it would return at once to its state of equilibrium without repeated oscillation. To effect this, as is well known, the law of friction between the clutch surfaces should follow the viscous law of fluid friction; this is secured by the employment of a wide extent of surface, separated by thin films of viscid oil.

35. *Crank Shaft Vibration as affecting Cam Shaft Drive.*—One of the consequent troubles that has been experienced as due to crank shaft vibration has been the difficulty of driving valve gear satisfactorily from the tail end of the crank shaft, that is to say from the end opposite to that on which the flywheel is placed. Since the introduction of the vibration damper, all difficulty on this score has entirely ceased. It will be understood, that although the vibration damper does not greatly affect the torsional yield of the crank shaft as due to any given impulse, it prevents the accumulation of amplitude, that is to say, we no longer have one disturbing impulse after another coming, so to speak, on the swing of the pendulum. In some cases undamped crank shafts have been known to acquire an amplitude of motion of several degrees owing to the accumulation of energy due to the synchronising with the exciting vibration periods.

APPENDIX I.

The present Appendix is a simple geometrical demonstration of the approximately harmonic character of the error introduced into the piston motion by connecting rod angularity. In Fig. 33 K represents the centre of the gudgeon pin, and KO and KP two positions of the connecting rod. Then the error due to the angularity of the rod will be represented by the quantity b . Let fall OQ perpendicular to KP , then the locus of the point Q will be on a semicircle struck from the centre point of the rod L ; that is to say, carrying out the construction described for different angularities of the rod, the point Q will describe a circular arc

FIG. 34.—Abscissae = value of crank angle α .

about the centre L , and the point P a circular arc about the centre K .

By Euclid, Book I. prop. 47, the square on OX is equal to QX multiplied by KX and KX may be taken as constant; or $s = QX$ and $x = OX$

$$s \propto x^2$$

Now x varies as $\sin \beta$ where β is the crank motion, or $s = \sin^2 \beta$.

We know as a matter of ordinary trigonometrical transformation

$$\sin^2 \beta = 1 - \cos 2\beta/2$$

and if we plot these functions as curves, as in Fig. 34, we have two sine curves one of which is twice the period of the other.

FIG. 33.

Now the quantity b is to a very close degree of approximation proportional to the quantity s (Fig. 33), in fact for small angles of rod we have $b = s/2$.

When the angle of rod becomes considerable, this approximation gets less and less accurate, and the extent of this further error is the measure of the variation between the curves c and d of Fig. 3.

APPENDIX II.

In dealing with the torque variation about the crank shaft axis due to piston inertia in the present paper, the main or fundamental component of the piston motion alone has been considered; the direct inertia effect of the octave components have been fully discussed, but the indirect or secondary effect as setting up torque variation has been regarded as negligible.

In most cases this secondary effect of the octave component is sufficiently small to be neglected, but in connection with the eight-cylinder radial type of engine described in section 21, investigation shows that the secondary effect of the octave component is a serious item.

FIG. 35.

As far as the main reciprocation is concerned, we know that when one set of four pistons is at its dead centre, the other set is in mid-stroke, and so on the "long-rod" basis the energy content of the pistons is constant. When, however, we consider the effect of shortness of rod, and focus our attention on the octave disturbance, we find that this component of the piston motion is zero for all pistons at each quadrant of the crank motion, and is maximum for all pistons at each intervening 45 degrees position.

The author has found by a rough computation that with an ordinary design of engine this secondary inertia torque may amount to about one-fifth or one-sixth of the mean full load

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torque, and so if considered as superposed on the driving torque will appear somewhat as indicated diagrammatically in Fig. 35, and is in no wise a negligible factor.

It is probable also that in a four-cylinder engine of the type described in section 21, or even in a two-cylinder engine of the type discussed at the conclusion of section 9, the secondary torsional effect might readily give trouble owing to its high frequency if its period happen to excite resonance in the chassis or other part of the vehicle; it is well to remember that just as the primary torsional effect based on the fundamental motion of the piston is twice the frequency of the piston motion, so the secondary torsional effect based on the octave motion of the piston has a frequency of twice the octave period, and therefore four complete torque reversals per revolution.

APPENDIX III.

In the application of the author's Anti-vibrator (section 19) to the correction of the main piston inertia torque variation, the arrangement shown in Fig. 16 is adopted, two balance weights being fitted to each spindle, one at each end. If these are arranged in the same plane, they are the equivalent of a single weight in the centre of the spindle of the same total moment. If, however, they are arranged in different positions, one leading and the other following, or if two additional weights are fitted at right angles to the main weights but on opposite sides of the spindle, torques will be set up of the same periodicity as the main piston inertia torque, which may be neutralised by this means. It is easy to show by a simple mathematical investigation that if it is desired to correct the *whole piston inertia torque*, the additional weights (if of the same mass as the main weight and set at right angles thereto) require to be separated by an *axial distance equal to the length of the connecting rod*.

DISCUSSION.

Mr. L. H. POMEROY, in opening the discussion, said: I feel that this is a difficult paper to discuss, because there is practically no controversial matter in it. The whole paper is really above criticism, and I think that the Institution is to be congratulated on obtaining from the author one of the finest and most lucid expositions of this very complex subject ever put before any scientific body. Most of us in our school days were taught the theory of balancing, but the questions of inertia balance and torsional balancing were kept separate, and the much more difficult subject of co-relating both inertia and torque variation with fluid pressures existing in the engine was not touched upon at all. The author has combined all these and put the whole matter in a very clear light.

I think one of the most important deductions to be drawn from the paper is the value of small cylinder units and high engine speeds. These result in unbalanced forces of a small absolute magnitude, which at high speed are sufficiently continuous to be dealt with by such a device as the author's Anti-vibrator.

It is very interesting to note how persistently the short stroke engine scores in matters of balancing, on account of the reduction of connecting rod angularity. This, of course, is well known, and it is somewhat astonishing that, in the face of it, designers should go out of their way to make trouble by producing long stroke engines, in which the connecting rod-crank ratio is necessarily small.

The author describes his Anti-vibrator very clearly, and in my opinion this marks a most important step in the development of the 4-cylinder automobile engine. I hope, possibly in the near future, to be able to report something to this Institution upon the use of this device on a very high speed engine.

The author makes the remark that the 4-cylinder engine has such a constitutional lack of balance that nothing short of a

missing piston is particularly noticeable. In these matters each maker must speak for himself.

Some two years ago I experienced a considerable amount of trouble with vibration in a type of car which was fitted with a comparatively large engine, running at a high speed; no reduction in the weight of the pistons and connecting rods seemed to have much effect. By running the crankshaft only in the bearings of the crank case, it was found that a very bad vibration was set up at certain periodic speeds, simply due to the crankshaft itself being unbalanced. The noise made was really extraordinary, and the removal of an ounce or so of material from the webs of the crankshaft along one side only, quite cured the defect.

Mr. L. BAIRSTOW: There are one or two cases in which problems of balancing have been of importance in practical work with which I have been connected, and one of the advantages not specially mentioned in the paper is that the stresses in the moving parts of an engine can usually be calculated with greater accuracy when the engine is balanced than when there are vibrations of unknown amount there. The investigation of vibration became of importance in one case in a research into the fracture of material under fatigue, and there it is quite necessary that, for an experimental piece of apparatus, the exact motion of any part should be known. In such cases the amount of vibration has to be reduced to a negligible amount, and one type of vibration, that due to the connecting rod, can be reduced by making the length of the connecting rod such that the length equal to the distance between the gudgeon pin and crank pin is equal to the length of the simple equivalent pendulum for the connecting rod oscillating about either centre. A machine making use of this principle is in existence at the National Physical Laboratory, and is perfectly balanced so far as can be felt from any motion of the frame. In that particular case there is the further advantage that the torque on the whole of the apparatus is constant, and there are no outside fluctuations of torque which give rise to possible troubles such as might occur in a completely balanced engine of the author's first type when driving a car with a varying resistance. In dealing with the problem of balancing, the author, on p. 200, points out the possibility of terms of higher order of periodicity becoming appreciable. It may possibly be of interest to have the next few terms of this Fourier's series for reference.

(Mr. L. Bairstow.)

If r be the length of the crank radius, and l the length of the rod, then the series which represents the displacement of the piston is—

$$\begin{aligned} x = r \cos \alpha + l \{ & r^2 \cos 2 \alpha / 4l^2 (1 + (r^2/4l^2) + (15r^4/128l^4 + \dots) \\ & - r^4 \cos 4 \alpha / 32l^4 (\frac{1}{2} + (3r^2/8l^2) + (35r^4/128l^4 \dots) \\ & + r^6 \cos 6 \alpha / 256l^6 (\frac{1}{8} + (5r^2/8l^2) + (315r^4/512l^4 \dots) \\ & - r^8 \cos 8 \alpha / 2048l^8 (\frac{1}{8} + (35r^2/32l^2) + (693r^4/512l^4 \dots) \\ & + \&c. \} \end{aligned}$$

It will be seen that besides the fundamental term $r \cos \alpha$, there are others of twice, four times, six times the frequency.

The importance of the terms is most readily seen from a numerical example. In the case of a connecting rod three times the length of the crank, the series giving the force corresponding with each term is—

$$\cos \alpha + 0.342 \cos 2\alpha - 0.010 \cos 4\alpha + 0.00033 \cos 6\alpha + \&c.$$

The term which is balanced by the author's ingenious device is $0.342 \cos 2\alpha$, and is more than one-third of the main out of balance force which is corrected by the looking glass symmetry arrangement of the pistons. The next term is only about 3 per cent of this latter, and it is therefore not surprising that no attempt is made to balance any terms more rapid than the octave.

MR. O. D. NORTH: In the second part of the paper the author speaks of the vibratory bending of the engine itself under the influence of reciprocating masses, that is, an engine with four or six cylinders as consisting of two halves, so that a bending moment is obtained about the centre line. It occurs to me that these vibratory bending moments are due not only to the reciprocating parts, but to the rotating parts, and that in point of fact the bending moment due to the unbalanced rotating masses is very much greater than the bending moment set up by reciprocating masses alone, and all the more serious as it occurs in the horizontal plane as well as the vertical plane, and it is obvious that the average motor car engine crank case is very much less stiff in the horizontal than in the vertical plane, especially in monobloc engines. There are in a 4-cylinder engine four rotating masses, whose centres are approximately in the centres of the big ends, and the two inner ones tend to pull the centre journal in one direction, and the outer ones pull the end journals in the other direction, like a skipping rope. I have made some fairly com-

plete calculations of the loads on the bearings of a big 4-cylinder engine at rather high speeds, the calculations consisting of plotting curves of the inertia and cylinder pressure forces, of which I will give some summaries which will show the effect of the unbalanced rotating masses. In one sense of the word they are not unbalanced, because the engine as a whole is in rotational balance.

Mr. LANCHESTER: But has not local balance?

Mr. NORTH: Yes. When running a standard engine at 2,000 revs. per minute the maximum load on the centre bearing in a vertical plane was $2\frac{1}{2}$ tons. The maximum horizontal load was $1\frac{1}{2}$ tons on the sides of the bearing. It is, of course, a split bearing, and not really adapted to take heavy side loads. Perhaps the magnitude of these forces is not realised. There was an average load of 2 tons due to the sum of the forces acting. The vertical bending moment was 27,000 inch-lb., and the horizontal 22,000 inch-lb. If weights are placed on the crankshaft to locally balance the rotating masses, the maximum vertical load, which was $2\frac{1}{2}$ tons, is reduced to 1 ton, rather a startling result; by simply putting lumps of metal on the crankshaft there is an actual reduction of load causing wear and friction on the bearing horizontally from $1\frac{1}{2}$ tons to 5 cwt., whilst the average load of 2 tons is reduced to less than $\frac{3}{4}$ -tons. The maximum vertical bending moment of 27,000 inch-lb. is brought down to 9,000 inch-lb., and the horizontal bending moment of 22,000 inch-lb. to practically nothing. In this particular engine we took the trouble to find out by experiment to what extent the crank case would deflect under the horizontal bending moment due to rotating masses. We put the crankshaft in place and rigged up cantilevers, and put a bending moment on the shaft similar to that set up when running at 2,000 revs. per minute, and we found that, neglecting resonance effect, which would tend to increase the amplitude of the vibration, the crank case will deflect twenty one-thousandths of an inch on either side of the centre line at each revolution of the crankshaft. I have made a rough calculation with regard to an imaginary, 6-cylinder engine, assuming reasonable proportions, and a bore and stroke of 80 mm. by 150 mm., and I found that the load on the seven bearings of the crankshaft due to the rotating masses is about 4 tons, at 2,000 revs. per minute, *i.e.*, $\frac{1}{2}$ -ton on each of the three end bearings, and 1 ton on the centre bearing, and it would be

(Mr. North.)

approximately true, not quite so, because the working pressures in the cylinders tend to reduce the loads somewhat, that the 4-ton load could be removed by fitting balance weights, and with it the corresponding horizontal and vertical bending moments. I should like to ask the author if he has made experiments to prove that his torsional vibration damper is of particular advantage on engines with two-bearing crankshafts. I understand that there is a great deal of trouble experienced with vibration in these engines, and if this torsional vibration damper was of any use, it would be of very great advantage, because from some points of view the two-bearing crankshaft is a very nice job.

Mr. THOMAS CLARKSON: I should like to say one word with regard to the charming lucidity and the excellent literary style of the paper. It is indeed a model, and we can see that the author is fairly revelling in his subject, and his expressions are so remarkably happy; for instance, the expression "looking glass" symmetry is one which appeals to us at once. The various important points which the author has dealt with as regards balancing an engine call up some interesting reminiscences. Not many years ago we used to try to balance a steam engine by hanging it up from the roof and supplying it with steam through a flexible tube; the problem was to balance it so that it would spin like a top, and have no vibration either on the casing of the engine or on any moving part, but there was one curious thing that baffled us for a time in connection with balancing that type of engine. It was that when there was an out of balance the shaft wobbled. We marked it very carefully, and the side which we marked we thought was the heaviest side, and we took metal off that side. But we found that was quite wrong, and a little reflection showed us why it was wrong. If we have a shaft with a piece of metal on one side it is thrown out of balance. The natural motion in revolving that shaft quickly is that it will not revolve about its geometrical axis, but about a point which represents approximately the centre of gravity of the combination, the result being that the light side is prominent when revolving because the centre is thrust away from it. That bothered us for quite a while, and anyone making tests in engine balance may find this experience useful. There was a point in section 7 of the paper in which I think there is a clerical error. It states there that "Attention will now be directed to a cause of vibration which, under certain conditions, may render itself most

apparent to users of automobile vehicles. It will be noted, in considering the problem as far as it has been developed, that the piston of our 1-cylinder engine has two positions per stroke when its kinetic energy is zero, and two positions when its kinetic energy is maximum." This appears to me to be not quite clear; either instead of the word "stroke" the word "revolution" should be substituted, or instead of saying two positions where the kinetic energy is maximum, it should be one position.

Mr. LANCHESTER: I agree; there is only one position.

Mr. CLARKSON: The author has devised a very ingenious and efficient arrangement for making two oppositely rotating masses balance a reciprocating mass, and the application of his *Anti-vibrator* ought to materially assist engine designers. With reference to the irregularity noticed in the Lanchester engine, when transmitting power to some outside mechanism—he referred to a rolling mill—it will be understood, of course, that the engine, being quite symmetrically self-contained, with equal and opposite revolving flywheels, does in itself tend to rotate by any change in the amount of energy transmitted. I would ask the author, supposing the power was taken from such an engine, not by a single propeller shaft—where, of course, there must be the reflex action—but suppose there are two propeller shafts, somewhat akin to Mr. Austin's transmission, one going to each of the driving wheels, and revolving in opposite directions, will not the difficulty which the author referred to be entirely eliminated?

Mr. LANCHESTER: It does not exist in the automobile engine.

Mr. CLARKSON: No. On the question of balancing flywheels, which the author refers to, it will be remembered by those who had an opportunity of going over some of the American works last year that the flywheels are generally forged over there. I do not know whether the forged flywheel has yet come into such favour on this side, but it seems to me a very excellent material, and more homogeneous. I believe they are rolled out of an ingot, and most leading firms there are now using these forged flywheels, which can be balanced almost perfectly. In the last part of the paper the author refers to a method of balancing the effect of the torsional irregularities in a long shaft, and to the fact that the front end cylinder furthest from the flywheel is the biggest offender in the cause of these irregularities. I have been studying the damping device, and I would like to ask whether it is not materially, in fact essentially, the same as the addition of a

(Mr. Thomas Clarkson.)

flywheel to the front end of the crankshaft. It is common knowledge, of course, that in gas engine construction, where greater steadiness of drive is required, it is quite usual to put a flywheel on both ends of the crankshaft instead of on one only. There may be some good reason why a flywheel may not conveniently be added to the front end of an engine, but it seems to me that it would tend certainly to steady these torsional stresses in the shaft, because, after all, that front cylinder is continually putting in power and taking it out from the flywheel at the other end. In a waterworks there may be a reservoir many miles away, but a service reservoir is often put quite close to the place of consumption. I think, therefore, that a smaller wheel on the front end of the engine which could respond to these irregularities on the part of the cylinders nearest to it, would be able, by taking from them their surplus energy, and giving it back during the compression period, to help to reduce the torsional irregularities to which the author has referred.

Mr. L. H. HOUNSFIELD: I would like to ask the author if he has tried running the engine, shown in Fig. 11, without gearing the two crankshafts together. I imagine that the transmission to the back axle is taken from one crankshaft only, and that the other shaft has to transmit its power through both sets of connecting rods to the driving shaft. As the connecting rods have to be strong enough to stand the maximum explosion and inertia forces, I should expect them to be capable of transmitting the less strenuous load of half the engine power to the driving shaft, and so to be capable of acting as transmission gear from one crankshaft to the other without the introduction of gear wheels. What is the reason for not doing this?

Mr. F. J. DYKES: I should like to mention one small point with regard to balancing, which may explain why an engine runs more smoothly at some speeds than at others. If we consider the explosion stroke, starting from the back end of compression, we have the ordinary explosion line on the diagram, and at high speed we get the inertia stress line, which can be represented as shown by the author in Fig. 2, and if we draw that line to the proper scale, the intercepts between the indicator diagram line and the inertia line will represent what may be termed the effective pressure on the piston for different speeds. The inertia line will always cross zero through the same point, but will have different inclinations at different speeds. Let us use this effective

pressure curve and draw another curve of torque on the crankshaft; supposing we had no inertia at all, and we found the moment of the pressure due to the piston about the centre of the crankshaft, we should get a curve which in ordinary engines rises very rapidly and then falls off again. Owing to the effect of the inertia, we have the effective pressure curve smoothed out, and therefore the torque diagram, considering inertia, will take a less sharply peaked form: we shall take something off the top of the diagram and increase it later in the stroke. In the case of 4-cylinder engines, this means that at certain speeds we shall get a fairly uniform torque on the crankshaft instead of an impulsive torque. If we have uniform resistance, we have the uniform resistance torque all the time, and the diagram shows the excess energy which is going to be stored up by the flywheel and come out again later. If we have the other type of diagram, at certain speeds we shall store up much less and give out much less, so that at certain speeds we shall get more smoothly-running engines than at other speeds. There is another small point which really refers to bygone days. A device was invented by Mr. Wilson, of Messrs. Wilson and Pilcher, years ago, for getting an engine completely balanced with regard to forces. Suppose we put a balance weight on the connecting rod so that the centre of gravity of this weight and connecting rod combined is at the crankpin, and then we put a balance weight on the other side of the crankpin. That engine is absolutely balanced for forces, but, of course, there is a very large bending moment on the connecting rod. In the case of the Lucas engine shown in Fig. 12, it is possible to connect the pistons together, and in that case all the side thrust on the cylinder is done away with, because, as the two crankshafts move in opposite directions at equal angular velocities, and are also similarly placed having regard to "looking-glass symmetry," we shall have equal and opposite side thrusts on the pistons at any moment. I remember a Lucas car in the days of accumulator ignition in which the commutator had shifted somewhat and the spark had got too far advanced; when we started up the engine and let in the clutch the car went backwards. The engine had actually started up in the wrong direction, but we had no evidence of the fact, and this shows how accurately it must have been balanced: the engine would run equally well backwards as it was a "two-stroke" engine.

Mr. H. J. GROVES: I should like to ask the author a question.

(Mr. H. J. Groves.)

If a single throw crankshaft were fitted with balance weights, properly balanced statically, the weights being mounted so that their mean weight is at a considerably greater distance from the centre of the crankshaft than the crankpin, what is the effect, if any, upon the shaft?

The PRESIDENT: It is now my pleasant duty to propose a vote of thanks for this excellent paper. The author has from time to time given us something to think about, but I think this time he has excelled himself, and in producing this, what I might almost call, text-book on engine balancing, not only has he gone into the theory of balancing, but he has shown us the application of his theories in a very lucid manner. The question of vibration has been the bugbear of motor engineers almost ever since we started building cars, and now that we have this clear explanation of the proposition, I am sure that we shall all derive very great benefit from what the author has done in this direction. Some time ago I had some crankshafts which repeatedly broke, and although the material was in every way satisfactory, and up to the strengths required, it was ultimately found that the fracture was simply due to excessive vibration. I daresay the author can throw some light on similar instances.

Mr. LANCHESTER, in replying on the discussion, said: Mr. Pomeroy expressed the opinion that the study of the question of engine vibration leads to the recognition of the advantage of small cylinder units; a reduction in the size of the cylinder units tends to smoothness; I think we are absolutely at one with him on that point. It is a mere question as to how far the sub-division of units can be carried. We have seen the 4-cylinder engine introduced and gradually replace engines of two and three cylinders; we have, in turn, seen the 6-cylinder engine come into being and gradually take the better class trade from the makers of the 4-cylinder. We do not know whether eight or twelve cylinders will come into general employment some day, but if so, I think it will be due to aeronautical experience; at present, I confess we are unable to see beyond the 6-cylinder as a commercial proposition. Mr. Pomeroy rather doubted my statement as to the 4-cylinder engine and the missing piston; to some extent I will admit that that statement was intended to be picturesque; at the same time, there are cases where it is in substance true; there are undoubtedly other cases where a few ounces will make an appreciable difference. A synchronising period in a motor car

in the spring or "whip" of the chassis or elsewhere is a very sensitive thing for detecting a want of balance, and enables ounces to be detected where otherwise pounds might escape notice.

Mr. Bairstow points out that if vibration is eliminated stresses are diminished. It is a point that perhaps I have not made the most of in my paper; in fact I do not appear to have mentioned it at all; it is, however, very true, and of great importance. The question of eliminating vibration is also important from the point of view of other annoying little matters such as loosened nuts, carburettor parts, and all sorts of little troubles which unquestionably are effects of vibration.

Mr. North mentions that he has found deflection and vibration due to the local want of balance of the crankshaft. His remarks, in effect, constitute a valuable contribution to the paper, and I hope that for the benefit of the proceedings he will give us the advantage of some of his experimental results in detail. One appreciates at once the disadvantage of a want of local balance of the crankshaft; at the same time, up to the present, it has not been found of sufficient importance by any manufacturer of note to do anything. Several firms, including the Lanchester Company, have tried local balancing, but the improvements that have been effected have not been considered of sufficient value to justify the means. Any scientific evidence on this point which Mr. North may be able to produce would be a most welcome addition. Mr. North has said that he obtained a deflection of 20 one-thousandths of an inch in his crankcase. I have just run out a mental calculation, and assuming his crankcase chamber to be 30 in. long and 10 in. wide (if it is more than 10 in. wide the case will be worse against him) to obtain a deflection of 20 one-thousandths the case has to be bent to such a curvature that stresses of seven or eight tons per square inch must be reached, and this would be three or four times as much as it would be desirable to put upon aluminium in the ordinary way, so that I should either doubt Mr. North's experimental figure, or I should think it would be well for him to look more closely into the question of the stiffness of the case.

Mr. NORTH: The crankcase does not bend as a pure bar. It is a quadrilateral; two bars at the end and one in the centre.

Mr. LANCHESTER: I evidently misunderstood. I now gather that it "lozenges"; that makes the matter clear.

In reply to Mr. North, I do not think that the crankshaft damper

(Mr. Lanchester.)

would be of any special value on a two-bearing crankshaft. It might improve the running, as we have found that on other 4-cylinder engines (for which it was not primarily designed) it has made a distinct improvement in the smoothness, but, generally speaking, I should not expect to find any wonderful advance.

I have to acknowledge a correction from Mr. Clarkson; in the narrow sense of the word there are not two positions, but one position; on page 200 I rather used the word "position" to include broadly not only locus but also state of motion. On the question of taking up power with two propeller shafts, the point I have raised as to the effect of a variable *resistance* torque is one of scarcely more than academic interest. I have never experienced it in practice. I fear that if any attempt is made to take the power off with two shafts special arrangements would have to be made to ensure that the proper amount of power is transmitted by each shaft. Obviously, no one would put an engine of the type in question to drive a rolling mill. The vibration damper with disk slipping clutch is a totally different thing from a second flywheel, and in reply to Mr. Clarkson I would remark that for a long time when two flywheels were fitted to gas engines there was much trouble from broken crankshafts. When there are two flywheels on a single shaft, torsional oscillations of very considerable amplitude are liable to be set up, and these oscillations give so much trouble that at one time the practice of fitting two flywheels entirely disappeared.

Mr. Hounsfield has asked whether I have ever run the old type Lanchester engine without gear. The engine was originally designed without any gear connection between the two cranks, and I ran it on a car for several hundred miles. At first it ran excellently, and then it took to vibrating, not in the ordinary way, but in a manner that would wake the dead; I could hardly sit on the car. This would happen quite intermittently, without any apparent cause, and to the present day I do not know precisely what really happened. The dead centre of one cylinder was out of centre with the other, and the whole connecting rod system made in effect a flexible coupling between the two shafts, and presumably the vibration was due to this. Without gearing, there are heavy lateral pressures on the pistons; whereas with gears the engine has all the virtues that Mr. Dykes pointed out as belonging to a modification of the Lucas engine; the piston has a straight line motion, and there is absolutely no lateral pressure on the cylinder

walls. We have had all sorts of things happen to these engines, and they have gone on just the same. We have had most of the brasses melt out on one occasion, and a piston broke and practically disappeared on another, and yet the engine remained serviceable. The difficulty was to get everything into line in the shop, because any want of accuracy in alinement was fatal to smooth running.

In reply to Mr. Dykes, the diagram which he referred to is really a paraphrase of what I have pointed out; the inertia forces roughly counterbalance cylinder pressure under certain conditions. The bob weight on an extension of the connecting rod, mentioned by Mr. Dykes as due to Mr. Wilson, does not give complete balance unless applied to a 2-cylinder engine; on a single-cylinder engine it can scarcely be said to have any useful effect; in any case, from a mechanical point of view, it is an absolutely hopeless and impossible scheme.

Mr. Groves has asked whether there is any difference if the balance weights are placed further away from the crankshaft and crankpin. I do not think that, if the balance weight is made of the proper size, it makes any difference where it is. The position of the balance weight is merely a matter of a certain weight at a certain distance, and if half the weight is placed at twice the distance the result, from the point of view of balance, is the same. The balance weight is not designed to perform the function of a flywheel, so its radial position is a matter of indifference provided that $\text{weight} \times \text{radius}$ is of the correct value.