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XII. *On the Conditions of Equilibrium of a Homogeneous Planet in a Fluid State.* By JAMES IVORY, K.H., F.R.S., &c. &c.*

THIS problem is treated in the *Traité de Mécanique* of Poisson, a work which is in everybody's hands; and, in order to abridge, I shall take the differential equation of a level surface from that work (Equat. (b), p. 536, vol. ii. edit. 2nd,) viz.

$$X dx + Y dy + Z dz + \alpha^2 (x dx + y dy) = 0 :$$

here x, y, z are the rectangular coordinates of a point in the level surface, z being parallel to the axis of rotation: X, Y, Z , are the attractions of the whole mass upon the particle, in the respective directions of x, y, z : and α is the angular velocity of rotation about the axis. By integrating the foregoing equation we obtain,

$$\text{Const.} = \int (X dx + Y dy + Z dz) + \frac{\alpha^2}{2} (x^2 + y^2) \dots (1.)$$

The whole mass being divided into two parts by the level surface, I shall put P, Q, R , for the attractive forces parallel to x, y, z , which the matter within the level surface exerts upon the particle in that surface; and P', Q', R' for the like attractions of the matter without the level surface: so that we shall have

$$X = P + P', \quad Y = Q + Q', \quad Z = R + R';$$

and the equation of the level surface will be

* Communicated by the Author.

$$\text{Const.} = \int (P dx + Q dy + R dz) + \frac{\alpha^2}{2} (x^2 + y^2) \\ + \int (P' dx + Q' dy + R' dz).$$

Now the least attention to the nature of this equation will show that the attraction of the matter without the level surface is entirely independent of the rest of the equation. As there is no definite relation between P , Q , R and P' , Q' , R' , these functions denoting the attractive forces of different quantities of matter, a level surface will have no determinate figure, unless we divide its equation into two separate parts containing the unrelated quantities, viz.

$$\text{Const.} = \int (P dx + Q dy + R dz) + \frac{\alpha^2}{2} (x^2 + y^2) \dots\dots (2.)$$

$$\text{Const.} = \int (P' dx + Q' dy + R' dz) \dots\dots\dots (3.)$$

Such are the equations of equilibrium of a homogeneous planet supposed fluid: but it will be more convenient to use the two following, of which equation (1.) is the sum of (2.) and (3.),

$$\text{Const.} = \int (X dx + Y dy + Z dz) + \frac{\alpha^2}{2} (x^2 + y^2) \dots (1.)$$

$$\text{Const.} = \int (P' dx + Q' dy + R' dz) \dots\dots\dots (3.)$$

Now these two equations are the same with those given in a paper in the Philosophical Transactions for 1824, and in two subsequent papers written for the purpose of obviating some objections (I had almost said, frivolous objections) of M. Poisson*. The foregoing very simple investigation proves the justness of the solution of the problem in the papers cited; and at the same time shows the insufficiency and incorrectness of making the figure of equilibrium depend on one equation. (Poisson, *Mécanique*, p. 550, vol. ii., edit. 2nd.)

What is said is sufficient for the present purpose: but the subject deserves a more extended discussion; because, by the procedure here followed, the greatest simplicity and clearness are introduced in one of the most perplexed and imperfect theories, that occur in the system of the universe.

July 10, 1838.

JAMES IVORY.

* See Phil. Mag., First Series, vol. lxiii. p. 339; vol. lxv. p. 241; vol. lxvi. p. 429; vol. lxvii. p. 31, 82, 439.—EDIT.