AN AXIOM FOR THE STRAIGHT LINE.

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Due to the initiative and untiring energy of Dr. George Bruce Halsted, whose researches in non-Euclidean fields have exposed the underlying fallacies of the definitions and postulates relating to the straight line as set forth in many modern text-books of plane geometry, the Mathematical Section of the Ohio Association of Teachers of Mathematics and Science have resolved that in teaching demonstrative geometry they will not use the phrase, "A straight line is the shortest distance or path between two points;" or the phrase: "The shortest distance between two points is measured on the straight line joining them." The reasons given for this action are:

1. That the idea of a minimum expressed by the term "shortest" is too complex in its ultimate analysis, and involves concepts not intended as assumptions.

2. That the definition of a straight line should be based upon its fundamental idea of quality rather than upon a specific property.

Finally the manner of defining a straight line with reference to its qualitative property is left to the judgment and ingenuity of the teacher as well as the task of formulating another axiom.

In texts which begin with the triangle it is an easy matter to avoid the use of this axiom by assuming that "one side of a triangle is less than the sum of the other two sides," but if they begin with the simple relations of lines and angles it seems necessary that some axiom be substituted for the one rejected. The following is suggested:

The broken line between two fixed points is longer than the straight line joining them.

This involves only the qualitative property of the straight line which the mind conceives in apprehending it, and no attention is drawn to any special property.

THE EQUATION OF A TANGENT TO A CURVE.

Though many methods have been given, the following, based upon my treatment of the angle-bisection theorems as given in *SCHOOL SCIENCE AND MATHEMATICS*, may be of interest.

(a.) It considers the tangent as the ray of instantaneous motion.

(b.) It does not involve the theory of limits.

(c.) It does not require the equation of the curve (except to reduce it to the standard form).

1. In the parabola let $P \equiv (x', y')$, the focus $F \equiv (a, 0)$. The equations of FP and the axis-parallel through P each being in normal form are

$$\frac{x' - a}{x' + a}y - \frac{y'}{x' + a}x + \frac{a}{x' + a}y' = 0 \quad (1)$$

y

—

y'

$$y' = 0 \quad (2)$$

Adding