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THE GRAPHICS OF THE EFFICIENCIES OF THE STEAM BOILER.

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A study of the several efficiencies of the steam boiler was made by the writer some years ago, and the results published originally in a paper presented to the American Society of Mechanical Engineers. This seems to have been the first attempt to frame a practically applicable theory of the various efficiencies with which the engineer has to deal in designing and constructing this class of apparatus, and the first presentation of the algebraic theory of the physical and the commercial aspects of the several problems which arise in the adaptation of the steam boiler to its location and work, having due regard to the costs and to the condition of the labor and money markets.* It was

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then shown that there are two principal efficiencies which the designing engineer must consider in proportioning a proposed boiler plant, and two subsidiary efficiencies—perhaps, more correctly, financial measures of efficient use of capital—which he may find to assume some importance when the boiler in question is one in actual operation. These so-called efficiencies are :

(1) The efficiency of the steam boiler in the accepted sense of that phrase; the ratio of the quantity of heat stored in making steam to the amount set free in the furnace.

(2) The commercial efficiency of the operation; which efficiency measures the approximation of the utilization of capital in the given case to that highest attainable productiveness which it is the principal duty of the engineer to seek in all cases.

(3) The commercial efficiency of a given boiler plant; which, when a maximum, involves the apportioning of such an amount of work to the plant as will give highest returns in value of product.

(4) The commercial limit of efficiency with increasing demand for steam.

The first two efficiencies are of interest to the designing engineer, who has for his task the proportioning of boilers to a stated, definitely prescribed, demand for steam; the second are of importance to the proprietor who desires to increase the product of a boiler plant previously constructed for a stated power. The first two enter the problem: Given: the quantity of steam required; what size and proportions of boiler shall be supplied to produce it? The second two relate to the problem: Given: a steam plant constructed and in operation; what amount of steam can be demanded of it with maximum profit to the owner? what amount can be made without exceeding the limit cost, plus a fair business profit? In the first set of problems, the variable is the size of boiler for a stated power; in the second it is the power to be demanded from a known extent of boiler heating surface.

The first of these efficiencies is the product of a number

of what we will call "efficiency-factors," as the efficiency of heat-development by more or less complete combustion; the efficiency of heat-transfer from furnace to the water and steam in the boiler; the efficiency of transfer from boiler to engine. The second on the list above involves the first and includes the consideration of the costs of construction, placing and operating, in such manner as to determine what proportion of boiler will give the highest total return on the capital invested or continuously expended. The last two efficiencies relate to use of capital with highest results, or up to a limit set by what would be considered fair and acceptable returns. The outcome of the investigation above referred to was in brief the following:

Expenses of Operation may be reckoned under three heads:

(1) Those costs of boiler and of maintenance, reckoned as annual expenses which depend upon original magnitude of investment in boiler, accessories and necessary incidental costs, as interest or loss of profit on money invested in boiler, house, land occupied, taxes, insurance, repairs, depreciation and sinking fund for ultimate replacement, etc.; which costs are all more or less directly variable with the size of boiler adopted.

(2) Those regular annual expenses which are incurred in the operation of the boiler, as costs of fuel, transportation, firing, removal of ash, storage, if any, freight in cases of locomotive and steamship operation; and, usually, some costs attendant upon incidental operations. These costs are variable with, and may commonly be taken as proportional to, quantity of fuel consumed.

(3) Expenses which do not vary with either size of boiler, in the particular case assumed, or with quantity of fuel burned, and which may, therefore, for the problem in hand in any given case, be taken as constant.

All variable expenses may be classified in this manner into the treasurer's account of investment expenditure and returns, and the working expenses of the boiler-room, and assigned, accordingly, to classes one and two. Examples of the first class are the interest and insurance accounts; of the

second class are costs of fuel, in the furnace; of the third are chief engineer's salary, fire risks on adjacent property, etc.

Efficiencies are computed as follows :

Let

A = constant in Rankine's expression for boiler efficiency.

B = ditto.

C = cost—Class II—of fuel per annum per pound burned per hour on one square foot, *i. e.*, of one pound of fuel per hour for a working year.

D = annual investment, Class I, per square foot of heating surface of boiler.

E_1 = water-evaporating power of the fuel for unity efficiency.

E = evaporating power for computed efficiency.

F = weight of fuel burned on the square foot of grate.

S = area of heating surface per square foot of grate.

The efficiency of the steam boiler may be computed with fair approximation by employing Rankine's formula

$$E = BE_1 \div (I + AF/S) \quad (1)$$

The costs of supplying steam, as an annual charge, will be

$$P = DGS + CFG; \quad (2)$$

and the profitable work done is measured by the weight of steam made,

$$W = FGE; \quad (3)$$

while their ratio, which is made a minimum by proper adjustment of the area of heating surface, is

$$y = P \div W = (CF - DS) + EF. \quad (4)$$

Introducing the value of E , from (1), and differentiating for a minimum, we obtain

$$S = F \sqrt{\frac{AC}{D}} = F \sqrt{AR}; \quad (5)$$

in which the quantity R is the ratio of fuel costs C , to investment account D . The constant A is usually about 0.5; B is from 0.8 to unity.

The area of heating surface per pound of fuel burned is seen to vary as the square root of the working costs, inversely as the square root of the investment costs, and directly with the square root of the annual working time—this varying inversely as the preceding item, fifty per cent. more can be paid for extended heating surface, where working night and day throughout the year, than where working but ten hours a day. A doubled cost of fuel and wages would justify increase of size of boiler by one-half for a stated power, the waste of fuel consequent upon inadequate size being such as to make this double expense for boilers, and the resultant saving, a paying operation. Common values of R range between fifteen and twenty-five. Where power is required to be concentrated, and forced draught is resorted to, its value falls greatly - often to three or four—as in the locomotive and in some marine practice. The following table gives some of these values:

COMMERCIAL EFFICIENCY OF BOILERS.
RATIO OF AREAS OF HEATING AND GRATE SURFACES.
Values of S.

F	6	10	12	15	20	30	40	50
R								
25	21	35	42	52	77	105	140	175
16	17	28	34	42	56	84	112	140
9	12	21	24	32	42	63	84	105
4	8	14	16	21	28	42	56	70

From the above equations, we obtain

$$E \text{ max.} = \frac{B E}{1 + \sqrt{\frac{A}{R}}} \quad (6)$$

The following are examples, in greater detail, of the application of the above:*

* *Ibid.*; also *Manual of the Steam Engine*, vol. ii, chap. viii.

EXPENSE ON BOILER ACCOUNT AND MAXIMUM COMMERCIAL EFFICIENCY.

CASES. Class 1 (D).	STATIONARY.		MARINE.	
	I.	II.	III.	IV.
	Cornish.	Tubular.	Tubular.	Tubular.
Total annual cost of boiler per unit of S	\$1 50	\$2 00	\$3 00	\$2 00
Interest	09	12	15	12
Repairs and depreciation	15	20	45	30
Rent, insurance and miscellaneous	10	07	1 00	20
Total value of D	34	38	1 60	62
Class 2 (C).				
Fuel (@ \$5 for I, II, IV; \$4 for III) per unit of F	7 50	7 20	12 00	2 00
Transportation and Storage	1 00	1 00	10 00	1 00
Attendance (variable cost)	—	50	50	—
Total	\$8 50	\$9 00	\$22 50	\$3 00
Value of $\frac{C}{D} = R$	25	23	14	5
Value of A	0'5	0'3	0'3	0'5
Value of $\sqrt{A R}$	3'5	2'7	2'0	1'6
Value of F	8	10	16	20
Value of $F \sqrt{A R} = S$	28	27	32	32
Value of $S \div F$	3'5	2'7	2'0	1'6

The engineer thus solves the most important problem in boiler design which may be thus enunciated: To determine the commercial efficiency of a steam boiler doing a fixed amount of work; or, given all variable expenses of boiler installation, maintenance and operation, to determine what proportion of heating surface to grate surface or fuel burned will give the required amount of power at least total cost.

A second commercial problem may sometimes be presented to the engineer: A steam boiler is in place and in operation; all constant expenses are known and all variable costs of maintenance and operation are determinable. The question arises, or may arise whenever additional steam is called for: How much can be obtained from the apparatus when driven to such an extent as to yield most steam per dollar of total cost of operation? The independent variable is now the quantity of fuel burned in the boiler, and

this is, in the established equation, represented by F , the fuel burned per unit of area of grate. This problem is thus stated:

Given: All expenses, constant and variable, and the method of variation of the latter and the proportions of the boiler as actually constructed, to determine that rate of combustion which will make the commercial efficiency of the given plant a maximum.

For this case, let K represent total annual expense, which is constant,

$$\text{and let } k = \frac{K}{G}.$$

Then the total cost of maintenance will be

$$P' = k G + D G S + C F G, \quad (7)$$

and

$$y = \frac{P'}{W} = \frac{k + D S + C F}{E F}, \quad (8)$$

F being taken as the independent variable.

This becomes a minimum when

$$F_1 = \sqrt{\frac{k S + D S^2}{A C}}, \quad (9)$$

and

$$E \text{ max.} = \frac{B E}{1 + A \sqrt{\frac{k + D S}{A C S}}} \quad (10)$$

The following cases illustrate this problem :

EXPENSES OF BOILER AND MAXIMUM ECONOMY OF PLANT.

<i>Cases</i>	STATIONARY.		MARINE.	
	<i>I.</i>	<i>II.</i>	<i>III.</i>	<i>IV.</i>
Cost of maintenance: D	\$0 34	\$0 58	\$0 88	\$0 62
Cost of operation: C	8 20	9 00	14 50	3 00
Cost of operation: K	30 00	25 00	10 50	10 00
For maximum fuel and work: F_1 .	16	13	17	21
For maximum efficiency, as before: F	8	10	16	20
$S \div F$:	1'8	2'1	1'9	1'6

Case No. 1 is that of a Cornish boiler; No. 2 that of a multitubular stationary boiler; No. 3 that of a sea-going, and No. 4 that of a yacht, as before.

It is seen that in all cases the weight of steam delivered from the boiler and the quantity of fuel burned at maximum commercial efficiency, for the case assumed, are less than where the boiler—once set and still capable of being forced to deliver more steam than originally proposed and calculated upon—is worked up to a maximum delivery per dollar of total expense.

“Maximum Commercial Efficiency of Boiler” and “Maximum Efficiency of a Given Plant” are, therefore, by no means identical conditions, and it will usually be found that when this maximum work can be put on the boiler, it might be done still more economically by a boiler specially designed, as in the first problem, to do the increased quantity of work; the conclusions from this fact being simply that economy dictates that as much steam-power as possible should be grouped into a single plant in order to diminish the proportional cost of the constant part of running expenses, *i. e.*, otherwise stated, there being given a certain necessary expenditure, invariable within certain limits with variation of size of boiler or of quantity of steam made, the larger the amount of work done without increasing this constant expense, the cheaper will the steam be made.

The larger the plant supervised by the engineer, the less the total cost per pound of steam made, other conditions of economy being unchanged.*

It will be noted that the variations above computed for F are much less than those for S ; the second being compared with the first case. Also, the harder the boilers are driven, the more nearly do the solutions of the two cases approximate the same limiting values. For the cases last assumed, the quantity of fuel burned, at highest economy, to the square foot of heating surface, is seen to range from 1.5 to 3.5; in usual practice this value is observed to range, as actually obtained with customary proportions and rates

* *Ibid.*

of combustion, from two to five, and to be gradually rising, where, as in marine practice, concentration of power is becoming more and more imperative.

A graphical construction may be readily made to exhibit to the eye the method of variation of costs with size of boiler. Thus, in *Fig. 1*, let it be assumed that the costs

of fuel, simply, are those of purchase, supply, and use of 90,000 pounds of steam per annum, thirty pounds per hour per horse-power of engine; that the evaporation of this quantity of steam in a boiler of unity area of heating surface and equal grate area would compel the expenditure of twenty pounds of fuel per hour; that this fuel costs \$1.33 $\frac{1}{2}$ per ton in the furnace; and that the period of operation is 3,000 hours. Let the ordinates of the curve represent the annual costs per horse-power in dollars, and the abscissas the necessary ratio of heating surface to grate area to give the required quantity of steam at the costs indicated by the value of corresponding ordinates. Then computing the relative costs with increasing areas of heating surface per unit of grate area, we obtain the curve marked "ideal case;" and the costs, for fuel simply, on the above basis, will decrease as indicated by that line. Costs of fuel thus rapidly decrease, with increasing areas of heating surface supplying the stated quantity of steam, until, at usual proportions, thirty to forty feet of heating surface to the foot of grate surface, the rate of combustion being reduced correspondingly, these expenditures fall from \$60 per annum per horse-power to \$10 or \$12.

But the costs of operation, outside the fuel account, must be added to obtain a measure of total costs. Let these be reckoned in terms of fuel costs and referred to the square foot of heating surface. Losses of heat occur, also, by conduction and radiation from boiler and connections, and *en route* to the engine; call these radiation losses and measure them in similar terms. Then the second line of the diagram will exhibit the costs of the horse-power, annually, when fuel and these wastes only are taken into account.

It is here assumed that the costs of operation, exclusive of fuel, will increase, so far as variable, with the area of heating surface, being proportional, nearly, to the cost of the boiler and to the interest account on that cost. Compute these costs, and take an equivalent value in fuel as the measure, per horse-power per annum, of the quantity to be added to the ordinates of the curve for such additional costs. We

thus obtain the upper line of the diagram, on the assumption that the measure thus taken, ten per cent. on \$4 per square foot of heating surface, corresponds to coal at \$1 per ton. With increasing relative costs of fuel, the lines representing this, and the total costs of operation per horsepower per annum, fall lower and lower, as shown in the

FIG. 2.—CURVE OF VARIATION OF "S"

Due to change in length of time of operation of the boiler.

illustration, and the curve, which, in the first case, is seen to pass a minimum ordinate at between ten and twelve square feet of heating surface to the square foot of grate area, passes that minimum at higher and higher values of that ratio, until, at \$8 to \$10 per ton, the best proportions of boiler are found to be far beyond the limits of usual practice.

The specially interesting fact illustrated by the diagram is the position of the best ratio of heating to grate surface in all common practice and at all usual prices of fuel and total costs. With coal at \$1 per ton, we find this point which gives maximum returns for capital expended, at about 11 to 1; with coal at \$2, it falls at 16; with fuel at \$4 per ton, it comes in at the ratio 24 to 1. Assuming coal to cost \$1 per ton, and labor to have half the value above taken, the line falls from *F* to *E*, giving the best ratio at 16; and with a value of labor and interest account at one-fourth that indicated by the line *F*, it drops to *D* while the best ratio rises to 24 and upward.

FIG. 3.—CURVE SHOWING VARIATIONS IN "S"

Due to changes in the price of coal. Changes in cost of labor shown by dotted line.

The next diagram, *Fig. 2*, exhibits the method of variation of the most economical proportions of boiler with change of time of working, through a range from an hour a day for the year up to continuous operation, night and day. The cost of fuel is assumed for convenience at \$1 per ton and wages at \$600 per year. Feed-water is taken as supplied at 185° F., and the rate of combustion as ten pounds per hour on the square foot of grate. The boiler is of 100 horsepower rated capacity, and capable, as is assumed, of supplying 3,000 pounds of steam per hour. The interest and other

investment accounts, D , amount to forty cents per square foot of heating surface; the fuel account, C , with its incidentals,

Ordinates A and B . Price per ton of coal or wages per day.

FIG. 4.—COST AND TIME VARIATIONS.

to \$15. The regular working time taken as the basis of comparison is ten hours per day. At twenty-four hours

working time per day, C becomes about \$35 and D about fifty cents. Points are thus worked out in the diagram and the curve drawn through them with the result here seen. Similarly, the diagram, *Fig. 3*, exhibits the variation in costs of fuel and of labor, and their influence upon the best proportions of boiler. The variations are taken as from a common point of departure, as above, which is assumed to be representative of a not unusual figure for fuel and for labor, at from \$1 to \$3 a day, where other costs are such as to give $S = 18$ when the figure is \$2.

The last set of curves, *Fig. 4*, exhibit still more fully the variations of proportions of boiler produced by variations of costs of fuel, wages, and time of operation, these costs being taken as ranging from one-half to five dollars per ton, and labor from one to five dollars a day. The time of operation varies between forty and six hundred hours per month, forty-eight to 7,200 hours per annum. Assuming fuel at \$2, wages at \$1.50, and 3,000 hours work per year, we find, for example, at these figures, taken as fairly representative of a very common practice, twenty-seven square feet of heating surface to the square foot of grate. It will be observed that, within the usual range of proportions, $S = 25$ to $S = 40$, the curves follow very nearly the paths of their tangents, and within this range of variation, we may take the size of boiler as variable directly with either fuel costs, wages, or time of operation without serious error.