XIV. The winding of voltmeters

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vibrations are applicable to circular bars of any cross section; but those from the longitudinal only to bars of small cross section. From a separate consideration of the torsional and longitudinal fixed-free vibrations of a thin circular bar, a discrimination might be made between the effects on the modulus of elasticity and on the rigidity due to definite physical changes.

By heating up the bar at one end, making that the fixed or free end, and employing (58) and (59) or (65) and (66), considerable insight might be gained into the structure. Then allowing the bar to cool and repeating similar observations, the residual effect after any interval might be determined. As to the advantages of determining modification of physical properties by observations on a single bar, remarks similar to those already made will apply.

By the test of reversion we can also select for our experiments a bar originally of great isotropy.

XIV. *The Winding of Voltmeters.* By Professors W. E. AYRTON, *F.R.S.*, and JOHN PERRY, *F.R.S.**

I. **T**HE use of voltmeters for measuring the potential-difference existing between the mains in electric light circuits, and for enabling us to maintain this potential-difference constant, is so wide in its possible application, that it is unnecessary to dwell on the importance of voltmeters giving accurate indications. Indeed, the sensibility of an incandescent lamp is so great (the light, as we have shown in a previous communication to this Society†, being in many cases proportional to the cube of the potential-difference minus a constant), that it is necessary that voltmeters should record with great accuracy the potential-difference, as well as quickly indicate any changes that from time to time take place in this potential-difference.

It is therefore desirable to consider what are the most important errors in commercial voltmeters as at present constructed, and how these errors may be practically diminished or altogether removed. They are:—

1. An error arising from the sensibility of the voltmeter varying with its resistance, and therefore with its temperature. This change of temperature is due partly to the variation of the temperature of the room and partly to the coils of the

* Communicated by the Physical Society: read June 13, 1885.

† "The most Economical Potential-Difference to employ with Incandescent Lamps," *Proc. Phys. Soc.* vol. vii. p. 40 [*Phil. Mag.* [5] vol. xix. p. 304 (April 1885)].

instrument becoming heated by the passage of the current through them.

2. An error arising from the sensibility of the instrument being temporarily varied by external magnetic disturbance.

3. An error arising from certain types of voltmeters altering in sensibility so that their constants vary from time to time.

4. An error arising from the volt standards employed by different makers of voltmeters differing several per cent. from one another.

The variation in the resistance of a voltmeter produced by changes of temperature of the room can of course be made very small by winding the instrument with German-silver wire, or better with platinum-silver, or, best of all, with wire made of platenoid, or it may be made absolutely nought by use of an outside resistance of carbon ; or, lastly, if the voltmeter be simply wound with copper wire, this error, although large, can be accurately allowed for if the temperature of the voltmeter be known. But the error arising from the heating of the voltmeter by the passage of the current through it is much more difficult to allow for, as this rise of temperature of the instrument will be unknown. Hence it is important to reduce this error to a minimum, and it does not at all follow that the employment of German-silver wire or even platenoid wire will effect this result; indeed we have already given, in a previous communication made to this Society*, general reasons showing that German silver is undesirable to be used in the winding of voltmeters. When, however, the diameter of the coil of the instrument exceeds a certain limit, it would appear that German-silver wire might be better for the outer coils: hence the following investigation has been made for the purpose of ascertaining whether in an instrument of the size of our Solenoid, or Long-Range Voltmeter, any portion of the coil should be wound with German-silver wire, and if so, how large a portion, also what should be the law of variation of sectional area of the copper and German-silver wire with the diameter of the convolution in which it is wound, so that the heating-error produced by the passage of the current through the voltmeter shall be a minimum.

Although the dimensions of the instrument adopted in the following investigation are those of this special form of voltmeter, the mode of considering the problem will apply equally well to any other galvanometer used as a voltmeter, or to the

* "Direct-reading Electro-Measuring Instruments, and a Non-Sparking Key," *Proc. Phys. Soc.* vol. vi. p. 69 [*Phil. Mag.* [5] vol. xvii. p. 304 (April 1884)].

is the rate at which heat is produced in a layer of thickness δr .

This rate is $\frac{\rho_0}{x_0^2} r^{b-2a}$ times the volume of the layer. Therefore the rate at which heat is produced *per unit volume* is $C^2 \frac{\rho_0}{x_0^2} r^{b-2a}$: therefore, by hypothesis, the increase of temperature is proportional to $C^2 \frac{\rho_0}{x_0^2} r^{b-2a}$.

The increase of resistance of this layer is the resistance of the layer multiplied by γ times the increase of temperature, and is therefore proportional to

$$\frac{2\pi\rho_0 l r^{1+b-2a}}{x_0^2} \delta r \cdot \gamma \cdot \frac{C^2 \rho_0}{x_0^2} r^{b-2a},$$

or to

$$\gamma \frac{C^2 \rho_0^2}{x_0^4} r^{1+2b-4a},$$

where $\gamma = \gamma_0 r^{c-b}$, and the whole increase of resistance is proportional to

$$\int_{r_0}^{r_1} \gamma_0 C^2 \frac{\rho_0^2}{x_0^4} r^{1+b+c-4a} dr,$$

or

$$\gamma_0 \frac{C^2 \rho_0^2}{p x_0^4} (r_1^p - r_0^p), \text{ if } p = 2 + b - 4a + c; \quad \dots \quad (7)$$

so that the fractional change of resistance is proportional to

$$\frac{\gamma_0 C^2 \rho_0^2}{R x_0^4 p} (r_1^p - r_0^p). \quad \dots \quad (8)$$

Now regarding M as constant, and substituting for C and R from (6) (5) and (8) we observe that the fractional change of resistance due to heating is proportional to

$$\gamma_0 \rho_0 \cdot \frac{n^2}{(r_1^n - r_0^n)^2} \cdot \frac{m}{r_1^m - r_0^m} \cdot \frac{r_1^p - r_0^p}{p} \quad \dots \quad (9)$$

and it is required to find the values of a , b , and c for which this expression is a minimum.

III. It may be seen that the error (9) is less as $\gamma_0 \rho_0$ becomes less, therefore $\gamma_0 \rho_0$ should be made as small as possible so that *when wires of two metals, as for example copper and German silver, are employed, the wire for which γp has the least value, that is the copper, should be used for the inner winding.* It is, however, when we consider the waste of electric energy

in the coil, that the great objection to the use of German silver for the internal winding becomes still more striking. For the rate of production of heat in the coil is proportional to C^2R , which, from (6) and (5), equals

$$\frac{x_0^2 n^2 M^2}{K^2 l^2 (r_1^n - r_0^n)} \times \frac{2\pi \rho_0 l}{m x_0} (r_1^m - r_0^m).$$

Hence to produce the same magnetic effect M in a coil of given length l , the rate of production of heat is proportional to

$$\frac{n^2}{(r_1^n - r_0^n)^2} \times \frac{r_1^m - r_0^m}{m} \rho_0. \quad . \quad . \quad . \quad (10)$$

Consequently when a given angular deflection is produced, the rate of wasting electrical energy in heat will be thirteen times as great if German-silver wire be used as it will be if copper is used, no matter what be the gauge of wire employed, that is, whether the voltmeter be wound to measure a maximum potential-difference of 50 or of 500 volts.

Eliminating r from (3) and (4) it follows that

$$\left(\frac{\gamma \rho}{\gamma_0 \rho_0}\right)^{\frac{1}{c}} = \left(\frac{\rho}{\rho_0}\right)^{\frac{1}{b}}, \quad . \quad . \quad . \quad (11)$$

where b may have any arbitrary value. Hence if there is a regular change possible in the value of $\gamma \rho$ such as given in (4), we must have

$$c = P b,$$

where P is some constant. Equation (11), therefore, becomes

$$\frac{\gamma \rho}{\gamma_0 \rho_0} = \left(\frac{\rho}{\rho_0}\right)^P. \quad . \quad . \quad . \quad (12)$$

If, therefore, our wire in the inside of the coil is of copper and on the outside of German silver,

$$\gamma \rho = \frac{13}{9} \gamma_0 \rho_0,$$

$$\text{or} \quad \rho = 13 \rho_0.$$

Hence from (11)

$$\frac{13}{9} = 13^P,$$

$$\text{or} \quad P = 0.143,$$

$$\text{and} \quad c = 0.143 \, b. \quad . \quad . \quad . \quad (13)$$

It is certain that generally the relationship between γ and

ρ cannot be of the simple character here assumed; but inasmuch as our results are only applied in cases where the windings are either of German silver or of copper, or of the two together, and in these cases the expressions used are quite correct, there is no practical objection to the use of such a law. The law is very nearly the same as the assumption that a wire of any material, lying electrically between German silver and copper, is made up of wires of German silver and copper.

Since from (7),

$$p = 2 + b - 4a + c,$$

it follows, using (13), that

$$p = 2 + 1.143b - 4a, \dots \dots \dots (14)$$

and the values of m and n are given respectively in terms of a , b , and d in (5) and (6), and are

$$m = 2 + b - 2a \dots \dots \dots (14)$$

$$n = d - a + 1. \dots \dots \dots (14)$$

In our Solenoid Voltmeter, r_0 equals 1, r_1 equals 8, and d equals -1 ; and since the greatest value of ρ is $13\rho_0$, it follows that r^b cannot be greater than 13, but b is a constant, and the greatest value of r is 8, therefore b must be less than 2. From (14), since b is always small, we see, after a has reached a certain large value, that p , m , and n increase numerically as a increases, and are positive when a is negative, and *vice versa*; also that, finally, p equals $2m$ or $4n$.

Substituting the values for r_0 and r_1 , the error (9) may be written

$$\frac{\gamma_0 \rho_0 \left(\frac{n}{8^n - 1} \right)^2 \left(\frac{m}{8^m - 1} \right)}{\frac{p}{8^p - 1}},$$

and it may be seen by inspection that each of the factors of which it is composed is always positive, whether m , n , and p be positive or negative. Therefore the error is always positive. For large values of m , n , and p , the error may be written

$$\frac{\left(\frac{n}{8^n - 1} \right) \left(\frac{2n}{8^{2n} - 1} \right)}{\frac{4n}{8^{4n} - 1}},$$

which equals

$$\frac{\left(\frac{n}{8^n-1}\right)^2}{\frac{2}{8^{2n}+1}},$$

or

$$\frac{n^2(2^n+1)}{8^{2n}+1-2 \times 8^n}.$$

This expression is always greater than n^2 whether n be positive or negative, and is therefore very great since n is great. Consequently for large positive or negative values of m , n , and p , the error (9) becomes very great, and therefore it can have no maximum. The values of a and b , therefore, found from equating to nought its differential coefficients with respect to a and to b , will give a minimum value of this error. And the error will be a minimum when its logarithm is a minimum. Now taking its logarithm, putting r_0 equal to 1, and r_1 equal to 8, differentiating with regard to a and b respectively, and equating to zero, we arrive at the equations

$$\phi(n) - 0.857 \phi(p) = 0, \quad \dots \dots \dots (15)$$

$$\phi(m) - 1.143 \phi(p) = 0, \quad \dots \dots \dots (16)$$

where

$$\phi(a) = \frac{\log_e 8}{1-8^{-a}} - \frac{1}{a}. \quad \dots \dots \dots (17)$$

To assist in the solution of these simultaneous equations we calculated the following Table :—

TABLE I.

a .	$\phi(a)$.
— 3	0.3293
— 2	0.4670
— 1	0.7029
0	1.0397
1	1.3764
2	1.6124
3	1.7502
4.5	1.8474
10	1.9794
100	2.0694

And in order to solve these equations we used the following artifice:—A curve was first plotted on squared paper with $\phi(a)$

as ordinate and a as abscissa, and by means of this curve for any value of $\phi(a)$ given we could at once find a approximately.

Table II. was then arrived at in the following way:—A value of p was chosen, and $\phi(p)$ written down. Then since $0.857 \phi(p)$ is equal to $\phi(n)$ by (15), n was known from this by the curve for $\phi(a)$; m was next found in the same way from (16). Thus columns 1 and 4 in Table II. contain values of p and n which satisfy (15), and which were obtained in this way, while in columns 1 and 6 we have values of p and m which satisfy (16).

TABLE II.

p .	$\phi(p)$.	$0.857 \phi(p)$, or $\phi(n)$.	n .	$1.143 \phi(p)$, or $\phi(m)$.	m .
2	1.6124	1.381	1.03	1.842	4.35
1	1.3764	1.18	0.39	1.572	1.78
0	1.0397	0.8913	-0.425	1.189	0.425
-1	0.70295	0.6026	-1.370	0.8035	-0.69
-2	0.467	0.4002	-2.38	0.5337	-1.69

Now first using the pair of columns 1 and 4, we chose corresponding values of p and n , say 2 and 1.03; then, since from (14)

$$2 = 2 + 1.143b - 4a,$$

and

$$1.03 = d - a + 1,$$

we could calculate a pair of values for a and b which satisfy (14) and (15), since d was assumed to be known. Next taking another pair of values of p and n from Table II., another pair of values of a and b was calculated, and so on. Plotting these values of a and b as coordinates of points on squared paper we obtained the curve sss , fig. 1*, the coordinates of every point of which are the values of a and b that satisfy (15). In the same way we drew the curve ttt for (16), and the coordinates of the point of intersection Z are the values of a and b which satisfy (15) and (16), and therefore make the error (9) a minimum.

Thus for example, taking $d = -1$, as seems to be the case in our solenoid instrument, we find that, with some approximation to accuracy,

$$a = 0.19 \text{ and } b = -0.82$$

* This and the two succeeding figures are only about one twelfth of the size each way of the figures actually employed in the calculations.

Fig. 1.

for minimum error. But if we have used our material of least resistance for the inner winding, b cannot in practice be less than 0, for ρ cannot be less than ρ_0 , and since r is never less than 1, ρ would be some fraction of ρ_0 if b were less than 0. Hence in practice we find that b must equal 0, that is, *a less heating-error is produced when all the wire is of copper than when we use German-silver windings on the outside of the coil.*

IV. Making b equal to nought as the least value of b attainable, we can now find the value of a to make the error a minimum. To do this we must put b equal to nought, take d equal to -1 in (9), differentiate with respect to a , and equate the result to 0.

Thus

$$\phi(m) + \phi(n) - 2\phi(p) = 0, \quad \dots \dots \dots (18)$$

where $\phi(a)$ has the signification given to it in (17).

Since $b=0$

and $d=-1$,

it follows from (14) that

$$\left. \begin{aligned} m &= 2-2a \\ n &= -a \\ p &= 2-4a, \end{aligned} \right\} \dots \dots \dots (19)$$

therefore, from (17)

$$\phi(2-2a) + \phi(-a) - 2\phi(2-4a) = 0.$$

This equation cannot be solved generally, but by giving various values to a we can find which of these values makes the expression on the left-hand side most nearly equal to nought, and to ascertain the value of the function for each of the values of a we can conveniently use the curve connecting $\phi(\alpha)$ and α previously employed.

For $a=0.5$ we have

$$\phi(1) + \phi(-0.5) - 2\phi(0) = 1.375 + 0.865 - 2 \times 1.04 \\ = +0.16;$$

for $a=0.4$ we have

$$\phi(1.2) + \phi(-0.4) - 2\phi(0.4) = 1.43 + 0.9 - 2 \times 1.182 \\ = -0.034;$$

for $a=0.3$ we have

$$\phi(1.4) + \phi(-0.3) - 2\phi(0.8) = 1.48 + 0.936 - 2 \times 1.311 \\ = -0.206;$$

for $a=0.2$ we have

$$\phi(1.6) + \phi(-0.2) - 2\phi(1.2) = 1.53 + 0.97 - 2 \times 1.43 \\ = -0.36.$$

Fig. 2.

Plotting these points, we find (fig. 2) that the curve for

the differential coefficient cuts the axis along which a is measured at a point 0, for which a has the value 0.42; or, in other words, *a equal to 0.42 makes the differential coefficient equal to nought, and hence is the value of a that makes the heating-error a minimum.* As a check on this we have drawn the curve for the values of the heating-error for different values of a (fig. 3), and from this also it is seen that the

Fig. 3.

heating-error has a minimum when a equals 0.42. Hence it follows that the law of increase of the sectional area of the wire with the radius of the convolutions should in our Solenoid Voltmeter follow the law

$$x = x_0 r^{0.42},$$

in order that the error due to heating may be a minimum.

Fig. 3 not only gives the value of a that makes the heating-error a minimum, but it shows by how much the heating-error is increased if a has a value greater or less than 0.42. For example, if a equals 0, that is to say if the voltmeter be all wound with wire of the same gauge, then, continuing the curve backwards, we find that the heating-error is increased by about one third.

V. One plan of diminishing the heating-error with voltmeters, and one that we have frequently employed, is to place in the same circuit with the instrument a coil wound with

German-silver wire, and having a resistance far greater than that of the voltmeter itself. By using wire of large gauge, wound on a bobbin of large volume, and with as efficient an arrangement for cooling as can be conveniently employed, the heating-error can be rendered extremely small. Such a bobbin, however, is both costly and bulky, and the employment of such an outside resistance-coil wastes energy, and can only be used with voltmeters in electric light and power installations where such a waste of energy is unimportant. The following investigation has therefore been made for the purpose of determining what must be the volume of this outside resistance-coil, and what must be its resistance relatively to that of the voltmeter-coil, so that the heating-error may not exceed any given value.

Let R be the resistance of the coil of copper wire wound on a voltmeter, let H be the rate of production of heat when the maximum deflection is obtained; then, as already shown, H will be constant for a voltmeter of a given size wound according to a definite law, whether it be wound with fine wire for a 500-volt instrument or with thicker wire for a 50-volt one. Let C be the current passing round it when this maximum deflection is being produced; then

$$C^2 R = H, \text{ a constant.}$$

Let the increase of resistance produced by the passage of the current for some time be r ; then $\frac{r}{R}$ is proportional to the product of γ , the coefficient of increase of the resistance of copper per degree of temperature, into H , since, as already explained, the voltmeter is so wound that the rate of production of heat is proportional to the increase of temperature. Hence

$$\frac{r}{R} = \text{a constant, } A \text{ say.}$$

Now let there be a resistance-coil of German-silver wire of uniform cross section, of volume V in series with the instrument, and let us suppose that for any coils thus added the rise of temperature is proportional to the energy wasted per second per unit volume, that is to say proportional to the rate of production of heat per unit volume.

Let R_1 be the resistance of this added coil, then $\frac{C^2 R_1}{V}$ is proportional to its rise of temperature; hence, if r_1 be the

increase of resistance and γ_1 the coefficient of increase of resistance of German silver per degree,

$$\frac{r_1}{R_1} \propto \gamma_1 \frac{C^2 R_1}{V},$$

or

$$r_1 = B \frac{C^2 R_1^2}{V} \text{ say,}$$

where B is a constant. Hence, if e is the error due to the proportional change of resistance,

$$e = \frac{r + r_1}{R + R_1},$$

$$= \frac{AR + \frac{BC^2 R_2}{V}}{R + R_1}.$$

Let

$$\frac{R_1}{R} = y,$$

then

$$e = \frac{A + \frac{B}{V} y^2}{1 + y} \dots \dots \dots (20)$$

To determine the value of the constants A and B we may proceed as follows :—Wind a voltmeter according to the law which is to be adopted with any or no resistance-coil outside ; find the potential-differences P_1 and P_2 required to produce the maximum reading of the instrument,

1st, immediately on making connection ;

2nd, after connection has been made for some time ; then

$$e = \frac{P_2 - P_1}{P_1}.$$

Let e be experimentally determined in two cases,—one where there is no resistance-coil outside, and the other where there is such an auxiliary resistance-coil, obtaining values e_1 and e_2 respectively. Measure y , that is the ratio of R_1 to R , and also V . Then from the equations

$$e_1 = A,$$

$$e_2 = \frac{A + \frac{B}{V} y^2}{1 + y},$$

the values of the constants A and B can be determined ; and when determined, the error arising with the employment of

an outside resistance-coil of any resistance and of any volume can be calculated from (20).

It is obvious that as V becomes greater the error becomes less and less, but there is a practical inconvenience in largely adding to the weight of the instrument.

VI. *To determine the best resistance to give to the outside coil of given volume.*—When V is fixed, e is a maximum or minimum, when

$$\frac{de}{dy} = 0.$$

Now

$$\frac{de}{dy} = \frac{\frac{B}{V}y^2 + 2\frac{B}{V}y - A}{(1+y)^2},$$

and

$$\frac{d^2e}{dy^2} = \frac{2\left(\frac{B}{V} + A\right)}{(1+y)^3},$$

y must be positive, therefore $(1+y)^3$ is positive, also A and B are both positive, therefore $\frac{d^2e}{dy^2}$ is positive, and therefore e is a minimum, when

$$\frac{de}{dy} = 0,$$

that is, when

$$\frac{B}{V}y^2 + 2\frac{B}{V}y - A = 0,$$

or

$$y = -1 \pm \sqrt{1 + \frac{AV}{B}};$$

but y cannot be negative, therefore only the positive sign can be used, or

$$y = \sqrt{1 + \frac{AV}{B}} - 1. \quad \dots \quad (21)$$

makes e a minimum.

Substituting this value of y in (20), we find that

$$e = 2 \frac{\frac{A+B}{V} - \sqrt{1 + \frac{AV}{B}}}{\sqrt{1 + \frac{AV}{B}}} \quad \dots \quad (22)$$

VII. *To determine the best volume, and the best resistance to give to the outside coil for a given permissible error.*—From the *Phil. Mag.* S. 5. Vol. 21. No. 129. Feb. 1886. I

preceding it is easy to see that if the maximum allowable value of e is given, the minimum value of the volume that can be employed is given from (22), and for this value of the volume, the best value of y , that is the best ratio of the resistance of the outside coil to that of the magnetizing coil of the voltmeter, can be calculated from (21).

VIII. *To determine the sectional area to be given to the inner wire of a voltmeter in terms of the maximum potential difference so as to make the heating-error a minimum.*—By experiment ascertain the potential difference that is necessary to produce the maximum deflection with any voltmeter of the shape and size to be employed and wound according to the law

$$x = x_0 r^a,$$

where a has the value that makes the heating-error a minimum, and which value will depend on the shape and size of the instrument, and may be calculated for any special type of voltmeter in the same way as it has been calculated in what precedes for the Long-Range Magnifying Spring-instrument. Then if it is desired to wind another instrument, of the same volume and shape, so that it will have its maximum deflection given by P_2 volts, it follows, since the heating will, as already proved, be the same in the two instruments for their maximum deflections, that their resistances must be proportional to the squares of P_1 and P_2 , and hence, from (5), since the same law of winding is followed, that the cross section of the innermost layer of wire in the two instruments must be inversely proportional to P_1 and P_2 .

To have the same heating-error in the two cases, with outside resistance-coils of the same shape and size, it follows from (21) that it is necessary to use the same ratio between the resistance of the outside coil and the resistance of the magnetizing coil. To have a less or greater heating-error in the second than in the first case for the maximum deflection, it is only necessary to determine from (22) (the constants of which have been deduced by experiment with the first instrument) the greater or less volume V of the outside resistance-coil needed with the second instrument, and calculate the ratio y of the resistance of the outside coil to that of the magnetizing coil by (21).

IX. As to the second error, arising from the sensibility of the instrument being temporarily varied by external magnetic disturbance referred to in § I. This can be overcome in three distinct ways : by

1. Shielding the instrument by putting it in an iron case with very thick walls.

2. Using a very powerful controlling field.

3. Using a motion of *translation*, and *not of rotation*, for the body in the instrument that is deflected by the current.

The first method to be successful requires that the iron case should be heavy and cumbersome, and makes it unsuitable for portable instruments.

A very powerful controlling field necessitates a very powerful deflecting force, and so either makes the instrument unsensitive or necessitates a considerable expenditure of electric energy in the instrument. The third method, consisting in giving the deflected body a *motion of translation*, is the best, since, no matter how strong the disturbing magnetic field may be, provided it is uniform throughout the small space traversed by the deflected body on its motion, no increase nor diminution of the deflection will be caused by the outside magnetic field. For this reason we have adopted this third method in our Magnifying Spring-instruments, and experiment shows that the shielding produced in this way is even better than in our older form of instruments provided with a powerful permanent magnet.

X. As to the third error, arising from certain types of voltmeters altering in sensibility so that their constants vary from time to time. This error arises from two causes. In some voltmeters, such as those in which a controlling permanent magnet is employed, there is an error which gradually comes on from the demagnetization of this magnet, an error which is greatly increased by a mistaken careful regular employment of a keeper for the magnet when the instrument is not in use. This error can be avoided by dispensing with a permanent magnet, and using a spring or a weight, or an electromagnet which is magnetized by the current to be measured. A second time-error arises from the employment of iron in the neighbourhood, and has the effect of making the indications of the instrument depend not merely on the current passing at the time of measurement, but also on the currents that have lately passed through the instrument. This error is especially marked in those instruments in which the controlling force, or the force tending to bring the needle back to zero in opposition to the deflecting force, is produced by the attraction of an electromagnet containing a core of soft iron wire, and which it is *assumed* is magnetized to saturation by the current which it is our object to measure. With small currents, however, which produce deflections in

the lower part of the scale, this iron core is not saturated, and differences in the values of the readings of as much as 30 to 40 per cent. are obtained, depending on whether it is an increasing or a decreasing current that is being measured with such an electromagnetic instrument.

XI. The fourth error, arising from the volt standards employed by various people differing several per cent. from one another, has been hitherto a very serious matter. The standards employed have usually been—

1. The E.M.F. of a Daniell's cell.
2. The E.M.F. of a Latimer Clark's cell.
3. A standard depending on the electrochemical equivalent of silver and the value of the legal ohm.
4. The indication of a voltmeter graduated by some well-known maker, but probably graduated in a way quite unknown to the user.

Standard 1 is vague unless the details of the method employed in setting up the Daniell's cell are more carefully specified than heretofore; for example, Carpart, in some recent tests of the E.M.F. of a Daniell's cell, made out of a U-tube, with a contraction in the lower part, found it to vary from 1.111 legal volts when the zinc-sulphate solution had 25 per cent. of ZnSO_4 , to 1.142 legal volts when the solution had only 3 per cent. of ZnSO_4 . His method of testing is based on the electrochemical equivalent of silver and the legal ohm, and as a proof of its accuracy he finds that the E.M.F. of a Latimer Clark's cell is 1.434 volts. If, however, we compare his results for a Daniell's cell with 1.072 volts as found by Sir W. Thomson when the zinc-sulphate solution had a specific gravity of 1.2, we see how Daniell's cells differ among one another*.

The E.M.F. of a Latimer Clark's cell has also some vagueness attached to it, unless the temperature-coefficient of variation of E.M.F. of the particular Latimer Clark's cell is known and taken account of. Generally the users of these cells are contented to say that the E.M.F. of a Latimer Clark's cell, as determined by Lord Rayleigh, is 1.435 true volts, or 1.438 legal volts, and they neglect the temperature variation altogether, which amounts to about 1 per cent. diminution per 12°C . elevation of temperature with certain Latimer Clark's cells.

* Since the reading of this paper this subject has been very fully treated by Dr. Fleming, in a paper "On the Use of Daniell's Cell as a Standard of Electromotive Force," read before the Physical Society on June. 27, 1885 [Phil. Mag. [5] vol. xx. p. 126 (August 1885)].

Standard No. 3 is the one that we have used for some time ; and as we have obtained very satisfactory results with it, it may be well to describe our mode of using it a little in detail. First, by means of a large number of direct comparisons with a copper voltameter, we determine the most probable value in amperes of the indications in different parts of the scale of an ammeter *fixed permanently in position* by being screwed to the table. A current is now sent through this ammeter in series with a long thick copper wire having about 6 ohms resistance and wound in a large open coil. This coil has attached to its two ends the standard voltmeter, which is also fixed *permanently in position*. Simultaneous readings of the ammeter and voltmeter are now taken by two observers, when a signal is given and the bridge-piece of a mercury commutator is moved so as to cut off the current and connect the copper wire together with the voltmeter, which is joined with it in parallel, to a differential galvanometer. Then *time* measurements, for about ten minutes, are taken by a third observer of the parallel resistance of the thick copper wire and the attached voltmeter. Next a curve is drawn connecting the resistance of the cooling coil with the time, and on completing the curve backwards we ascertain with great accuracy, by interpolation, the resistance at the moment of stopping the current, that is at the moment of making the simultaneous observations on the ammeter and voltmeter. Our copper wire is sufficiently thick that for a current of some 15 amperes passing through it, corresponding with about 80 volts maintained at its ends, the time-curve of its resistance, obtained after stopping the main current, is nearly a straight line with hardly any inclination to the axis along which the time is measured ; or, in other words, the change of resistance due to cooling while the measurements of resistance are being made is very small and easily allowed for. And this resistance, in legal ohms, multiplied by the current passing in amperes, gives of course the potential difference in legal volts.

In using the copper voltameter to calibrate the ammeter it is of course necessary, in order that a firmly adherent deposit should be obtained, that the area of the immersed surfaces of the anode and cathode opposed to one another should exceed the limit given by Dr. Hammerl, viz. one square decimetre per 7 amperes, or rather more than two square inches per ampere ; and in our experiments we have taken care to use a larger area in proportion to the current. Our negative plates consisted of a number of separate sheets of very thin hard copper, so that the weight of the deposit could be accurately

measured ; the current was supplied by accumulators and increased or diminished by a carbon resistance worked with a screw. By watching the ammeter and slightly turning this screw one way or the other, the current could be kept very constant for a long time. The value of the electrochemical equivalent employed for copper was 0.3295 milligrammes per coulomb, which is deduced from the mean of the electrochemical equivalents of silver, 1.118 milligrammes per coulomb as given by Lord Rayleigh, and 1.1183 by F. and W. Kohlrausch, and by using 63.47, 107.56, 32.07, and 16 as the atomic weights of copper, silver, sulphur, and oxygen respectively. The solution of copper sulphate employed was a saturated one, the crystals of copper sulphate used in making it being chemically pure.

To avoid errors being introduced by oxidation of the copper that had just been deposited on the copper plates, they were, on being taken out of the sulphate of copper bath, on the stoppage of the current, immediately and quickly washed, first in water, secondly in alcohol, thirdly with ether poured over them, and then dried.

The currents that a given set of accumulators will send through a copper voltameter depends of course partly on the distance between the alternate positive and negative plates, and partly on the depths to which they are immersed in the saturated solution of copper sulphate. The proper arrangements of the plates to allow any particular current to pass cannot conveniently be determined with the plates that are going to be used for the experiment, as an unknown weight of copper would be deposited on them during the process of adjustment, and great delay would be introduced if the plates had to be dried and weighed after the adjustment and before making the final experiment. Hence after the proper plates to be used as the anodes had been very carefully cleaned and weighed, a spare set of exactly the same size and shape were put into the bath, and their distance, as well as the carbon resistance, adjusted until the desired current was passing; then the spare set was removed and the weighed set substituted for them, and the experiment carried out.

Having determined our volt-standard in the way above explained, we tested various voltmeters of different makers, and we found that whereas the indications of some of them agreed very well with our standard, the indications of others differed by several per cent.