

# THE CHARGE ON THE ELECTRON AND THE VALUE OF PLANCK'S CONSTANT $h$ .\*

BY

IRVING LANGMUIR, Ph.D.

Research Laboratory, General Electric Company, Schenectady, N. Y.

LEWIS AND ADAMS (*Phys. Rev.* (2), 3, 92, (1914) from a theory of ultimate rational units, *i.e.* by a kind of dimensional reasoning, have derived the following relation between Planck's constant  $h$  and the electron charge  $e$ .

$$h = \frac{e^2}{c} \times 32\pi^3 \sqrt[3]{\frac{\pi^2}{15}} \dots \dots \dots (1)$$

where  $c$  is the velocity of light ( $2.9986 \times 10^{10}$  cm. per sec.). If we take for  $e$  the value  $4.774 \times 10^{-10}$  e.s. units as determined by Millikan (*Phys. Rev.* (2), 2, 143, 1913) this gives

$$h = 6.560 \times 10^{-27} \text{ erg seconds.}$$

In a recent paper R. T. Birge (*Phys. Rev.* 14, 361, 1919) has calculated  $h$  in seven different ways and obtains as the most probable value

$$h = 6.5543 \times 10^{-27}.$$

Every method of determining  $h$ , however, involves the use of  $e$  so that any error in this quantity produces a corresponding uncertainty in  $h$ . Sommerfeld in his recent book on atomic structure (*Atombau und Spectrallinien*, Nov., 1919) has given a new relation between  $h$  and  $e$  in terms of the Rydberg constants obtained with hydrogen and helium. From Bohr's theory, taking into account the masses of the nuclei of the atoms, and by comparing the spectra of hydrogen and helium, he is able to calculate the value of  $e/m$  with very great accuracy (here  $m$  is the mass of the electron). He obtains

$$e/m = 1.7686 \times 10^7 \text{ e. m. units,}$$

$$\text{or } e \times 1.7686 \times 10^7 \text{ e. s. units.}$$

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\* Communicated by the Author.

Besides being based upon the spectroscopic wave lengths, this value involves a knowledge of the ratio of the atomic weights of helium and hydrogen (4.000:1.008). The relative error in  $e/m$ , however, is only about one-third as great as that in the ratio of the atomic weights. Sommerfeld has furthermore derived the relations

$$e = \frac{\alpha^3 c^2}{4\pi N_\infty (e/m)} \dots \dots \dots (2)$$

where

$$\alpha = \frac{2\pi e^2}{hc} \dots \dots \dots (3)$$

Here  $N_\infty$  is the wave number corresponding to the Rydberg constant for an infinitely heavy nucleus. It has the value

$$N_\infty = 109737.11 \text{ cm.}^{-1} \dots \dots \dots (4)$$

These equations together with the Lewis and Adams relation enable us to calculate a new value of the electron charge  $e$ . Combining Equations 1 and 3 we find

$$\alpha = \frac{1}{16\pi^2} \sqrt[3]{\frac{\pi^2}{15}} = 0.0072798 \dots \dots \dots (5)$$

Substituting this together with the values of  $N_\infty$  and  $e/m$  in Equation 2 gives

$$e = 4.745 \times 10^{-10} \text{ e.s. units} \dots \dots \dots (6)$$

while this value substituted in (1) gives

$$h = 6.481 \times 10^{-27} \text{ erg seconds.}$$

This value of  $e$  is about 0.6 per cent. lower than that given by Millikan, who estimates the accuracy of his result as 0.2 per cent. However, Millikan's determination has never been checked by any independent method of comparable accuracy, so it seems possible that a constant error of the above magnitude might have affected his results. The value of  $h$  given by Equation 7 is 1.2 per cent. lower than the value calculated by Birge but this results merely from the different value for  $e$ . The following table gives the values of  $h$  calculated from the seven sets of data used by Birge: first, using Millikan's value  $e = 4.774$ ; and second, using the value  $e = 4.745$ .

*Determinations of  $h$ .*

Method	Dependence on $e$ .	$e=4.774$	$e=4.745$
Stefan-Boltzmann Constant.....	$e^4/3$	6.551	6.497
Wien Constant $c_2$ .....	$e$	6.557	6.517
Bohr-Sommerfeld theory, Equations 2 and 3.....	$e^5/3$	6.548	6.481
Einstein's photo-electric equation.....	$e$	6.578	6.538
Quantum relation for X-rays.....	$e^4/3$	6.555	6.501
Lewis and Adams theory.....	$e^2$	6.560	6.481
Quantum relation for ionization potentials.....	$e$	6.579	6.539
	Average	6.555	6.507

The values of  $h$  calculated from  $e = 4.745$  are consistently smaller than those based on Millikan's value, but they agree among themselves within the probable experimental error. In any case the results do not warrant the conclusion that the Lewis and Adams or the Bohr-Sommerfeld theories are not rigorous. Unless one of these theories is incorrect or incomplete, however, the values of  $e$  and  $h$  represented by Equations 6 and 7 should have a much higher accuracy than any of the other experimental determinations given in the above table. Under these conditions it seems desirable that new determinations of the electron charge should be made, especially if new methods can be developed.

**An Experimental Determination of the Inertia of a Sphere Moving in a Fluid.** GILBERT COOK. (*Philosophical Magazine*, March, 1920.)—Sir G. G. Stokes showed that a solid body moving in a fluid of infinite extent has its inertia apparently increased by the effect of fluid pressure. This effect became of interest in war time in connection with the theory of oscillating mines. A spherical mine-case 38.2 inches in diameter, ballasted so that its weight in water was only one pound, was allowed to fall freely through the water in a tank 15 feet in diameter and 30 feet deep. The motion was automatically recorded and the velocity and acceleration were calculated. When the acceleration is plotted against the square of the velocity the points lie close to a straight line. The faster the mine sinks the less is its acceleration. When the apparent inertia was computed it came out equal to 1.46 times the ordinary mass of the case.

G. F. S.