therefore

$$S\eta \left(\frac{\rho_1}{\sin r_1} + \frac{\rho_2}{\sin r_2} + \frac{\rho_3}{\sin r_3} \right) = 3 \sin \kappa - \cos \kappa (\cot r_1 + \cot r_2 + \cot r_3) .$$

But

$$\frac{\rho_1}{\sin r_1} + \frac{\rho_2}{\sin r_2} + \frac{\rho_3}{\sin r_3} = \frac{\rho}{\sin r} ,$$

therefore

$$\frac{S\eta\rho}{\sin r} = 3 \sin \kappa - \cos \kappa (\cot r_1 + \cot r_2 + \cot r_3) .$$

But

$$S\eta\rho = - \cos (\kappa - r) ,$$

therefore

$$- \frac{\cos(\kappa - r)}{\sin r} = 3 \sin \kappa - \cos \kappa (\cot r_1 + \cot r_2 + \cot r_3) ,$$

therefore

$$4 \sin \kappa = \cos \kappa (\cot r_1 + \cot r_2 + \cot r_3 - \cot r) .$$

therefore

$$\tan \kappa = \frac{\cot r_1 + \cot r_2 + \cot r_3 - \cot r}{4} = \frac{\tan R}{2} .$$

3. On some Remarkable Changes, Additions, and Omissions of Letters in Certain Cognate European Words. By the Hon. Lord Neaves.

The subject of comparative philology has always interested scholars, but latterly the study has been carried on in a more scientific manner, and I may also say with more success, than at any former period. One great object in prosecuting the study is to detect the various disguises which words radically the same are apt to assume in different languages or dialects. The great scholars of two centuries ago were fully alive to the importance of this inquiry, and although they sometimes indulged in too great a latitude of conjecture, there is scarcely an etymological affinity now generally admitted of which traces and indications are not plainly to be found in the works of those learned men, and more particularly in the writings of Salmasius, the greatest of them all.

But it cannot be denied that a strong impetus to this science has latterly been given, arising partly from a more extended knowledge of the forms of speech since Europeans began to study the cognate languages of the East. Comparative philology has thus assumed a more definite shape within the last fifty years, as for instance in the law of sound-change first pointed out by Rask, and afterwards confirmed and extended by Grimm.

Other phenomena of change have still more recently been made prominent, and to some of these I now wish shortly to direct attention.

An opinion prevails among several eminent philologists that the letter and sound of *l* did not originally occur in the Aryan family to which our chief European languages belong. Its introduction, if it is not original, is certainly not recent, for it would be difficult to maintain that it has not existed for several thousand years, as it plays so conspicuous a part in the Homeric writings. But it appears that the Zend language—that is, the old Persian or Bactrian—had no such letter as *l*, and that European words which have that sound have frequently Zend forms where *r* supplies the place of *l*. It is said also that in the oldest Indian writings the same peculiarity appears, though the *l* has been freely introduced into the later Sanscrit.

Be this as it may, it must be admitted that there is a great affinity between the smooth and the rough liquids, *l* and *r*, and that they are frequently interchangeable. We see much of this in Greek and Latin, and it is not easy to say that either of the two languages shows a preference for one of those letters over the other. Let us take some plain and undoubted examples:—*λειριον*, Gr., = *lilium*, L.; *ρακος*, Gr., = *λακος*, Gr., a ragged garment; in connection with which it has been specially observed that the Cretan form of Doric frequently confounded *ρ* and *λ*. The terminations *-pos*, Gr., and *-lus*, L., seem cognate, as in *τρομερος* and *tremulus*. In Latin itself we have two terminational forms that seem identical—*alis* and *aris*—the use of which seems in a great measure determined by euphony, in this way, that where *l* occurs in the radical word, the termination *-aris* is used for the sake of variety; and when *r* occurs in the radical, *-alis* is used. Thus from *populus* comes *popularis*; and from *natura*, *naturalis*.

It is remarkable that in the later Romance languages, *l*, when it is found in Latin, sometimes disappears, and is replaced by *r*: as apostolus, apôtre; epistola, épître; capitulum, chapitre, &c. Indeed *l* does not very well stand its ground in modern times. In Italian it often becomes an *i*; in French it becomes an *u*; and in the lower German dialects, such as our own Scottish, it is similarly changed or lost.

Let us now assume as an interim hypothesis that *r* and *l* are interchangeable in Greek and Latin, and see if that assumption will afford us results that tend to confirm its truth.

The names for the *swallow* in those two languages are respectively χελιδων and hirundo. Upon the hypothesis suggested, χελιδων may be changed into χεριδων; and then, by well-known tendencies of the Latin language, the final *ν* will be dropped, leaving χεριδω, while an *n* may be inserted before the *d* to strengthen the syllable, as in tundo, tudi; fundo, fudi; findo, fidi; frago, frango; tago, tango, &c. We thus get χερινδω and hirundo, the identity of which is manifest.

Χαλαζα and grando, the words for *hail*, may be assimilated nearly in the same way. Χαλαζα becomes χαραζα: this when contracted becomes χραζα, as χαρις becomes gratia. Z is = to ds or di, and with an inserted *n*, χαραζα is equal to grandia, which is close upon *grando*.

Upon this footing we see the identity or near affinity of κρυπτω and καλυπτω; and with these, perhaps, κλεπτω may be connected.

Κυκλος, the Greek for a wheel or ring, may in its more primitive form be set down as κυκρος, which seems cognate to the Indian form chakra, with the same meaning. But κυκρος with a slight metathesis leads easily to the Latin circus, circulus; and it is again possible that by aspirating and modifying the consonants, circus becomes identified with the Teutonic ring = hring, while κυκλος is thought to be cognate to the Teutonic wheel; so many diversities of form may thus be derived from the same elements of a guttural twice repeated, and a liquid *r* or *l* variously arranged.

Ἑλμινθς, by changing the *l* into *r* and prefixing a digamma, becomes vermis, the relation of the aspirate and digamma being the same as in ἑσπερος and vesper. The Greek ἑδρα would be easily changed in Latin into sedla, which by assimilation becomes

sella. *Balbus* and *barbarus* seem in like manner to be connected, the meaning of *barbarus* being one who speaks unintelligibly.

We may here give an example of the same radical word appearing in two different forms in the same language with diversified but kindred meanings. The Greek ἀμέργω has the general meaning of pressing or squeezing, while ἀμέλω has the special meaning of pressing out milk from the udder. The first of these has not been adopted by the other European languages, but ἀμέλω is very widely diffused as *mulgeo* in Latin, and *milk* in the Teutonic languages.

A somewhat similar example may be found in the Greek words γραφω and γλαφω. These two words mean different methods of a kindred operation, that of marking intelligible forms by some sharp or cutting instrument, the one designating the process of writing or painting, and the other that of carving or modelling. Another cognate seems to be γλυφω. But of these words, and some others connected with them, I shall have occasion afterwards to speak more fully.

Examples of the interchange of *r* and *l* might be further multiplied, but those already given will sufficiently illustrate the subject, and direct attention to this mode of discovering latent affinities in words.

It has often been surmised that a similar relation subsists between other liquids, as between λυμφα and *nympha*, and it seems clear that the Attic dialect frequently changed an *ν* into *λ*, as in νιτρον, λιτρον, &c. But this hypothesis has not as yet been sufficiently matured, and I refrain from entering on it.

The next point that I shall notice is the peculiarity that undoubtedly exists, of leaving out or adding an initial sibilant in cognate words, so as partially to disguise them. In some languages combinations of letters are found to be admissible, and even frequent, which are not found in other languages, though nearly allied. Neither Greek nor Latin seems to admit of the initial combinations of *sl* or *sn*, and, accordingly, it is probable that an affinity subsists between words in other European languages which show these combinations, and Latin and Greek words that show no sibilant. Thus *laxus* may be a cognate of *slack*, *limus* of *slime*, &c. *Nix*, undoubtedly, is identical with *snow*, and

nervus is supposed to be the same as snare, which is a word for a string.

The Greek makes use of the initial combination *sm*, which the Latin rejects; but the Greek is not constant to that combination of letters, and the *s* is often dropped, so that we have *σμικρος* and *μικρος*, *σμαραγδος* and *μαραγδος*. The word *μειδιάω* does not show any initial *σ*, but it may be conjectured that the *σ* was once there, and has been dropped. The prosody and form of Venus's epithet *φιλομμειδῆς* would be well explained by considering it as a corruption of *φίλοσμειδῆς*. If we adopt this view, we then establish an affinity between the Greek *μειδιαω* and the English *smile*, which is further supported by the well-known tendency of the Greek *δ* to become an *l*.

In those words in other languages which have an initial *s* and a consonant, the *s* is often dropped in Latin, while the rest of the word is retained. We see examples of this in comparing Greek and Latin words. Thus we have the Greek *σφαλλω* becoming in Latin *fallo*; *σφογγος* becoming *fungus*; and *σφαιδόνη*, *funda*. The Greek *στέγω* and *στέγος* seem to be identical with *tego* and *tectum*, and in other languages the *s* seems also to be lost, as in the Gaelic *teach*, the English *thatch*, and the German *decken*.

In some cases we find an initial *s* in the Teutonic languages, where it is wanting in Latin and Greek. Thus the Latin *taurus* and the corresponding Greek *ταυρος* seem to be represented by the Teutonic word *steer*, of which a diminutive is *stirk*. The Greek *κείρω* appears in English in the form of *shear*, an *s* being prefixed, and the other consonant thereby softened. This root in the Teutonic languages is very productive, there being many forms of it connected with the process of cutting or dividing, as *shear*, *share*, *ploughshare*, *scar*, *score*, *sharp*, &c. Probably, also, *short* comes from this source, and fully represents the Latin *curtus* with a sibilant prefixed. The Latin *caveo* seems to have a cognate in an Anglo-Saxon word *scevean*, from which come our common words of *shy* and *shun*, for the Latin *caveo* has completely the idea of shunning or being shy of an object. "Hunc tu Romane caveto." The Latin *carus* with *careo* may possibly be connected with the English *scarce*, for the radical idea in *carus* is that of

scarcity as adding to the value of a thing, just as the physical dearth adds to the moral dearness of an object; *parco* also in Latin may be cognate with the English spare.

The initial *l* in Latin seems often to disturb the formation of words, and to sacrifice some letter that had preceded it. Besides other examples already referred to, we may notice the word *lien*, which appears to have lost the older initials *sp*, which would identify it with *spleen*. But the most remarkable instance of this is found in the word *lis*, which from the old grammarians we know to have been originally *stlis*. But acting on the principle which identifies *l* and *r*, we see that the original form would give us *strits* instead of *stlis*, and thus we should have the word commencing with the same letters as our Teutonic words *strife*, *sturt*, *streit*, *G.*, &c., which undoubtedly are cognate in their meaning with the Latin *lis*, though this word by a strange metamorphosis has lost all trace of that struggle or violent contention which it really represents.

It is a peculiarity of the Greek language that all words beginning with ρ are supposed to have a prefixed aspirate which is analogous to a Latin sibilant. If we transferred a Greek word of this kind to a Teutonic form, we should prefix a sibilant to the *r*, but as *sr* is not a combination favoured by the Teutonic languages, a *t* might be inserted for the sake of euphony. Upon this footing we may plausibly consider the Greek $\rho\epsilon\nu\mu\alpha$ as identical with the Teutonic *stream*. It is remarkable, however, that the Gaelic has no objection to the initial combination of *sr*, and accordingly we find the word *sruth*, pronounced *srhu*, meaning a stream or current, and occupying an intermediate position between the Greek $\rho\epsilon\omega$, $\rho\epsilon\nu\sigma\omega$, and the Teutonic *stream*. If we could get over the change of vowel, we might in the same way connect the Greek $\rho\iota\nu$, a nose, with the Gaelic *sron*.

It seems a remarkable circumstance that in Greek and Latin words beginning with a sibilant and another consonant there is a tendency to confound or corrupt the second consonant so as to change one for another in a somewhat arbitrary way. In Latin, in particular, where the consonant succeeding the sibilant is always a tenuis, one tenuis is frequently changed for another in comparing Greek and Latin words. Thus $\sigma\pi\omicron\nu\delta\eta$ and *studium*

seem equivalent, also perhaps *σταδιον* and *spatium*, *σκεπω* and *specio*, *σκυλευω* and *spolio*.

In conclusion, on this modification of original roots by the adjection of an initial sibilant, I shall revert to a set of words already noticed—viz., *γραφω*, *γλαφω*, *γλυφω*. These words are of analogous meaning; and it is interesting to see whether we can find corresponding and cognate words in Latin. I think the Latin words *scalpo* and *sculpo* are in this situation. These words seem to be formed by prefixing a sibilant to the radical elements of the Greek forms which consist of a guttural *γ* or *c*, a liquid *λ* or *r*, and a labial *φ* or *π*. Now these elements, with a slight metathesis, may become *γραπ* or *glap*, *γαρπ* or *galp*; prefixing an *s* we have *scalp* and *sculpo*, which words indicate operations of a kindred kind, that of carving or embossing. Some philologists, of whom Salmasius is one, consider that by a similar process *γραφω* becomes *scribo*.

I now conclude these few observations, not unconscious that some of the conjectures and speculations which have been ventured may be overstrained, or in some respects mistaken. But I feel considerable confidence that the general views I have expressed, most of which are derived from higher sources than my own opinions, are correct in substance, and are calculated to afford considerable aid in the continuous progress which philology is making.

4. De l'interpolation des fonctions irrationnelles en général, et des fonctions logarithmiques en particulier, à l'aide des tables numériques. Par F. Lefort, inspecteur général des Ponts et chaussées, membre correspondant de l'Académie des Sciences de Naples.

Introduction.

Pour faire un usage intelligent des tables des fonctions irrationnelles en général, et des tables de logarithmes en particulier, il est indispensable de connaître les formules fondamentales de l'interpolation, et de savoir se rendre compte du degré d'approximation que l'on peut obtenir dans les différents cas. Je traite dans ce mémoire ces deux points qui sont à peine indiqués dans la plupart des