To the present writer it has always appeared that the suggestion is an extremely interesting and ingenious one, the chief objection being the immense difficulty in proving that it actually exists or has ever existed as an operative principle. If it exists, then much that is claimed for it would follow. We know that geographical isolation is followed by differentiation, and many evolutionists are prepared to admit that some of the minor differences thus produced may be independent of natural selection. Most naturalists would probably be inclined thus to explain the differences between the land-shells of adjacent valleys in the Sandwich Islands as described by Gulick. Those who believe in physiological selection consider that what is here brought about through the prevention of interbreeding by geographical barriers, is produced on continuously peopled areas by the physiological barrier of infertility.

This conclusion is capable of being tested to a certain extent by an investigation of the degree of infertility between species which are known to have been produced in the former manner and those (of approximately equal differentiation) which are believed to have been produced in the latter. Apart from its application to the present controversy such an inquiry would be of great interest in itself.

The results of such an investigation would be far more convincing than the elaborate and often very ingenious arguments of the writer, many of which are capable of an immediate and satisfactory answer. For instance on pp. 48, 49, he asks how it is that the reproductive system is always affected "in the same peculiar way," viz. so as to produce mutual sterility, between different species of all kinds, animal and vegetable, separated by morphological differences of infinite variety. It is evident that he regards this question as unanswerable except on the view that the infertility is the invariable precursor and condition of the differentiation. But the facts can readily be explained otherwise. Mutual fertility depends upon the exact relotionship of two extraordinarily complex bodies, the germ-cells of male and female; it depends upon a reciprocal adjustment of almost infinite precision. Single individual variations receding from the necessary precision continually arise, but are infallibly exterminated. Such variations are not to be looked upon as due to a single and uniform change in the complex material of the germ-substance. The opposite point of view is the truer: mutual fertility is due to a single and uniform constitution rigidly kept within the narrowest limits, while a minute change of constitution *in any direction* means infertility. Mutual infertility is, in fact, but the single external indication of numberless changes of constitution. The necessary precision of adjustment of the male to the female germ-substance is only kept up in the species by unremitting selection, and there is no cause for surprise that it should cease when selection is no longer forthcoming for its support. These considerations seem at first sight to indicate that mutual fertility between domestic breeds is a matter for greater wonder than the infertility between natural species. We cease to wonder, however, when we reflect upon the length of time which must have elapsed since the separation of natural species such as the horse and ass, which are nevertheless fertile when crossed, although their hybrid progeny is sterile. Since this is the case the mutual fertility of our modern domestic races, so far as it has been proved to exist, is only what we should have been led to expect. In this relationship the present writer has often considered that further experiments upon these latter would be of great value, especially in the case of races in which the morphological differences have been carried to a very high degree—so much so, indeed, that artificial fertilisation would probably be necessary.

The argument which has been met in the last paragraph is evidently one on which the greatest stress is laid. Thus we again read on p. 51 of "this one peculiarity of the reproductive system," viz. mutual infertility; and on pp. 52, 53, it is made a chief support for the hypothesis of physiological selection, a good brief account of which will here be found. Throughout the whole work we meet with the same insistence on "this constant primary distinction," "the same peculiar change," &c., as one main foundation for the hypothesis.

If space had permitted, many other interesting points raised in this volume might have been discussed. It is of great service to the student of evolution that the hypothesis of physiological selection, the arguments for it and evidence which supports it, should have been brought forward in so readable a form. The work is printed in pleasant type, and has been so carefully seen through the press, that there are practically no printer's mistakes. E. B. P.

THE THEORY OF GROUPS.

Theory of Groups of Finite Order. By W. Burnside, M.A., F.R.S. Pp. xvi + 388. (Cambridge University Press, 1897.)

THE theory of groups of finite order is one to which in very recent times the attention of mathematicians has again and again been directed. Until a little time ago any one who wished to become acquainted with the elements of the theory would have been referred to J. Serret's "Algebra," to C. Jordan's comprehensive "Traité des substitutions," and to E. Netto's introductory work on the same subject. No one who takes account of the time at which it was written will wish to depreciate the merits of the first of these, but the chapters dealing with groups consist of a series of extracts reproduced from the original memoirs of the masters to whom we owe this branch of mathematics: these extracts are chosen with all the knowledge of an expert as to what should be chosen, but are not worked up. C. Jordan brings together an overpowering wealth of material, which for the most part has its origin in his own researches; but there can scarcely ever have been any one who would be in a position to work through the treatise from beginning to end; or who could without guidance from some other source separate the fundamental portions from the mass of detail. Lastly, Netto's book on substitutions is now fifteen years old; it still forms a useful introduction to the subject, but it goes no further than that.

The most modern treatises on algebra mete out varying treatment to the theory of groups. Netto leaves it on one side altogether; Drach makes the interesting experiment of attempting to build up the elements of

algebra by a consecutive formulation of Kronecker's abstractions without using the notion of a limit, and restricts himself in the process to those parts of the group-theory that are necessary for his purpose ; Vogt provides for French readers an equivalent of what Netto, in his older book, and Netto's translators, Battaglini and Cole, had given to Germans, French, and English ; in Weber's great treatise the theory appears as an important aspect, but still only an aspect, of the problem of algebra. For all these books the theory of groups is a means to an end, or rather to one of multifarious ends which it can be called upon to serve, viz. the algebraic solution of equations.

But in course of time the theory of groups has ever more and more emancipated itself from algebra, whose servant it originally was. Years ago Cayley threw out, though he did not develop, the fundamental idea that the notion of a group, in and for itself, is in no way bound up with permutations and substitutions, but arises whenever the effect of two operations performed successively upon an object is the same as that of a single operation of the same kind. In the general theory of groups thus founded, groups of finite order form a well-marked division. To bring together the great mass of single results yielded by exploring this division of the theory from different sides, to take a comprehensive view of them, and to exhibit them in a well-digested form, is the problem that has been attempted for the first time in the book before us, and it is solved in the happiest manner.

The author does not push abstraction to the point of banishing from his book all concrete methods of representation of groups. He frequently uses properties of groups of substitutions, in particular, not merely with a view to making results plain to intuition, but also for the deduction from them of properties of groups in the abstract. He himself asks the question why other particular methods of representation of a group, *e.g.* by means of homogeneous linear transformations, are not employed in a similar way, and he answers it, as I think rightly, in the words "that, while in the present state of our knowledge, many results in the pure theory are arrived at most readily by dealing with properties of substitution groups, it would be difficult to find a result that could be most directly obtained by the consideration of groups of linear transformations."

Believing that familiarity with symbolical calculations concerning interchanges of letters is not to be assumed on the part of his countrymen, the author gives a sketch of this theory in a short first chapter. The second chapter begins with Cayley's abstract definition, cited above, and carries the development of the general properties of a group as far as Dyck's theorem to the effect that every group of finite order N can be represented as a group of interchanges of N symbols. The third chapter develops the notions: sub-group, self-conjugate sub-group, simple group, isomorphism, factor groups. Then follow in chapters four and five special investigations relating to Abelian groups and to groups whose order is a prime number ; in the latter the author has placed a series of results of his own researches. The sixth chapter brings us back with Sylow's theorem to the general theory, and the pivot of the theory is found, in chapter seven, in the theorems on the composition-series of a group. The three following chapters are especially concerned with groups of substitutions, with the questions of their transitivity and primitivity ; in addition to the general theorems, they contain a great number of completed researches on special groups and types of groups ; some of these are of intrinsic interest, and others serve as vivid illustrations of the general theory. The eleventh chapter treats of the isomorphism of a group with itself on the lines followed by Hölder and Frobenius. In the twelfth and thirteenth chapters is explained the method of representation of a group which was developed in a general manner by Dyck, viz. the representation by means of geometrical transformations of a surface divided into regions, and especially by the transformations effected by linear relations between complex variables. An older method, due to Cayley, of representation by coloured diagrams leads in simple cases to results easier to appreciate at a glance ; this method also is here expounded and illustrated by a beautifully executed coloured plate. The fourteenth chapter treats of the representation of a group by means of systems of linear congruences. There is a certain incongruity in the fact that while this is inserted, the representation by means of systems of linear equations is omitted ; but the theory of this method would by itself furnish material for a second book as large as the one before us. Perhaps the author will some day give us such a book. On the other hand we find here the extension of the theory named to congruences holding among Galois' imaginaries. This was cultivated in his time by E. Mathieu, and has been taken up recently by E. H. Moore and the author of this treatise, in whose hands it has led, among other remarkable results, to the knowledge of a new series of types of finite groups. Finally the last chapter gives an account of the most modern enumerations of types of groups, in particular of simple groups of an order which can be expressed in a prescribed manner as a product of primes.

The author has not attempted to give historical references concerning the discovery of the older theorems ; but for the more modern literature, of about the last twenty years, and right up to the time of publication, his references are full and trustworthy.

In respect of the completeness and exactness of its matter, the work fills in a very acceptable manner a gap in the literature of mathematics, and not merely of mathematics in England. It brings to the special subject a great wealth of material for the widening and deepening of knowledge ; while at the same time beginners, under expert guidance, will be able to make a profitable use of it.

H. BURKHARDT.

OUR BOOK SHELF.

Radiation: an Elementary Treatise on Electro-magnetic Radiation, and on Röntgen and Cathode Rays. By H. H. Francis Hyndman, B.Sc. (Lond.). With a Preface by Prof. Silvanus P. Thompson, D.Sc., F.R.S. Pp. xviii + 307. (London : Swan Sonnenschein and Co., Ltd. New York : The Macmillan Co., 1898.)

THE author considers chiefly those portions of the subject which are somewhat neglected in most books, and in addition deals with the results of some of the