17. The angular elements of Calcite are—

$$100, 110 = 60^\circ; 010, 011 = 45^\circ 23'4; 001, 101 = 29^\circ 40'.$$

Those of Aragonite are—

$$100, 110 = 58^\circ 5'; 010, 011 = 40^\circ 50'; 001, 101 = 35^\circ 47'.$$

These angular elements differ considerably. Aragonite and Calcite will therefore continue to form distinct mineral species.

18. The author hopes soon to be able to publish the results of investigations which appear to show that the crystals included in the oblique and anorthic systems are formed by the combination of hemihedral and tetartohedral forms of the prismatic system.

8. Preliminary Note on the Connection between the Form and Optical Properties of Crystals. By Alfred R. Catton, B.A., F.C.P.S., Scholar of St John's College, Cambridge.

1. It is the object of this note to give an account of the results of investigations, which have had for their object the discovery of the connection between the form and optical properties of crystals.

It is believed that in the results here given, some of the principal difficulties of this important problem have been overcome.

2. The first step towards the solution of this problem was made by Sir David Brewster in 1818. He discovered that crystals belonging to the prismatic, oblique, and anorthic systems, are biaxial; those belonging to the pyramidal and rhombohedral systems uniaxial, while crystals of the cubic system do not possess double refraction (a fact which had been previously stated by Haüy).

In the prismatic system, the optic axes always lie in a plane containing two of the crystallographic axes, and make equal angles with each of these axes.

Now the theoretical investigations of Fresnel have shown that the optic axes lie in a plane containing the axes of greatest and least elasticity, and make equal angles with each of these axes.

Hence it follows, that, in the prismatic system, the axes of elasticity coincide with the crystallographic axes.

In the oblique system, one axis of elasticity coincides with that crystallographic axis which is at right angles to each of the other two.

In the anorthic system, the position of the axes of elasticity is subject to no known law.

3. These facts show that there is a fundamental connection between crystalline form and optical properties. Of late, nevertheless, many eminent physicists, following the views of De Senarmont, have doubted the existence of such a connection. We shall, therefore, show hereafter that the experiments of De Senarmont as well as those of Des Cloizeaux are not at all opposed to, but important confirmations of, the views here adopted, which are founded on the facts stated above, and in addition on the remarkable relations that have been observed between hemihedral forms and rotatory polarisation in quartz, and dextro- and lævo- tartaric acids.

4. In this note we shall consider in detail the connection between the form and optical properties of crystals belonging to the prismatic system. For crystals belonging to the oblique and anorthic systems, the investigation is still in progress.

5. We proceed to consider the following problem.

“Given the angular elements of a crystal belonging to the prismatic system, to find the angle between the optic axes.”

We have before observed, that, in the prismatic system, the optic axes always lie in a plane containing two of the crystallographic axes, and make equal angles with each of these axes.

First, let the optic axes lie in the plane containing the parameters c and a , and each make an angle ω_c with the axis c . Our object is to find an expression for some function of ω_c , say $\tan \omega_c$, in terms of the parameters a , b , c .

Now crystals belonging to the pyramidal system have only one optic axis, which coincides with the axis c , if $a=b$. Hence, when $a=b$, the angle which each optic axis makes with the axis c vanishes; or, in other words, both optic axes coincide with the axis c .

Hence, ω_c and $\therefore \tan \omega_c$ vanishes when $a=b$.

$\therefore \tan \omega_c$ is divisible by $a-b$.

6. In an abstract of a paper on the “Rhombohedral System,” published in the present number of the “Proceedings,” it is shown that the crystals hitherto included in the rhombohedral system are merely the particular forms which crystals of the prismatic system assume when one angular element is equal to 60° .

But crystals belonging to the rhombohedral system have only one optic axis, and this, as is shown in the paper referred to, coincides with the axis c if the angular element $100, 110 = 60^\circ$; *i.e.*, if

$$\frac{a}{b} = \sqrt{3}.$$

Hence when $a = b\sqrt{3}$, the angle which each optic axis makes with the axis c vanishes.

Hence, ω_c and $\therefore \tan \omega_c$ vanishes when $a = b\sqrt{3}$.

$\therefore \tan \omega_c$ is divisible by $a - b\sqrt{3}$.

But we have before shown that it is divisible by $a - b$.

$\therefore \tan \omega_c$ is divisible by $(a - b)(a - b\sqrt{3})$.

But all uniaxal crystals belong either to the pyramidal or rhombohedral systems.

Hence these are the only factors containing a and b , by which $\tan \omega_c$ is divisible.

7. Again, if $\omega_c = \frac{\pi}{2}$, the optic axes coincide with the axis a .

But when this is the case, by the same reasoning as before, either $b = c$, or $b = c\sqrt{3}$; $\therefore \omega_c$ must $= \frac{\pi}{2}$, or $\tan \omega_c$ must become infinite when $b - c = 0$, or $b - c\sqrt{3} = 0$.

Hence the expression for $\tan \omega_c$, in terms of a, b, c , must be a fraction, the denominator of which contains the factors $b - c$ and $b - c\sqrt{3}$; and, for the same reason as before, these are the only factors in the denominator containing b and c .

Hence, since $\tan \omega_c$ is of no dimensions, the expression for $\tan \omega_c$ in terms of a, b, c , must be of the form

$$\tan \omega_c = \frac{C(a-b)^m(a-b\sqrt{3})^n}{A(b-c)^m(b-c\sqrt{3})^n}$$

where C is independent of a and b , and A of b and c .

8. Crystals belonging to the cubic system do not possess double refraction; in other words, they have an infinite number of optic axes.

Hence, $\tan \omega_c$ ought to become indeterminate, when $a - b = 0$ and $b - c = 0$, which is the case with the above expression.

9. We have shown from physical considerations, that the factors $a - b$, and $a - b\sqrt{3}$, must enter into the expression, for $\tan \omega_c$ at least, to the first degree; there is, however, no physical reason for

supposing that these factors enter to a degree higher than the first, and we are therefore led to assume that they enter to the first degree only. Hence the expression for $\tan \omega_c$ becomes

$$\tan \omega_c = \frac{C(a-b)(a-b\sqrt{3})}{A(b-c)(b-c\sqrt{3})} \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

Similarly, if the optic axes lie in the plane containing the axes a and b , and if ω_a be the angle which each makes with the axis a ,

$$\tan \omega_a = \frac{A(b-c)(b-c\sqrt{3})}{B(c-a)(c-a\sqrt{3})} \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

Also, if the optic axes lie in the plane containing the axes b and c , and if ω_b be the angle which each makes with the axis b ,

$$\tan \omega_b = \frac{B(c-a)(c-a\sqrt{3})}{C(a-b)(a-b\sqrt{3})} \quad . \quad . \quad . \quad . \quad . \quad (3.)$$

We shall show, by the comparison of these formulæ with observation, that $\frac{A}{a} = \frac{B}{b} = \frac{C}{c}$.

10. These formulæ can be easily expressed in terms of the angular elements $\alpha_1, \alpha_2, \alpha_3$.

$$\text{For } \tan \alpha_1 = \frac{a}{b} \quad \tan \alpha_2 = \frac{b}{c} \quad \tan \alpha_3 = \frac{c}{a}$$

Hence, putting $\frac{A}{B} = \frac{a}{b}$, the formula (2.) may be written in the form,

$$\tan \omega_a = \frac{(\cot \alpha_2 - \cot 45^\circ)(\tan \alpha_3 - \tan 60^\circ)}{(\cot \alpha_3 - \cot 45^\circ)(\tan \alpha_1 - \tan 60^\circ)}$$

and so for the others.

11. The angular elements of a crystal are to a certain extent arbitrary; thus the parameters may be changed from a, b, c , to pa, qb, rc —where p, q, r are positive integers none of which are zero—provided the symbols of the simple forms are altered accordingly.

With the new parameters formula (2.) becomes

$$\tan \omega_a = \frac{pa(qb - rc)(qb - rc\sqrt{3})}{qb(rc - pa)(rc - pa\sqrt{3})}$$

We conclude, therefore, that finite and integral values of p, q, r may be found such that, with the angular elements given in

Miller's "Mineralogy," or elsewhere, the calculated agrees with the observed angle between the optic axes.

We proceed to the comparison of these formulæ with observation.

12. *Chrysoberyl*.—In this mineral, the optic axes lie in the plane bc , and make angles of $13^\circ 55'$, with axis c .

Let ω be this angle.

$$\text{Then } \tan \omega = \cot \omega_b = \frac{C \left(1 - \frac{b}{a}\right) \left(1 - \frac{b}{a} \sqrt{3}\right)}{B \left(\frac{c}{a} - 1\right) \left(\frac{c}{a} - \sqrt{3}\right)} \text{ by formula (3.)}$$

Angular elements,—

$$100, 110 = 64^\circ 49'; \quad 010, 011 = 39^\circ 1'; \quad 001, 101 = 30^\circ 7'.$$

$$\therefore \frac{b}{a} = \cot 64^\circ 49' = .4702.$$

$$\frac{c}{a} = \tan 30^\circ 7' = .5801.$$

$$\begin{aligned} \therefore \tan \omega &= \frac{.5298 \times .1856}{.4199 \times 1.152} \times \frac{C}{B} \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (a.) \\ &= .20326 \text{ if } \frac{C}{B} = 1. \end{aligned}$$

On this supposition, ω is somewhat less than $11^\circ 30'$, which differs by $2^\circ 25'$ from the observed value of $13^\circ 55'$; this difference is too great to be ascribed either to errors in the determination of the angular elements, or of the angle between the optic axes.

Hence $\frac{C}{B}$ is not equal to unity.

The value of $\frac{C}{B}$ must be such that the value of ω given by equation (a) is equal, or nearly so, to $13^\circ 55'$.

We find this to be the case on trial, if $\frac{C}{B} = \frac{c}{b}$.

For chrysoberyl $\frac{c}{b} = \cot 39^\circ 1' = 1.2337$.

Hence, equation (a) becomes

$$\tan \omega = .20326 \times 1.2337.$$

$$= .2507.$$

$$\therefore \omega = 14^\circ 4' 5'';$$

which only differs from $13^{\circ} 55'$ by $9' \cdot 5$, a difference quite within the limits of errors of observation.

Hence, from the optical properties of chrysoberyl, we deduce that, if the optic axes lie in the plane bc ,

$$\tan \omega_b = \pm \frac{b(c-a)(c-a\sqrt{3})}{c(a-b)(a-b\sqrt{3})}.$$

The sign $\pm$ is used, for in the reasoning by which the above formula has been deduced, there is nothing to show whether the factor corresponding to $a=c$ is $a-c$, or $c-a$, or that corresponding to $c=a\sqrt{3}$, $c-a\sqrt{3}$, or $a\sqrt{3}-c$, &c.

13. *Aragonite*—

In this mineral the optic axes lie in the plane ca , and for the fixed line B make angles of $9^{\circ} 2' 41'' \cdot 5$ with the axis c , and of $9^{\circ} 20' 20''$ for the fixed line H.*

Hence by formula (1.),

$$\tan \omega_c = \frac{(pa - qb)(pa - qb\sqrt{3})}{(qb - rc)(qb - rc\sqrt{3})} \frac{rc}{p^a}$$

Angular elements—

$$100, 110 = 58^{\circ} 5'; \quad 010, 011 = 40^{\circ} 50'; \quad 001, 101 = 35^{\circ} 47'.$$

$$\therefore \frac{a}{b} = 1 \cdot 6055 \quad \frac{c}{b} = 1 \cdot 1571 \quad \frac{c}{a} = \cdot 72078.$$

In the above formula put $p=q=1$ and $r=2$.

The form denoted in Miller's "Mineralogy" by the symbol 201 is thus taken as the form 101.

$$\begin{aligned} \text{Hence } \tan \omega_c &= \frac{\cdot 6055 \times \cdot 1266}{2 \cdot 3142 \times 3 \cdot 0084} \times 2 \times \cdot 72078 \\ &= \cdot 110106 \times 2 \times \cdot 72078 \\ &= \cdot 15872 \\ \therefore \omega_c &= 9^{\circ} 2' \text{ nearly,} \end{aligned}$$

which differs from Kirchhoff's determination for mean rays, viz., $9^{\circ} 9' 35''$ by $7' 35''$.

14. *Karstenite* (Anhydrite).—Optic axes in plane ab , and make angles of $20^{\circ} 18' \cdot 5$, with axis b . Here,

* Kirchhoff (*Pogg. Annalen.* cviii. (1859), p. 574).

$$\tan \omega = + \frac{(c-a)(c-a\sqrt{3})}{(b-c)(b-c\sqrt{3})} \frac{b}{a}$$

$$\text{putting } \frac{B}{A} = \frac{b}{a}.$$

Angular elements,—

$$010, 011 = 42^\circ 17'; \quad 001, 101 = 44^\circ 25'.$$

If these be altered thus,—

$$010, 011 = 42^\circ 30' \text{ (diff. of } 13'); \quad 001, 101 = 44^\circ \text{ (diff. of } 25'),$$

we have $\tan \omega = \cdot 37134$

$\therefore \omega = 20^\circ 30'$ nearly, which differs by $11' \cdot 5$ from the observed value.

15. *Nitre*.—Optic axes in plane ca , and make angles of about 3° with axis c .

Angular elements,—

$$100, 110 = 59^\circ 25'; \quad 010, 011 = 40^\circ 8'.$$

If these be altered thus,—

$$100, 110 = 59^\circ 40' \text{ (diff. of } 15'); \quad 010, 011 = 40^\circ \text{ (diff. of } 8').$$

The value of ω given by the formula,

$$\tan \omega = \pm \frac{(a-b)(a-b\sqrt{3})}{(b-c)(b-c\sqrt{3})} \frac{c}{a}$$

is $3^\circ 20'$ nearly, differing by about $20'$ from the observed angle.

16. *Cerussite*.—In this mineral the optic axes lie in the plane bc , and make angles of $4^\circ 8'$ with axis c .

Angular elements,—

$$100, 110 = 58^\circ 36' \cdot 5; \quad 010, 011 = 40^\circ 9' \cdot 5; \quad 001, 101 = 35^\circ 52'.$$

$$\therefore \frac{b}{a} = \cdot 6102 \quad \frac{c}{a} = \cdot 723 \quad \frac{c}{b} = 1 \cdot 1851.$$

Hence equation (3.) becomes

$$\begin{aligned} \tan \omega &= \frac{\cdot 3898 \times \cdot 0569}{\cdot 277 \times 1 \cdot 0091} \times \frac{C}{B} \\ &= \cdot 08 \text{ if } \frac{C}{B} = 1. \end{aligned}$$

$\therefore \omega = 4^\circ 36'$ nearly, which differs by $28'$ from the observed value.

If $\frac{C}{B} = \frac{c}{b} = 1 \cdot 1851$, the equation becomes,

$$\tan \omega = \cdot 08 \times 1 \cdot 1851 = \cdot 0948.$$

$\therefore \omega = 5^\circ 24'$, which differs from the observed value by $1^\circ 16'$.

17. By making small alterations in the angular elements of Glaserite, Brookite, Epsomite, Goslarite, Cordierite, the calculated may be made to agree with the observed angle within the limits of errors of observation.

18. Hence if $\frac{A}{a} = \frac{B}{b} = \frac{C}{c}$, the expressions (1), (2), (3), for the inclination of the optic axes to the crystallographic axes c , a , b , when they lie in the planes ca , ab , bc , respectively, agree with observation, in some cases, without making any, or only slight changes, in the angular elements; but in other cases it is necessary to make considerable changes. It is therefore important to consider whether the changes required are greater than the possible errors in the angular elements.

19. Now, the measurements of the angles between the faces of different specimens of the same mineral, by the same or by different observers, often vary very considerably, sometimes as much as 1° or 2° . In Karstenite, for instance, the angle between the faces of the form 110, which are truncated by the face 010, is variously stated as $96^\circ 36'$ or $98^\circ 54'$, which differ by $2^\circ 18'$. This difference is too great to be ascribed to errors of observation. It only appears possible to account for these differences in the measurements of the same or of equally accurate observers, by supposing that the angular elements of a given mineral are liable to vary slightly in different specimens. The question is, to what causes is this variation due? Now, in isomorphous substances, the angular elements often differ by a degree or more. For instance, the angular elements of the minerals isomorphous with Aragonite are,—

	100, 100	010, 011
Aragonite,	$58^\circ 5'$	$40^\circ 50'$
Cerussite,	$58^\circ 36' \cdot 5$	$40^\circ 9' \cdot 5$
Strontianite,	$58^\circ 39'$	$40^\circ 5'$
Witherite,	$59^\circ 15'$	$38^\circ 45'$
Alstonite,	$59^\circ 25' \cdot 5$	$38^\circ 39'$

It will be observed that the angular elements of Alstonite differ respectively by $1^\circ 20' \cdot 5$ and $2^\circ 11'$ from those of Aragonite, those of the other minerals differing by less. Now Alstonite may be considered as Aragonite, in which half the carbonate of lime is replaced by carbonate of baryta. Hence we may say, that when half the

lime in Aragonite is replaced by baryta, the angular elements of the latter are altered from $58^{\circ} 5'$ and $40^{\circ} 50'$ to $59^{\circ} 25' \cdot 5$, and $38^{\circ} 39'$ respectively.

Reasoning of a similar kind may be applied in a number of other cases.

Hence, we conclude that the angular elements of a mineral vary when the normal constituents are replaced by isomorphous substances.

20. Again, all crystals, except those of the cubic system, expand, in general, differently in different directions under the action of heat. This is always the case in crystals belonging to the prismatic, oblique, and anorthic systems.

Hence a change of temperature alters the angles between the faces, and therefore the angular elements, of crystals belonging to these systems, and the angular elements will in general be permanently altered if the temperature be sufficiently raised.

Hence the angular elements of a mineral depend upon the temperature to which it is or has been exposed.

21. We conclude, therefore, that the angular elements, instead of being constant in all specimens of the same mineral, vary within narrow limits, according to the amount of isomorphous replacement and the temperature to which it is or has been exposed;* and further, that the amount of this variation is quite as great as the differences which must be made in the angular elements, given in Miller's "Mineralogy," in order to make the calculated agree with the observed angle between the optic axes. It is possible that, in a number of cases, the angular elements of the crystal employed

* Many apparent cases of dimorphism may possibly be hence explained. Thus, Aragonite is not pure carbonate of lime, but is always associated with variable quantities of the carbonates of strontia, lead, or manganese. Calcite sometimes contains a considerable quantity of foreign substances,—such as protocarbonate of iron, oxide of zinc, &c. Hence, if isomorphous replacement produces changes in the angular elements, these differences in the composition of Calcite and Aragonite would be sufficient to account for the differences in their angular elements (supposing them both referred to the *prismatic* system). Graphite sometimes contains as much as 10 per cent. of iron; titanite is always associated in anatase, rutile, and brookite, with variable quantities of sesquioxide of iron. Differences in the angular elements may also be due to differences in the temperature at which the crystals were formed.

for determining the angle between the optic axes, were, from the causes mentioned, different from those given by Professor Miller.

22. But if the angular elements of a mineral vary within certain limits, the angle between the optic axes, calculated by the formulæ given in this note, must also vary within corresponding limits.*

The causes, therefore, which produce in a given mineral the variations in the angle between the optic axes are the same as those which produce the variations in the angular elements. That the angle between the optic axes of a crystal varies by isomorphous replacement, is established by the experiments of De Senarmont;† that it varies with the temperature to which the crystals are or have been exposed, is established by those of Des Cloizeaux.‡

The existence of this variation, therefore, far from being opposed to the existence of a connection between the form and optical properties of crystals, as argued by De Senarmont,§ is strongly in support of it.

23. Suppose that from either of the causes mentioned, the parameters a, b, c , become $a + x, b + y, c + z$, where x, y, z , are small quantities. Let the optic axes lie in the plane ca , and make an angle ω with the axis c , and let ω' be the angle corresponding to the new parameters. It is easily seen that

$$\tan \omega' = \tan \omega \left[1 + \frac{\frac{x-y}{a-b} - \frac{y-z}{b-c} + \frac{x-y\sqrt{3}}{a-b\sqrt{3}} - \frac{y-z\sqrt{3}}{b-c\sqrt{3}}}{1 + \frac{y-z}{b-c} + \frac{y-z\sqrt{3}}{b-c\sqrt{3}}} \right]$$

neglecting $\frac{c+z}{a+x}$

Hence, if 100, 110 be nearly 45° or 60° , and $x - y$ not very small in the former case, nor $x - y\sqrt{3}$ in the latter, while 010, 011 differs considerably from 45° or 60° , $\tan \omega' - \tan \omega$ will be large. In this case, therefore, the angle between the optic axes will vary considerably.

* Topaz and mica are well known examples of this variation of the angle between the optic axes.

† Annales de Chemie, third series, xxxiii. p. 391.

‡ Annales des Mines, sixth series, ii. p. 327.

§ Ibid., p. 433.

If 010, 011 be nearly 45° or 60° , and $y - z$ not very small in the former case, nor $y - z\sqrt{3}$ in the latter, while 100, 110 differs considerably from 45° or 60° , the value of $\tan \omega' - \tan \omega$, though considerable, will not be so great as before.

If 100, 010 and 010, 011 are both nearly equal to 45° or 60° , $\tan \omega'$ will differ very little from $\tan \omega$, except when $x - y$ and $y - z$, or $x - y\sqrt{3}$ and $y - z\sqrt{3}$ differ considerably.

Hence are explained the variations of the angle between the optic axes produced by an increase of temperature in Brookite and Chrysoberyl, as observed by Des Cloizeaux.

24. We find that the expression for $\tan \omega_c$ in terms of a, b, c , contains the factor $\frac{c}{a}$. For the existence of this factor no *a priori* physical reason can be assigned. Now $c = 0$ represents an infinite plane, containing the crystallographic axes a and b , and $\tan \omega_c$ vanishes when $c = 0$. The physical interpretation of this factor therefore is, that such an infinite plane has only one optic axis which coincides with the axis c . We may compare this result with the fact, that if a ray of light is incident on a crystal of the prismatic system, in the direction of one of the principal planes, one of the refracted rays, which is polarised in that principal plane, follows the ordinary law of refraction.

25. Again, the expression for $\tan \omega_c$ becomes indeterminate if

$$\begin{aligned} a - b &= 0, \text{ and } b - c\sqrt{3} = 0; \\ \text{or, } b - c &= 0, \text{ and } a - b\sqrt{3} = 0; \\ \text{or, } a - b\sqrt{3} &= 0; \text{ and } b - c\sqrt{3} = 0. \end{aligned}$$

Hence, if the angular element of a crystal of the pyramidal system were equal to 60° , it would not possess double refraction; the same would be the case if one of the angular elements of a crystal of the rhombohedral system were equal to 60° .

26. If a', b', c' be the optical constants in descending order of magnitude, and ω the angle which each optic axis makes with the axis of least elasticity, we have,

$$\tan^2 \omega = \frac{a'^2 - b'^2}{b'^2 - c'^2},$$

assuming Fresnel's expression for the angle between the optic axes

in terms of the optical constants,* but if the optic axes lie in the plane ca , we have also,

$$\tan^2 \omega = \frac{(a-b)^2 (a-b\sqrt{3})^2 c^2}{(b-c)^2 (b-c\sqrt{3})^2 a^2};$$

$$\therefore \frac{a'^2 - b'^2}{(a-b)^2 (a-b\sqrt{3})^2 c^2} = \frac{b'^2 - c'^2}{(b-c)^2 (b-c\sqrt{3})^2 a^2},$$

which is a relation between the optical and crystallographic constants.

Similarly, if the optic axes lie in the plane ab , and $b'c'a'$ be in descending order of magnitude,

$$\frac{b'^2 - c'^2}{(b-c)^2 (b-c\sqrt{3})^2 a^2} = \frac{c'^2 - a'^2}{(c-a)^2 (c-a\sqrt{3})^2 b^2},$$

and if the optic axes lie in the plane bc , and $c'a'b'$ be in descending order of magnitude,

$$\frac{c'^2 - a'^2}{(c-a)^2 (c-a\sqrt{3})^2 b^2} = \frac{a'^2 - b'^2}{(a-b)^2 (a-b\sqrt{3})^2 c^2}.$$

27. A serious objection may be made to formulæ such as the above, expressing relations between a', b', c' , and a, b, c .

In the formula

$$\frac{a'^2 - b'^2}{b'^2 - c'^2} = \frac{(a-b)^2 (a-b\sqrt{3})^2 c^2}{(b-c)^2 (b-c\sqrt{3})^2 a^2}$$

for instance, the left side of the equation is a function of the wavelength.

But since a, b, c depend only on the angles between the faces of crystals, which are of course invariable, the right side of the equation is independent of the wave length.

The only way of overcoming this difficulty appears to be by supposing that the apparent angles between the faces of crystals of the prismatic system, as determined by the reflective goniometer, may vary with the kind of light employed. Some experiments

* The measurements of Kirchhoff (*Ibid.*) have shown that this formula agrees closely with observation in the case of Aragonite.

Also, if $a' = b'$ the crystal becomes uniaxal, and the optic axes coincide with axis c' ; hence $\tan^2 \omega$ ought to vanish when $a' = b'$.

If $b' = c'$ the optic axes coincide with axis a' ; hence $\tan^2 \omega$ ought to become infinite when $b' = c'$. Fresnel's expression fulfils these conditions.

of Sir David Brewster on crystalline reflexion appear to countenance such a supposition ; but the problem of crystalline reflexion is involved in obscurity.

28. Since all the properties of crystals, as the values of the principal refractive indices, the wave velocities, the form of the wave surface, &c., depend on the values of a' , b' , c' , it follows that the complete solution of the problem of the connection between the form and optical properties of crystals consists in determining a' , b' , c' as functions of a , b , c .

$$\text{Let } \frac{a'^2}{f_1 abc} = \frac{b'^2}{f_2 abc} = \frac{c'^2}{f_3 abc}$$

where a' b' c' are in descending order of magnitude,

$$\text{then } f_1 abc - f_2 abc = (a - b)^2 (a - b\sqrt{3})^2 c^2$$

$$f_2 abc - f_3 abc = (b - c)^2 (b - c\sqrt{3})^2 a^2$$

$$\begin{aligned} \therefore \frac{a'^2}{(a - b)^2 (a - b\sqrt{3})^2 c^2 + (b - c)^2 (b - c\sqrt{3})^2 a^2 + f_3 abc} \\ = \frac{b'^2}{(b - c)^2 (b - c\sqrt{3})^2 a^2 + f_3 abc} \\ = \frac{c'^2}{f_3 abc} \end{aligned}$$

Hence only one function $f_3 abc$ remains to be determined. Symmetry would suggest the value $(c - a)^2 (c - a\sqrt{3})^2 b^2$ for $f_3 abc$. We have not as yet, however, attempted to determine the value of $f_3 abc$ by the comparison of this formula with observation.

29. The investigation of the optical properties of crystals belonging to the oblique system is still in progress. The following is the method employed in this investigation. Each crystal is referred to three rectangular axes, one axis being perpendicular to the plane of the optic axes, the other two being the internal and external bisectors of the angle between the optic axes. The new parameters are calculated by means of formulæ investigated in a paper on the "Rhombohedral System;" and thence the angle between the optic axes is found, as if the crystal belonged to the prismatic system, by means of the formulæ given in this note. The angle between the optic axes of one mineral belonging to the oblique system has been calculated by this process; and the calculated has been found

to agree approximately with the observed angle. If this should prove to be generally the case, it will not only be a solution of the problem which forms the subject of this note for crystals of the oblique system, but it will prove that these crystals are formed according to the same laws of symmetry as crystals of the prismatic system.

9. A Contribution to the History of the Oxides of Manganese. By W. Dittmar, Esq.

It is known that oxides of manganese, when ignited in air, either absorb or lose oxygen, until they attain the composition expressed by the formula Mn_3O_4 . Schneider found that when they are heated in pure oxygen, they leave the oxide Mn_2O_3 .

This latter result is not what one might have expected, for we know of no positive action exerted on manganese oxides or on pure oxygen by nitrogen. Thus air should act like dilute oxygen, producing the same oxide.

To explain Schneider's result, we must suppose either that artificial oxygen differs from that contained in air, or, what is more likely, that the amount of oxygen which any given quantity of manganese can take up when heated in it, depends not only on the temperature, but also *on the tension of the oxygen* employed.

If, then, it is true, that at a red heat, in presence of oxygen of the tension of one atmosphere, Mn_2O_3 is a stable compound, while the same oxide, exposed to the same temperature in presence of oxygen of the tension of one-fifth of an atmosphere, is reduced to Mn_3O_4 , the question naturally arises, What oxides will be formed in presence of oxygen of different tensions from those two above mentioned? Is there a continuous series of oxides corresponding to a continuous series of oxygen-tensions under which they have been formed?

These questions seemed to me to merit investigation, especially as their results would form an addition to our very imperfect knowledge of the influence of physical conditions on chemical reactions.

In the first series of experiments, pure peroxide of manganese, in a porcelain or platinum boat, was placed in a porcelain tube, kept at a bright red heat, while a current of nitrogen or oxygen, or of a mixture of these two gases, was passed over it. The