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LIII. *Theory of the Spirit-Level.* By J. NIXON, Esq.*

Definitions†.—1. **T**HAT part of the straight (or perpendicular) line in which the plummet hangs, and bodies fall to the ground, which lies above any given point (on the earth's surface) through which it passes, is termed the vertical (line) of that point, and terminates upwards at another point called the vertex or zenith. 2. A straight line, or plane passing through the given point at right angles to its vertical, is termed a horizontal line or plane. 3. As no one vertical is parallel to another, the horizontal lines or planes of points situated in the same vertical, although parallel to each other, are nevertheless inclined to those of points lying in any other vertical. 4. Planes which pass through the given point and that of its zenith (in the direction of the vertical line in which they are situated), cut its horizontal plane at right angles, and are called vertical planes. 5. As the angle formed at the given point by the intersection of its vertical and a straight line from any other point will lie in a vertical plane, it is termed a vertical angle, and is equal to the zenith distance of that point or line. 6. The vertical angle formed at the given point by the intersection of one of its horizontal lines and a straight line produced from any other point, is equal to the horizontal inclination of that line. 7. This vertical angle is also termed the elevation or depression of the same line or point, accordingly as it is situated above or below the horizontal plane (or horizon) of the given point. 8. The zenith distance of a horizontal line or plane, and the elevation of the zenith of any given point being equal to each other and to a right angle, it follows that the angle of elevation of any other point or line is equal to 90° minus its zenith distance, and that of depression to the zenith distance minus 90° .

9. Fluids gravitate in straight lines in the direction of gravity. 10. When at rest, and subjected to the sole action of gravity, their order of superposition, with curved surfaces of contact, is inversely as their specific gravities. 11. The horizontal lines or plane of any given point situated on the surface of a fluid, having other fluids superincumbent or not, are tangents to that surface. 12. When the surface of the fluid is of inconsiderable extent, it is sensibly a horizontal plane, perpendicular to the vertical and parallel to the horizontal lines and plane of any other point above or below it.

* Communicated by the Author.

† The exact figure of the earth's surface is supposed to be unknown.

If we place in a vertical position the sides of a (glass) vessel *W*, formed of two equal parallel circular planes, held together by a rim *R*, perpendicular to the planes, the surface of contact of the incompressible fluid (or liquid), with the superincumbent elastic fluid, together filling the vessel, will be sensibly a horizontal plane. A vertical plane passing in the direction of the centres of the circles, (through their vertices and that of the rim,) will divide at right angles a straight line *Ll*, drawn on this horizontal surface parallel to the circles, into two equal parts. The vertex or zenith *v* of the circumference of either circle or the rim will therefore be the point of bisection of such part of the arc of either as is situated *above* this horizontal surface.

Having marked this zenith-point, if the vessel be made to describe in a vertical plane, any part of a revolution about the horizontal line or axis *C*, passing through the centres of the circles, the mark moving along with the vessel, will pass over an equal arc of revolution. The zenith-distance of a straight line drawn from the mark, which we will now call *v'*, to *C*, will therefore be equal to that arc or to the angle formed by the intersection at *C* of this straight line, and a vertical line passing through the *new* zenith-point of the rim, &c. found by bisecting, as before, the arch of the rim, &c. *now* extended over the horizontal surface *Ll*.

When the interior of the rim is perfectly circular, the arc passed over (or zenith-distance of *v'*) may be measured at once on its graduated parallel exterior. But should the figure of the rim be that of *any* other curve, the length of *Ll* will vary in different parts of the curve;—the zenith-points will seldom be vertical to the point of bisection of *Ll*, or be situated at the middle point of the arch extended over it. The points *v* and *v'* must now be found exclusively by drawing straight lines through the centre of revolution (or axis) of the vessel perpendicular to *Ll*; their angular opening or zenith-distance of *v'* being measured on a graduated circle described

on either vertical side of the vessel concentric to the axis of rotation.

Vertical lines passing through points situated near each other, differ so slightly in parallelism, that the vessel might have been moved forty horizontal feet in the interval of marking the two zenith-points, without introducing an error in the zenith-distance of v' equal to half a second. If the expansion of the liquid should exceed that of the glass of the vessel, the increase of temperature which augments its volume will elevate its horizontal surface, and cause a sensible diminution in its area and the length of the line Ll . Decrease of temperature will therefore augment the horizontal surface, and elongate Ll . In either case the zenith-point will be invariably at the point of bisection of the arch situated above the surface of the liquid, without regard to its extent.

When the temperature of every part of the vessel is not the same (which may be the result of handling it, or breathing on it), the circular figure is destroyed, and the length as well as probably the figure of the surface of the liquid undergo alterations. The graduations of the distorted rim are therefore rendered unserviceable from their inequality; and should the partial temperature affect the vessel where in contact with the surface of the liquid; in such case the true zenith-point cannot be found otherwise than by drawing a line through C perpendicular to Ll . Generally the surface of the liquid will appear to advance towards that part of the rim bulged out by the partial increase of temperature.

In the construction of a spirit-level the upper interior surface of a straight (hollow) cylinder of glass is ground in the direction of its axis to a perfectly circular arch. Either end being permanently closed, the cylinder is *nearly* filled with spirits of wine or ether, and the other end hermetically sealed. Hence it is evident that any section of our circular vessel perpendicular to its sides, when nearly filled with the proper liquid, and securely closed up, would be equivalent to a similar spirit-level. In this instrument the atmospheric air incumbent on the ether, &c. or rather their surface of contact, is termed the air-bubble, or simply the bubble; and is represented in our circular vessel by the horizontal surface of the liquid on which is drawn the straight line Ll .

Having learned from our experiments with the circular vessel, that the length (or figure) of the bubble Ll would not alter in a constant temperature, we may restrict ourselves, in lieu of finding the zenith-points v and v' , to the marking of either end of the bubble, as L or l , *before*, and the *same* end
subsequent

subsequent to any degree of revolution of the vessel. The zenith-distance of v' , equal to its change of inclination, may now be ascertained at once, by observing on the graduated rim, &c. the angular distance of the two marks. To guard against any change of temperature in the interval of the operation, it will be advisable to mark the rim at both ends of the bubble (L and l) and to consider the half-sum of the degrees, &c. on the rim corresponding to each mark as the zenith-point of v (or v'). The difference of these two half-sums, granting the change of temperature to have been uniform, will give the zenith-distance of v' . When the graduations are sufficiently large to admit of being read off without vernier, &c. we may dispense with the marks by noting the degrees, &c. exactly over the ends of the bubble.

In order to graduate in a similar manner the upper or convex surface of a spirit-level, we must ascertain in the first place the linear space passed over by its bubble, corresponding to a certain angular change of inclination, as one minute, one second, &c. This may be effected after various methods*; as by fixing the level to a long straight bar of a known length, and having elevated either end a quantity equal to the given angle, to note the inches, &c. passed over by the bubble; generally termed its displacement. The tube or the ivory scale laterally attached to it, may now be divided into equal spaces†, each equivalent to a change of inclination of one second, &c.; and so numbered as to give, without risk of mistake, the middle point of the bubble, regardless of its varying length, and consequently minute differences of vertical angles‡.

* The French verify the scale of the great level of their repeating-circle, by measuring on its graduated vertical circle a sufficient multiple of the minute angle subtended by two well-defined fixed objects situated in the same vertical plane, and comparing the result with the corresponding measurement by the divided scale of the level.

In lieu of the repeating-circle we might have recourse to the micrometer of a telescope.

† When the bubble does not pass over equal spaces for equal angles of inclination throughout the length of the tube, it proves that the arch is not perfectly circular.

‡ In general the middle division is considered as zero, whence the numeration, alike for the divisions on each side of it, is carried on progressively to each end of the scale.—The divisions on the one side of zero being considered as positive, and those on the other side as negative, the distance of each end of the bubble from zero, with the proper signs prefixed, are registered; and half their sum, when the signs are alike, or half their difference (with the sign of the greater quantity), when the signs are unlike, is considered the middle point of the bubble, or vertex of the level. Hence the difference or sum, as their signs are like or unlike, of two similar middle points, of which one was noted before, and the other after an alteration of inclination of the level, indicate its angular value.

The radius of curvature of a spirit-level is found by multiplying the linear displacement of the bubble, answering to a change of inclination of one second, by 200,000; and vertical angles are measured on the divided scale of a spirit-level as correctly as by a plumb-line of the length of the radius of curvature of its tube*. (Had the cylindrical tube of the level been without curvature, and closed at the ends with (circular) planes perpendicular to the axis of the cylinder, the bubble, if we may designate the horizontal surface of the liquid as such, would extend from one end of the tube to the other, and small angles of inclination, measured on a divided vertical line or scale passing through the centre of each end, would possess a degree of accuracy equal only to similar observations made with a plumb-line of the length of the cylinder.)

The circular vessel *W*, is represented in the figure as fixed to a pedestal (*P*) of a triangular shape; and we might imagine, that increased temperature, inasmuch as it would elevate *T* without affecting the height of *U*, would produce a greater inclination of the plane *TU*, and consequently throw *vC* out of perpendicular. But as the sides of the triangle will elongate in one and the same ratio, the angles they subtend, and therefore the horizontal inclination of *TU*, must be constant. Hence the *exterior* sides of the tube, instead of being parallel, might be inclined to the cylindrical interior without disturbing, during variations of uniform temperature, the position of the bubble. Supposing even the *interior* of the tube to be conical, it does not follow, as might at first be conjectured, that change of temperature would affect the inclination of the level.

Let *ABCD* represent the vertical section of a conical tube resting on the plane *E*, which forms an angle with the horizon equal to the inclina-

tion of the sides of the cone (or *BfD*), so that the upper side of the cone *AB* will be horizontal. Then if the four sides of the section augment (from expansion) in the same ratio, the

* The linear displacement per second of the bubble of a spirit-level seldom exceeds the one-twentieth part of an inch; but occasionally, especially on the continent, they have been constructed with a much greater radius of curvature. In a level by Reichenbach it amounted to 200 miles!

angles at A, B, C, D will be constant, and the upper side AB preserve its parallelism to the horizon*.

When the temperature of the level is not uniform, the bubble (as in the parallel case of the circular vessel) will be displaced and move towards the warmer end of the tube. Its ground arch, as well as the divisions of the scale being distorted, the half-sum of the divisions at each end of the bubble cannot correspond with the true vertex of the level.

The tube of the level is generally mounted in a thin case of brass, a metal which expands in a greater ratio than glass. When the bottom of the case and that side of the tube in contact with it are not strictly parallel, it may occur in great variations of temperature that the tube will rest on some different part of the case, and cause a sensible variation in its inclination to the horizon. The difference of expansion may also affect the radius of curvature, or alter the perfectly circular figure of the arch of the tube.

In our circular vessel, as the graduated rim is perpendicular to its horizontal axis, either end of the bubble (or horizontal line *LZ*) must describe, as the vessel revolves, arcs of a circle lying in a vertical plane: and were a hollow (glass) sphere, nearly filled with any liquid, made to complete a revolution about a horizontal line (or axis) passing through its centre, then would the centre of the surface of the liquid (or bubble) have described a great circle, also lying in a vertical plane perpendicular to the axis, and on which, when graduated, zenith-distances, &c. might be measured, precisely the same as on the rim of the circular vessel. And if we divide the interior of the sphere into two *unequal* parts by means of a circular plane (inferior in diameter to the sphere) also placed at right angles to the axis of rotation, and nearly fill the two divisions with any liquid, then will a vertical plane passing in the direction of the axis of the sphere, divide at any period of its revolution the semicircular bubble of the lesser division, and the circular one of the larger division (moving parallel to each other) into two equal parts. Hence differences of inclination, &c. measured on the great circle, or on the similarly graduated rim of the parallel circular plane would always be equal.

Let a graduated hollow glass ring, nearly filled with any liquid, be made to closely incircle the sphere (the partition being withdrawn) in the direction of any one of its great circles except the one perpendicular to its axis. When the point of intersection of these two circles is made to coincide with the

* It is nevertheless *certain* that change of temperature alters the vertex, or reversing point of most spirit-levels; which the artists attribute to the tubes not being perfectly cylindrical.

vertex of the sphere, then will the centres of the bubble of the ring and that of the sphere also coincide, or be in the same vertical; yet it will be found after any partial revolution of the sphere, that although the bubble of the ring will always come to rest at the most elevated point of the ring, or that part of it the nearest to the vertex of the sphere (or centre of its bubble), its distance from its initial mark, as measured on its graduated scale will, however, fall short of the correct zenith-distance of that mark or arc of revolution;—the discrepancy augmenting with the inclination of the ring to the circle described by the bubble of the sphere. When the inclination equals 90° , in which case (the plane of) the ring passes through (the centre of) the sphere in the direction of its axis, the arc of revolution may amount to 90° , without displacing the (unserviceable) bubble of the ring from between its marks*.

Let $a'b'a'b'$ be the great circle of the sphere parallel to the horizon; bb' the vertical circle perpendicular to the axis of rotation aa' , described by the bubble of the sphere now at rest at the vertex v ; and let v' be the bubble of the (oblique circle, or) ring rr' , inclined to bb' at an angle equal to $\angle vmv'$, m being their point of intersection or initial mark where the bubbles of the ring and sphere coincided when

in the same vertical. Then as the bubble of the ring will be stationary at that point of the ring the most elevated above the horizontal circle, v' will be equidistant, or 90° , from r and r' , and also touch the nearest of the small circles of equal altitude described round v as a centre, so that $\angle m v' v$ must be right-angled at v' . We shall, therefore, have given in the right-angled spherical triangle vmv' the leg $v'm$ (or zenith distance of m as given by the bubble and graduations of the ring) and the angle (of inclination of the ring to the vertical

* The bubble will nevertheless pass over an arc of 90° of the minute circle on which we measure the interior diameter of the ring.

circle)

circle) $v m \nu$, to find the hypothenuse $m v$ (or true zenith-distance of the mark m).

Were fluids indeed subjected, as we have hitherto supposed, to the sole action of gravity, our explanation of the theory of the spirit-level might be considered as complete; but from the effect of the mutual attraction of glass and the liquid of the level, the figure &c. of the bubble, as we shall proceed to demonstrate, must suffer material alterations.

[To be continued.]

LIV. *Observations on the Crystalline Form, &c. of the Gaylussite.* By W. PHILLIPS, F.L.S. G.S. &c.*

IN the *Ann. de Chim.* for March 1826, is inserted an account and analysis of a mineral newly discovered in a natron-lake in Colombia, by M. Boussingault, followed by a description of its crystalline forms, by M. Cordier. It appears to be a hydrous carbonate of lime and soda; consisting of Carbonate of soda 33.96, Carbonate of lime 31.39, Water 32.20, Carbonic acid 1.45, and Alumine 1.0, according to M. Boussingault. It has received the name of Gaylussite, in honour of the celebrated French chemist M. Gay-Lussac.

Five crystals of this substance have been presented to me by my brother, who lately received them from Robert Stephenson, a gentleman connected with the establishment of the Columbian Mining Company. One only of these crystals is what may be termed symmetrical in its form, the rest being elongated and channelled on their surfaces in a very remarkable manner. M. Cordier also appears to have possessed only one regularly-formed crystal; but as this was not, as he observes, sufficiently bright for the use of the reflective goniometer, he was compelled to rely on the common one for the measurement of its planes. Mine, on the contrary, is remarkably brilliant, and even transparent: I submitted it therefore to the former instrument, which confirms the most important measurements by M. Cordier.

The primary form adopted by M. Cordier is an irregular octohedron; but finding, as he observes, that "it is not easy to make it clearly appear how the planes of these crystals relate to that form as their primary," he has substituted, "as being more simple, and as Haüy had done in analogous cases, an oblique prism." The measurements and cleavages of these crystals have led me to the conclusion, that the primary is in reality an oblique rhombic prism, but of different measure-

* Communicated by the Author.