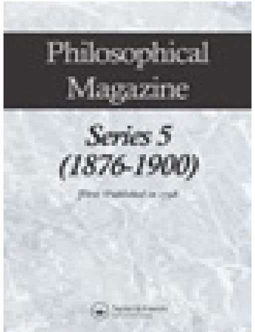

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# XXV. The study of transformers

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electrolytes increase in mean amount of their specific gravities during mixture or dilution; these statements agree with the usual views entertained on the subject.

By comparison of these results with those in Table I., and those of Table I. with one another, it appears, 1st, that increase of mean specific gravity of the two liquids during dilution or mixing occurs not only in cases in which heat is evolved and lost, but also in those in which heat is absorbed and becomes latent; 2nd, that increase of mean electromotive force of the two liquids also occurs not only in those cases in which heat is absorbed and becomes latent, but also in those in which it is set free. And 3rd, that in 17 cases out of 20 an increase of mean specific gravity of the two liquids was attended by an increase of the mean amount of their electromotive force. We may therefore conclude that *in cases of mere physical mixture, the changes of mean specific gravity and of mean electromotive force of electrolytes are probably related to each other* as concomitant effects of the same cause, change of molecular motion. From the known general relation of increase of specific gravity to decrease of specific heat, it further suggests the inference that in cases of simple dilution the mean electromotive force of electrolytes is related to their mean specific heat.

As in all such cases, each phenomenon, whether it be physical mixture, dilution, specific gravity, specific heat, latent heat, or electromotive force, involves the mutual action of two substances, it is essentially necessary in every case to ascertain the *mean* amount of change occurring in the two substances.

In a separate and more extensive research, not yet published, I have measured the losses and gains of electromotive force by means of different positive metals in different classes of mixtures of electrolytes, including cases of chemical union as well as of mere physical mixture.

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## XXV. *The Study of Transformers.*

*By* Prof. JOHN PERRY, *F.R.S.*, *D.Sc.**

[The following paper was written in February last, at a time when I had been compelled to listen to many discussions on the Transformer. It has been nearly forgotten till now; but I see that it ought to be published now, so that it may precede my paper read four days ago before the Physical Society on 'Mr. Blakesley's method of Measuring Power in Transformers."—26th May, 1891.]

I HAVE been engaged on quite other matters during the last few years, and thought a few weeks ago that I must be hopelessly in arrear concerning the subject of

* Communicated by the Author.

transformers. Every year I give to students in an "Applied Mathematics" class the old-fashioned formulæ concerning two or more circuits; my students have generally worked a few numerical examples on transformers, and we have then gone on to other subjects.

I venture now to think that I have lost almost nothing by remaining for so long ignorant of the vague but reckless statements made by writers of papers on this subject, and that students will gain something by coming back to the old-fashioned method of treating it.

I say, on the assumption of constancy of magnetic permeability, of no hysteresis, and no heating of either iron or copper by eddy currents, what are the laws of a transformer?

Let this be found out first.

On comparing the theoretic and experimental results, we shall be in a position—we have never yet placed ourselves in this position—to theorize on the effect of the new phenomena, and even this it will not be wise to do until we observe whether magnetic leakage will not account for some of the discrepancy.

If one gives numerical values to resistances, sizes of iron, numbers of windings, &c. it is quite easy—a matter of a few hours at most—to calculate everything about all the currents for a transformer with two, three, or more coils.

This year, instead of asking my students to work a few numerical exercises, I asked them to work out quite a number of exercises on a transformer with two coils, and I venture to think that the numbers given by them in the following Tables are of even more interest—just now—than experimental results. The tables give results that no experimenter could give. I have had some experience myself, and really I cannot say that I see my way, with any ordinary dynamometer, to distinguish between a lag of $175^\circ$ and $180^\circ$; whereas in these tables the lag sometimes varies for no load and full load between $179^\circ{\cdot}940$ and $179^\circ{\cdot}942$. The graphical method of working cannot distinguish differences so small as these.

As for the trouble of working,—my students have much other home-work to do, and they do it, and yet one of them has brought me two complete tables worked out at home between Friday and Monday.

Let V be the primary voltage, C the primary current, R primary resistance, L the primary self-induction, C′ the secondary current, R′ the secondary resistance, L′ the secondary self-induction, and M the mutual induction. Let $\rho$ be a non-inductive part of the resistance of the secondary, the part external to the transformer. Let V′ be the voltage at the terminals of $\rho$. Let P be the average power given to the primary, and P′ that given out by the secondary. Then, taking

$$\left.\begin{aligned} V &= a \sin\frac{2\pi}{\tau}t, \\ C &= A \sin\left(\frac{2\pi}{\tau}t-\epsilon\right), \\ C' &= A' \sin\left(\frac{2\pi}{\tau}t-\epsilon'\right), \\ V' &= a' \sin\left(\frac{2\pi}{\tau}t-\epsilon'\right)=\rho C', \\ P &= \tfrac{1}{2}aA\cos\epsilon, \quad P'=\tfrac{1}{2}\rho A'^2, \\ \text{Percentage efficiency} &= 100\,\frac{P'}{P}=E, \text{ say,} \end{aligned}\right\} \quad . \quad . \quad . \quad (1)$$

it is known that to calculate all the necessary values, the simplest plan is to calculate the following magnitudes first :—

$$\left.\begin{aligned} & I'=\sqrt{R'^2+\frac{4\pi^2L'^2}{\tau^2}}, \quad r=R+\frac{4\pi}{\tau^2}\frac{M^2R'}{I'^2} \\ & l=L-\frac{4\pi^2}{\tau^2}\frac{M^2L'}{I'^2}, \quad i=\sqrt{r^2+\frac{4\pi^2l^2}{\tau^2}} \\ \text{Then} \quad & A=\frac{a}{i}, \quad \tan\epsilon=\frac{2\pi l}{\tau r}, \\ & A'=\frac{2\pi}{\tau}a\frac{M}{I'i}, \quad \tan\epsilon'=\frac{\tau}{2\pi}\cdot\frac{\frac{4\pi^2}{\tau^2}L'l-R'r}{L'r+R'l}. \end{aligned}\right\} \quad . \quad . \quad (2)$$

As an example of a transformer with which we have experimented electrically and arithmetically, take R=10 ohms, R′=·1+$\rho$, $a$=1000 volts, L=10 secohms, L′=·1 secohm, and take as is usual M=1 secohm. That is, assume no magnetic leakage. We have the following interesting results obtained by altering $\tau$, beginning with a frequency of 160 per second and ending with the rather absurd case of $\tau$= more than 6 seconds. It is to be observed that when I say in any case that $\frac{2\pi}{\tau}=1000$, or 100, I really mean that $\frac{2\pi}{\tau}L=10000$ or 1000 respectively. It will be observed that L, M, and L′ only enter into the calculations in combination with $\frac{2\pi}{\tau}$. Hence, instead of saying that my tables show the effect of diminishing frequency in a given transformer, I might say that they really show the effect of keeping frequency constant and diminishing the section or increasing the length of the iron magnetic circuit.

Thus, for example, the table for $\frac{2\pi}{\tau}=100$ may mean

$$\frac{2\pi}{\tau}=100 \text{ and } L=10,\ M=1,\ L'=\cdot 1\ ;$$

but this table is correct for

$$\frac{2\pi}{\tau}=1000, \text{ and } L=1,\ M=\cdot 1,\ L'=\cdot 01.$$

I shall only assume, in fact, that in any table the ratios of L to M and L′ remain constant.

For the sake of beginners it is well to state that, using amperes, volts, and ohms, if P and S are the numbers of windings of the primary and secondary respectively; if $a$ is the cross section of the iron in square centimetres, and $\lambda$ the average length of the complete iron magnetic circuit and $\mu$ the permeability (being about 1500 in ordinary transformer working), we may take it that

$$\left.\begin{aligned} L &= P^2\frac{a\mu}{\lambda}\frac{4\pi}{10}, \\ L' &= S^2\frac{a\mu}{\lambda}\frac{4\pi}{10}, \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (3)$$

and if there were no magnetic leakage—that is, if all the field due to a primary current through every winding of the primary passed through every single winding of the secondary, then

$$M=\sqrt{LL'}, \quad \text{or } M=PS\frac{a\mu}{\lambda}\frac{4\pi}{10}.$$

If two or three tables be compared for which the values of $\frac{2\pi}{\tau}$ L differ even greatly, it will be noticed that A′, $\epsilon'$, $\rho'$ are practically the same from the very smallest to the greatest loads. Also, except for small loads A, $\epsilon$, P, and E, are practically the same. This is the more striking as the frequency is greater. Now this is really the same as saying that if the ratios of L, L′, and M remain constant, considerable changes in their absolute values do not greatly affect the results ; that is, considerable changes in $\mu$ do not affect the results ; that is, even hysteresis need not be expected to greatly affect the results.

$$\frac{2\pi}{\tau}\mathrm{L}=10000.$$

| $\rho$. | A. | A′. | $\epsilon$. | $\epsilon'$. | $a'$. | P. | P′. | E. |
|---|---|---|---|---|---|---|---|---|
| ∞ | ·100 | 0 | 89°·9 | 179°·939 | 100 | ·085 | 0 | 0 |
| 99·9 | ·141 | ·999 | 44·97 | 179·939 | 99·82 | 50·06 | 49·85 | 99·59 |
| 49·9 | ·223 | 1·997 | 26·5 | 179·940 | 99·61 | 99·95 | 99·45 | 99·49 |
| 9·9 | ·996 | 9·913 | 0·05 | 179·940 | 99·14 | 497·8 | 486·4 | 97·70 |
| 4·9 | 1·963 | 19·61 | 0 | 179·941 | 96·08 | 981·7 | 942·3 | 95·98 |
| 0·9 | 9·09 | 90·95 | 0 | 179·947 | 81·85 | 4545 | 3722 | 81·70 |
| 0·4 | 16·67 | 166·6 | 0 | 179·952 | 66·6 | 8335 | 5551 | 66·61 |
| 0·1 | 33·33 | 333·3 | 0 | 179 961 | 33·3 | 16667 | 5555 | 33·33 |
| 0 | 50 | 500 | 0 | 179 971 | 0 | 25000 | 0 | 0 |

$$\frac{2\pi}{\tau}\mathrm{L}=4000.$$

| $\rho$. | A. | A′. | $\epsilon$. | $\epsilon'$. | $a'$. | P. | P′. | E. |
|---|---|---|---|---|---|---|---|---|
| ∞ | ·25 | 0 | 89°·85 | 179°·9857 | 100 | ·055 | 0 | 0 |
| 99·9 | ·269 | ·9965 | 68·05 | 179·8601 | 99·55 | 51·52 | 49·49 | 96·05 |
| 49·9 | ·320 | 1·996 | 51·20 | 179·8573 | 99·6 | 100·1 | 99·4 | 99·28 |
| 9·9 | 1·02 | 9·902 | 13·89 | 179·8584 | 98·03 | 495·2 | 484·2 | 97·77 |
| 4·9 | 1·976 | 19·61 | 7·03 | 179·8597 | 96·10 | 980·4 | 942·3 | 96·07 |
| 0·9 | 9·09 | 90·88 | 1·25 | 179·8967 | 81·8 | 4544 | 3717 | 81·79 |
| 0·4 | 16·66 | 166·7 | 0 69 | 180 | 66·67 | 8332 | 5554 | 66·51 |
| 0·1 | 33·33 | 333 | 0·19 | 179·9047 | 33·3 | 16667 | 5555 | 27·02 |
| 0 | 50 | 500 | 0·17 | 179·8713 | 0 | 25000 | 0 | 0 |

$$\frac{2\pi}{\tau}\mathrm{L}=2000.$$

| $\rho$. | A. | A′. | $\epsilon$. | $\epsilon'$. | $a'$. | P. | P′. | E. |
|---|---|---|---|---|---|---|---|---|
| ∞ | ·5 | 0 | 89°·72 | 179°·713 | 100 | 1·275 | 0 | 0 |
| 99 9 | ·5094 | ·999 | 78·30 | 179·714 | 99·8 | 52·09 | 49·94 | 95·88 |
| 49·9 | ·5373 | 1·995 | 67·90 | 179·714 | 99·55 | 101 | 99·33 | 98·33 |
| 9·9 | 1·107 | 9·896 | 26·27 | 179·716 | 97·97 | 496 | 485 | 97·68 |
| 4·9 | 2·019 | 19·59 | 13·75 | 179·719 | 96·09 | 980 | 940 | 95·90 |
| 0·9 | 9·108 | 90 96 | 2·60 | 179·723 | 81·86 | 4550 | 3724 | 81·91 |
| 0·4 | 16·674 | 166·67 | 0·20 | 179·761 | 66·66 | 8336 | 5554 | 66·64 |
| 0 | 49·997 | 499·97 | 0·15 | 179·857 | 0 | 24998 | 0 | 0 |

$\frac{2\pi}{\tau}$ L=1500.

| ρ. | A. | A′. | ε. | ε′. | α′. | P. | P′. | E. |
|---|---|---|---|---|---|---|---|---|
| ∞ | ·667 | 0 | 89°·6 | 179°·962 | 100 | 2·328 | 0 | 0 |
| 99·9 | ·673 | ·999 | 81·08 | 179·817 | 99 82 | 52·16 | 49·85 | 95·6 |
| 49·9 | ·695 | 1·995 | 72·92 | 179·782 | 99·56 | 102·1 | 99·33 | 97·5 |
| 9·9 | 1·199 | 9·97 | 33·30 | 179·623 | 98·70 | 501 | 492 | 98·2 |
| 4·9 | 2·066 | 19·60 | 18·06 | 179·617 | 96·85 | 982 | 961 | 97·8 |
| 0·9 | 9·105 | 90·88 | 3·45 | 179·538 | 81·79 | 4553 | 4130 | 90·9 |
| 0·4 | 16·67 | 166·5 | 1·55 | 179·667 | 66·6 | 8333 | 5554 | 66·5 |
| 0·1 | 33·34 | 333·4 | 0·55 | 179·779 | 33·34 | 16660 | 5555 | 33·3 |
| 0 | 50 | 500 | 0·183 | 179·809 | 0 | 25000 | 0 | 0 |

$\frac{2\pi}{\tau}$ L=1000.

| ρ. | A. | A′. | ε. | ε′. | α′. | P. | P′. | E. |
|---|---|---|---|---|---|---|---|---|
| ∞ | ·999 | 0 | 89°·427 | 179°·427 | 100 | 4·997 | 0 | 0 |
| 100 | 1·004 | ·998 | 83·725 | 179·427 | 99·82 | 54·93 | 49·82 | 90·7 |
| 50 | 1·018 | 1·993 | 78·133 | 179·428 | 99·65 | 104·7 | 99·33 | 94·9 |
| 10 | 1·393 | 9·806 | 44·717 | 179·433 | 98·06 | 495 | 481 | 97·1 |
| 5 | 2·159 | 19·23 | 26·48 | 179·438 | 96·15 | 966 | 925 | 95·7 |
| 4 | 2·57 | 23·81 | 21·73 | 179·441 | 95·24 | 1195 | 1133 | 94·85 |
| 3 | 3·27 | 31·25 | 16·67 | 179·445 | 93·75 | 1567 | 1464 | 93·47 |
| 2 | 4 64 | 45·44 | 11·31 | 179·453 | 90·88 | 2277 | 2065 | 90·73 |
| 1 | 8·383 | 83·32 | 5·75 | 179·475 | 83·32 | 4171 | 3470 | 83·25 |
| 0·5 | 14·3 | 142·8 | 2·94 | 179·509 | 71·39 | 7141 | 5097 | 71·39 |
| 0·1 | 33·34 | 333·3 | 0·77 | 179 618 | 33·33 | 16668 | 5556 | 33·33 |
| 0 | 50 | 500 | 0·29 | 179·714 | 0 | 25000 | 0 | 0 |

$\frac{2\pi}{\tau}$ L=500.

| ρ. | A. | A′. | ε. | ε′. | α′. | P. | P′. | E. |
|---|---|---|---|---|---|---|---|---|
| ∞ | 1·999 | 0 | 88°·85 | 179°·862 | 100 | 20·07 | 0 | 0 |
| 100 | 2·001 | ·998 | 85·98 | 178·887 | 99·79 | 70·06 | 49·8 | 71·1 |
| 50 | 2·006 | 1·992 | 83·57 | 178·833 | 99·58 | 111·9 | 99·2 | 88·65 |
| 10 | 2·234 | 9·914 | 62·53 | 178·845 | 99·14 | 515 | 491 | 95·4 |
| 5 | 2·519 | 17·66 | 49·1 | 178·877 | 95·30 | 825 | 780 | 96·74 |
| 1 | 8·531 | 83·33 | 11·35 | 178·95 | 83·33 | 4182 | 3471 | 83 |
| 0·5 | 14·39 | 139·2 | 5·83 | 178·99 | 69·6 | 6844 | 4679 | 68·3 |
| 0·1 | 33·37 | 333·4 | 1·52 | 179·24 | 33·34 | 16679 | 5557 | 33·45 |
| 0 | 50·08 | 500 | 0·58 | 179·43 | 0 | 25041 | 0 | 0 |

$$\frac{2\pi}{\tau}L=250.$$

| ρ. | A. | A′. | ε. | ε′. | a′. | P. | P′. | E. |
|---|---|---|---|---|---|---|---|---|
| ∞ | 3·996 | 0 | 87°·7 | 179°·77 | 100 | 20·14 | 0 | 0 |
| 99·9 | 3·994 | ·999 | 86·25 | 177·712 | 99·82 | 130·7 | 49·85 | 38·1 |
| 49·9 | 3·996 | 1·997 | 84·85 | 177·713 | 99·65 | 179·2 | 99·5 | 55·5 |
| 9·9 | 4·078 | 9·893 | 73·7 | 177·731 | 95·83 | 572 | 484 | 84·6 |
| 4·9 | 4·381 | 19·6 | 61·18 | 177·767 | 95·04 | 1055 | 941 | 89·2 |
| 2·9 | 5·035 | 32·23 | 47·98 | 177·782 | 93·48 | 1686 | 1507 | 89·4 |
| 0·9 | 9·346 | 86·78 | 20·7 | 177·916 | 78·1 | 4371 | 3388 | 77·5 |
| 0·4 | 16·90 | 165·6 | 11·23 | 178·008 | 66·24 | 8287 | 5486 | 66·2 |
| 0·1 | 33·44 | 324·1 | 9·18 | 178·367 | 32·41 | 16500 | 5250 | 31·8 |
| 0 | 50 | 500 | 1·2 | 178·898 | 0 | 25000 | 0 | 0 |

$$\frac{2\pi}{\tau}L=50.$$

| ρ. | A. | A′. | ε. | ε′. | a′. | P. | P′. | E. |
|---|---|---|---|---|---|---|---|---|
| ∞ | 19·6 | 0 | 79°·69 | 168°·69 | 100 | 1923 | 0 | 0 |
| 100 | 19·6 | ·98 | 78·42 | 168·70 | 97·85 | 1967 | 47·9 | 2·43 |
| 50 | 19·57 | 1·95 | 78·14 | 168·72 | 97·65 | 2011 | 95·4 | 4·74 |
| 10 | 19·45 | 9·62 | 75·97 | 168·80 | 96·15 | 2359 | 452 | 19·6 |
| 5 | 19·36 | 18·86 | 73·3 | 168·90 | 94·3 | 2778 | 890 | 32·03 |
| 4 | 19·31 | 23·36 | 72·81 | 168·94 | 93·44 | 2982 | 1092 | 36·6 |
| 3 | 19·27 | 30·68 | 69·87 | 169·03 | 92·04 | 3315 | 1412 | 42·6 |
| 2 | 19·28 | 44·65 | 65·8 | 169·19 | 89·30 | 3951 | 1994 | 50·5 |
| 1 | 19·81 | 82 | 55·17 | 169·61 | 82 | 5656 | 3359 | 59·38 |
| 0·5 | 21·99 | 141 | 40·47 | 170·27 | 70·4 | 8364 | 4957 | 59·25 |
| 0·1 | 35·58 | 330 | 14·21 | 172·41 | 33·0 | 17250 | 5457 | 31·64 |
| 0 | 50·74 | 498 | 5·6 | 174·29 | 0 | 25240 | 0 | 0 |

$$\frac{2\pi}{\tau}L=10.$$

| ρ. | A. | A′. | ε. | ε′. | a. | P. | P′. | E. |
|---|---|---|---|---|---|---|---|---|
| ∞ | 70·7 | 0 | 45° | 174°·269 | | 25000 | 0 | 0 |
| 99·9 | 70·6 | ·706 | 45 | 135·033 | 70·53 | 24980 | 24·95 | 0·1 |
| 49·9 | 70·5 | 1·41 | 45 | 135·05 | 70·36 | 24970 | 49·61 | 0·2 |
| 9·9 | 70·4 | 7·04 | 44·75 | 135·3 | 69·7 | 24980 | 297·5 | 1·276 |
| 4·9 | 70·0 | 14·0 | 44·4 | 135·5 | 68·6 | 25010 | 480 | 1·92 |
| 0·9 | 67·6 | 67·6 | 42·0 | 135·7 | 60·8 | 25130 | 1634 | 6·5 |
| 0·4 | 65·0 | 130 | 38·7 | 139·98 | 52·0 | 25360 | 3380 | 13·3 |
| 0·1 | 62·1 | 282 | 29·7 | 146·3 | 28·2 | 26980 | 3976 | 14·74 |
| 0 | 50 | 357 | 14·01 | 143·8 | 0 | 24250 | 0 | 0 |

An examination of these Tables will suggest many experiments, and also will suggest simple general rules for all periodic currents.

It is to be observed that in the tables, 1000, A, A′, and $a'$ divided by $\sqrt{2}$ give the readings on dynamometers of the primary voltage and primary current and secondary current and secondary voltage respectively, or their *effective* values.

Note that the lag $\epsilon$ varies considerably from no load to small ordinary loads, but the lag $\epsilon'$ is always nearly 180°. And hence the secondary voltage and currents are almost identically the same kind of function of time (only of opposite sign) as the primary voltage. To make this certain:—the general term in V, whatever V may be, may be written

$$a_i \sin ikt + b_i \cos ikt,$$

and the general term in $v'$ is

$$\frac{a_i}{s} \sin (ikt - \pi) + \frac{b_i}{s} \cos (ikt - \pi),$$

where $s$ is the same for all terms. We are here considering only the $i$th term.

Evidently this is

$$-\frac{a_i}{s} \sin ikt - \frac{b_i}{s} \cos ikt,$$

and it is to be noted also that the lag is more nearly 180° as $i$ is greater. I have here used $k$ to represent $\frac{2\pi}{\tau}$, the reciprocal of $\tau$ being the frequency. This is true also of the primary current when R′ is not great.

With these tables before us, certain generalizations may be made in the formulæ given above. I assume, as in the tables, no magnetic leakage, so that $M^2 = LL'$.

1st. When R′ is very great. Let $\frac{2\pi}{\tau}$ be written as $k$.

$$\left.\begin{aligned}
& I' = R', \quad r = R + k^2 \frac{M^2}{R'}, \\
& l = L, \\
& i = \sqrt{\left(R + \frac{k^2 M^2}{R'}\right)^2 + k^2 L^2}, \\
& A = \frac{a}{\sqrt{\left(R + \frac{k^2 M^2}{R'}\right)^2 + k^2 L^2}}, \quad \tan \epsilon = k \frac{LR'}{R'R \times k^2 M^2}, \\
& A' = k \frac{MA}{R'} = \frac{Ma}{R'L} \text{ nearly}, \\
& \tan \epsilon' = -k \cdot \frac{RR'}{L'R + R'L + \frac{k^2 M^2 L'}{R'}}, \text{ or } \epsilon' \text{ nearly } 180^\circ.
\end{aligned}\right\} \quad (4)$$

Of course, if $R'=\infty$, there is further simplification:

$$r=R,\ i=\sqrt{R^2+k^2L^2},\ A=\frac{a}{\sqrt{R^2+k^2L^2}}=\frac{a}{kL}\text{ nearly;}$$

$$\tan\epsilon=\frac{kL}{R},\quad \therefore\ \epsilon=90^\circ\text{ very nearly,}$$

$$A'=\frac{Ma}{R'L}=0,\quad \tan\epsilon'=-\frac{1}{k}\cdot\frac{R}{L}=-\frac{R}{Lk},$$

or $\epsilon'$ is nearly 180°.

Hence, if the periodic function is not merely a simple sine function; if R′ is very great, the primary current is a very different function of the time from the primary voltage, whereas the secondary current and secondary voltage are the same kinds of function of the time as the primary voltage.

2nd. When R′ very small:

$$I'=kL',\ l=0,\ r=R+\frac{M^2R'}{L'^2},\quad i=R+\frac{L}{L'}R',$$

and

$$A=\frac{a}{R+\frac{L}{L'}R'},\quad \tan\epsilon=0,$$

$$A'=\frac{aM}{L'R+LR'},\quad \tan\epsilon'=0, \qquad . \ . \ . \ (5)$$

Hence, in general, whatever be the law of current, both the primary and secondary currents, and therefore the secondary voltage, are the same functions of the time as the primary voltage, and to calculate their dynamometer readings is very simple.

3rd. As a help to the memory, it may be remarked that the primary current is just what it would be if there were no secondary circuit and if the resistance and self-induction were $r$ and $l$ respectively.

When R′ is great, we have practically $r=R$ and $l=L$.

When R′ is small,

$$r=R+\frac{M^2R'}{L'^2}\quad\text{and}\quad l=\frac{LL'-M^2}{L'}; \quad . \ . \ . \ . \ (6)$$

and as these do not involve the periodicity, we may say generally, whatever be the law of change of primary voltage in an ordinary transformer,

$$V=r\,C+l\dot{C},$$

using the above values for almost no loads and for great loads.

4th. If we imagine no magnetic leakage, $l=0$, and we have

$$C=\frac{V}{R+\frac{M^2}{L'^2}R'} \text{ or } =\frac{V}{R+\frac{P}{S}R'} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

for all kinds of current variation unless R′ is large.

5th. We see that, except when R′ is great, $A'=\frac{M}{L'}A$, and when R′ is infinity, $A'=0$, and $A=\frac{a}{KL}$; so that, except for a very small range of small loads, we might expect the law

$$A = \frac{a}{kL}+\frac{L'A'}{M}$$

to be nearly true.

This law is very easily tested by means of the Tables.

Having seen, then, from the Tables the small inaccuracies of such a law, we might expect that generally

$$\textit{Effective } C=m+n\times\text{effective } C_1, \quad . \quad . \quad . \quad . \quad (8)$$

where $n$ is constant and nearly equal to $\frac{S}{P}$, however the frequency &c. may alter, but $m$ is a small constant which alters if the frequency alters. We should expect this law to be true when the secondary circuit is open, and also when the secondary current changes from small loads to the very greatest loads, and when it is short-circuited; but for very small loads it is somewhat untrue. For all practical purposes it is true.

6th. As, unless when R′ is very great,

$$A'=\frac{aM}{R'L+RL'}=\frac{a\frac{S}{P}}{R'+R\frac{S^2}{P^2}}.$$

If $R'=R_0+\rho$, then

$$a'=a\frac{S}{P}\cdot\frac{\rho}{R_0+\rho+R\frac{S^2}{P^2}}. \quad . \quad . \quad . \quad . \quad (9)$$

This is Dr. Hopkinson's rule for the drop in the secondary volts as the load increases, when the currents are sine functions of the time. An examination of my Tables will show that this law is wonderfully true for all loads. From what I have already said, it is obviously the rule whatever function of the time the current may be, if it is rapidly alternating. That is,

$$\text{Effective Secondary Voltage} = \text{Effective Primary Voltage} \times \frac{\frac{S}{P}\rho}{R_0+\rho+R\frac{S^2}{P^2}}. \quad (10)$$

In practice it is found that the secondary voltage "drops" more rapidly than this law would indicate—a result which might have been expected, as we know that there always is leakage.

*May 26th.*—When the above paper was being prepared in February last, not for publication, but to be handed about among my students to connect the results of their calculations, I asked some of them to repeat their calculations on the assumption that there may be magnetic leakage. That is, taking the same values of all their coil quantities as before, assuming that there may be a little magnetic leakage. I did not expect very great differences from the old numbers, but to my astonishment there were very great differences.

Thus, taking $R=10$ ohms, $R'=\cdot 1+\rho$, $a=1000$ volts, $L=15$ secohms, $L'=\cdot 15$ secohm, two students have worked out what is the effect of magnetic leakage, 1st, when the frequency is 106 per second; 2nd, when the frequency is 10·6 per second. In the first case, two tables were calculated; in the second case, four.

No general mathematical expressions will show so forcibly the necessity for caution in neglecting leakage.

Frequency 106 per second. M=1·50.
No magnetic leakage. (Mr. Eliott.)

| $\rho$. | A. | A′. | $\epsilon$. | $\epsilon'$. | $a'$. | P. | P′. | E. |
|---|---|---|---|---|---|---|---|---|
| ∞ | ·1000 | 0 | 89°·9 | 179·9394 | 100 | ·0850 | 0 | 0 |
| 99·9 | ·1412 | ·9991 | 44·97 | 179·9395 | 99·82 | 50·06 | 49·85 | 99·59 |
| 49·9 | ·2232 | 1·9965 | 26·5 | 179·9395 | 99·61 | 99·95 | 99·45 | 99·49 |
| 9·9 | ·9956 | 9·913 | 0·05 | 179·9401 | 99·14 | 497·8 | 486·4 | 97·70 |
| 4·9 | 1·9630 | 19·61 | 0 | 179·9409 | 96·08 | 981·7 | 942·3 | 95·98 |
| 0·9 | 9·09 | 90·95 | 0 | 179·9468 | 81·85 | 4545 | 3722 | 81·70 |
| 0·4 | 16·67 | 166·6 | 0 | 179·9519 | 66·64 | 8335 | 5551 | 66·61 |
| 0·1 | 33·33 | 333·3 | 0 | 179·9610 | 33·33 | 16667 | 5554 | 33·33 |
| 0 | 50 | 500 | 0 | 179·9708 | 0 | 25000 | 0 | 0 |

Frequency 106. M = 1·48.
$1\frac{1}{3}$ per cent. magnetic leakage. (Mr. Eliott.)

| ρ. | A. | A′. | ε. | ε′. | a′. | P. | P′. | E. |
|---|---|---|---|---|---|---|---|---|
| ∞ | ·1000 | 0 | 89°·9 | 179°·939 | 98·67 | ·085 | 0 | 0 |
| 99·9 | ·1412 | ·9808 | 46·4 | 181·5 | 98·49 | 48·61 | 48·54 | 99·70 |
| 49·9 | ·2228 | 1·968 | 29·5 | 182·98 | 95·98 | 96·96 | 96·65 | 99·68 |
| 9·9 | ·9627 | 9·458 | 20·4 | 194·8 | 93·63 | 451·2 | 442·8 | 98·13 |
| 4·9 | 1·741 | 17·15 | 30·4 | 204·85 | 84·05 | 750·8 | 720·7 | 96·01 |
| 0·9 | 3·370 | 33·28 | 68·2 | 247·6 | 29·95 | 628·4 | 498·2 | 79·27 |
| 0·4 | 3·658 | 36·11 | 77·6 | 257·33 | 14·44 | 392·8 | 260·7 | 66·37 |
| 0·1 | 3·726 | 36·78 | 83·7 | 263·55 | 3·678 | 204·4 | 53·75 | 26·29 |
| 0 | 3·739 | 36·91 | 85·8 | 265·7 | 0 | 136·9 | 0 | 0 |

Frequency 10·6 per second. M = 1·50.
No magnetic leakage. (Mr. Howitt.)

| ρ | A. | A′. | ε. | ε′. | a′. | P. | P′. | E. |
|---|---|---|---|---|---|---|---|---|
| ∞ | ·9995 | 0 | 89°·42707 | 179°·42707 | 100 | 4·997 | 0 | 0 |
| 100 | 1·004 | ·9982 | 83·725 | 179·42764 | 99·82 | 54·93 | 49·815 | 90·7 |
| 50 | 1·018 | 1·993 | 78·133 | 179·428203 | 99·65 | 104·7 | 99·33 | 94·9 |
| 10 | 1·393 | 9·806 | 44·7166 | 179·43268 | 98·06 | 494·9 | 480·7 | 97·1 |
| 5 | 2·159 | 19·23 | 26·4833 | 179·43808 | 96·15 | 966·2 | 924·5 | 95·68 |
| 4 | 2·57 | 23·81 | 21·73 | 179·441 | 95·24 | 1195 | 1133 | 94·85 |
| 3 | 3·27 | 31·25 | 16·67 | 179·445 | 93·75 | 1567 | 1464 | 93·47 |
| 2 | 4·64 | 45·44 | 11·31 | 179·453 | 90·88 | 2277 | 2065 | 90·73 |
| 1 | 8·383 | 83·32 | 5·75 | 179·4748 | 83·32 | 4171 | 3470 | 83·25 |
| 0·5 | 14·3 | 142·78 | 2·9416 | 179·5089 | 71·39 | 7141 | 5097 | 71·39 |
| 0·1 | 33·34 | 333·33 | ·766 | 179·61803 | 33·33 | 16668 | 5556 | 33·33 |
| 0 | 50 | 500 | ·28646 | 179·71352 | 0 | 25000 | 0 | 0 |

Frequency 10·6 per second. M = 1·485.
Magnetic leakage 1 per cent. (Mr. Howitt.)

| ρ. | A. | A′. | ε. | ε′ | a′. | P. | P′. | E. |
|---|---|---|---|---|---|---|---|---|
| ∞ | ·9995 | 0 | 89°·427 | 179°·427 | 99 | 4·977 | 0 | 0 |
| 100 | 1·004 | ·988 | 83·836 | 179·54 | 98·8 | 53·85 | 48·8 | 90·55 |
| 50 | 1·017 | 1·972 | 78·37 | 179·65 | 98·6 | 102·6 | 97·25 | 94·75 |
| 10 | 1·393 | 9·705 | 45·83 | 180·56 | 97·05 | 485·35 | 470·95 | 97·03 |
| 5 | 2·157 | 19·02 | 28·65 | 181·63 | 95·1 | 946 | 905 | 95·63 |
| 4 | 2·571 | 23·56 | 24·45 | 182·156 | 94·2 | 1170·5 | 1109·5 | 94·79 |
| 3 | 3·268 | 30·89 | 20·23 | 183·005 | 92·7 | 1533 | 1432 | 93·4 |
| 2 | 4·629 | 44·85 | 16·48 | 184·63 | 89·7 | 2219·5 | 2012 | 90·63 |
| 1 | 8·281 | 81·51 | 15·18 | 188·9 | 81·5 | 3997 | 3321·5 | 83·1 |
| 0·5 | 13·79 | 136·3 | 18·85 | 195·41 | 68·15 | 6525 | 4646 | 71·2 |
| 0·1 | 27·87 | 275·8 | 34·44 | 213·29 | 27·58 | 11490 | 3803·5 | 33·11 |
| 0 | 35·53 | 351·8 | 45·28 | 224·71 | 0 | 12500 | 0 | 0 |

Frequency 10·6 per second. M = 1·425.
Magnetic leakage 5 per cent. (Mr. Howitt.)

| $\rho$. | A. | A′. | $\epsilon$. | $\epsilon'$. | $a'$. | P. | P′. | E. |
|---|---|---|---|---|---|---|---|---|
| ∞ | ·9995 | 0 | 89°·427 | 179°·427 | 95 | 4·997 | 0 | 0 |
| 100 | 1·004 | ·948 | 84·268 | 179·985 | 94·8 | 50 | 44·94 | 89·8 |
| 50 | 1·017 | 1·892 | 79·253 | 180·54 | 94·6 | 94·9 | 89·55 | 94·35 |
| 10 | 1·389 | 9·279 | 50·18 | 184·9 | 92·8 | 444·5 | 430·6 | 96·86 |
| 5 | 2·101 | 17·99 | 37·097 | 190·07 | 89·9 | 847·5 | 808·5 | 95·42 |
| 4 | 2·512 | 22·08 | 34·83 | 192·54 | 88·3 | 1031 | 975 | 94·57 |
| 3 | 3·138 | 28·47 | 33·66 | 196·436 | 85·4 | 1305·5 | 1216 | 93·12 |
| 2 | 4·261 | 39·62 | 35·3 | 203·44 | 77·2 | 1738·5 | 1569·5 | 90·26 |
| 1 | 6·536 | 61·72 | 45·05 | 218·776 | 61·7 | 2308·5 | 1904·5 | 82·49 |
| 0·5 | 8·379 | 79·45 | 57·59 | 234·155 | 39·7 | 2245·5 | 1578·5 | 70·31 |
| 0·1 | 9·822 | 93·42 | 74·01 | 252·865 | 9·3 | 1353 | 436·3 | 32·17 |
| 0 | 10·06 | 95·54 | 78·97 | 258·4 | 0 | 962 | 0 | 0 |

Frequency 10·6 per second. M = 1·35.
Magnetic leakage 10 per cent. (Mr. Howitt.)

| $\rho$. | A. | A′. | $\epsilon$. | $\epsilon'$. | $a'$. | P. | P′. | E. |
|---|---|---|---|---|---|---|---|---|
| ∞ | ·9995 | 0 | 89°427 | 179°·427 | 90 | 4·997 | 0 | 0 |
| 100 | 1·003 | ·8981 | 84·808 | 180·514 | 89·81 | 45·41 | 40·33 | 88·82 |
| 50 | 1·017 | 1·793 | 80·308 | 181·596 | 89·65 | 85·6 | 80·3 | 93·77 |
| 10 | 1·372 | 8·690 | 55·287 | 190·00 | 86·90 | 390·7 | 378 | 96·63 |
| 5 | 2·033 | 16·31 | 46·59 | 199·57 | 81·55 | 698·5 | 664·5 | 95·14 |
| 4 | 2·353 | 19·51 | 46·17 | 203·87 | 78·36 | 814·5 | 768 | 94·25 |
| 3 | 2·825 | 24·29 | 47·51 | 210·29 | 72·87 | 954 | 884·5 | 92·72 |
| 2 | 3·532 | 31·11 | 52·36 | 220·5 | 62·22 | 1078·5 | 968 | 89·73 |
| 1 | 4·494 | 40·22 | 63·85 | 237·57 | 40·22 | 990·5 | 808·5 | 81·64 |
| 0·5 | 4·962 | 44·57 | 73·01 | 249·71 | 22·28 | 719 | 496·6 | 69·07 |
| 0·1 | 5·205 | 46·84 | 82·02 | 261·02 | 4·68 | 354·8 | 109·7 | 30·91 |
| 0 | 5·237 | 47·13 | 84·56 | 263·99 | 0 | 248·2 | 0 | 0 |

To show the effect of even a magnetic leakage so small as *one-tenth of one per cent.* Let $y = {\cdot}001$, where

$$M = \sqrt{LL'}(1-y).$$

Taking, as before, $L=15$, $L'={\cdot}15$, $LL'-M^2=4{\cdot}5\,y$,

$$I'=KL',\quad r=R+\frac{M^2R'}{L'^2}=R+100\,R' \text{ nearly},$$

$$l=L-\frac{M^2L}{L'^2}=30y={\cdot}03.$$

Take $V=1000\sin kt$, where $k=1000$, and therefore the frequency is 159, we have the following results :—

| R′. | C if there is no magnetic leakage. | C if there is a leakage of one-tenth of one per cent. |
|---|---|---|
| 1 | 9·0909 sin $kt$. | 8·79 sin $(kt-15°)$. |
| ·5 | 16·6667 sin $kt$. | 14·907 sin $(kt-27°)$. |
| ·2 | 33·333 sin $kt$. | 23·57 sin $(kt-45°)$. |
| ·1 | 50 sin $kt$. | 27·74 sin $(kt-56°)$. |

So that even when the leakage is so insignificant as this, its effect is very marked when there is a heavy load on the transformer. Of course, the secondary current and secondary voltage would exhibit the same kind of discrepance.

I do not know how much magnetic leakage there may be in an ordinary transformer; and indeed my present purpose is only to show students that as there is always some leakage, it ought to be taken into consideration.

I think it impossible that the leakage should be less than one one-thousandth of the whole induction. That is, that ·001 of the whole induction produced by the primary escapes the secondary coil, and ·001 of the whole induction produced by the secondary escapes the primary. The result is practically the same as if I said that ·0005 of the primary escapes the secondary, and ·0015 of the secondary escapes the primary. Now, taking a current of frequency 159 with the above-mentioned transformer, let the primary voltage be

$$V=a_1 \sin(1000\,t+\alpha_1)+a_2\sin(2000\,t+\alpha_2)+a_3\sin(3000t+\alpha_3)$$
$$+a_4\sin(4000\,t+\alpha_4)+ \;\&c.$$

If there were no leakage, the primary current would be with great exactness, if $R'=1$ ohm,

$$C=\frac{a_1}{110}\sin(1000\,t+\alpha_1)+\frac{a_2}{110}\sin(2000\,t+\alpha_2)$$
$$+\frac{a_3}{110}\sin(3000\,t+\alpha_3)+\frac{a_4}{110}\sin(4000\,t+\alpha_4)+\;\&c.$$

Whereas with only one-tenth of one per cent. of leakage, the primary current is

$$C=\frac{a_1}{112{\cdot}7}\sin(1000\,t+\alpha_1-15°)+\frac{a_2}{125{\cdot}3}\sin(2000\,t+\alpha_2-28°{\cdot}6)$$
$$+\frac{a_3}{142}\sin(3000\,t+\alpha_3-39°)+\frac{a_4}{163}\sin(4000\,t+\alpha_4-48°)+\;\&c.$$

The higher harmonics diminishing more and more rapidly, and having greater and greater lag.

In the light of our general expressions (2), it will be seen in the third statement and (6) that $\frac{LL'-M^2}{L'}$ represents a resultant coefficient of self-induction in the primary coil, unless for exceedingly small loads, and the resistance is $R+\frac{M^2R'}{L'^2}$. Hence, unless the leakage is very small indeed, or very great, it is obvious that the primary current cannot be a periodic function of the same kind as the primary voltage. The Tables and expressions show that this is also the case with the secondary current. They also show that the secondary and primary currents are nearly the same functions of the time, although opposite in sign, and that they are nearly proportional to one another.

All necessary general rules suggested by the Tables are easily worked out from the formulæ. But the suggestions are such that it is evidently worth while to treat the subject more generally, and those who are interested in symbolic methods, as employed in linear differential equation work, may prefer to see the equations written as

$$V=(R+L\theta)C+M\theta C', \quad . \quad . \quad . \quad . \quad (11)$$

$$0=M\theta C+(R'+L'\theta)C', \quad . \quad . \quad . \quad . \quad (12)$$

rather than in the usual way. Hence

$$C'=\frac{-M\theta}{RR'+(RL'+R'L)\theta+(LL'-M^2)\theta^2}V, \quad . \quad (13)$$

$$C=\frac{(R'+L'\theta)}{RR'+(RL'+R'L)\theta+(LL'-M^2)\theta^2}V, \quad . \quad (14)$$

where $\theta$ stands for $\frac{d}{dt}$, and $\theta^2$ for $\frac{d^2}{dt^2}$.

These values are true for all kinds of currents, and any two circuits, whether there is iron present or not. We know that on a transformer L and L′ are practically proportional to $P^2$ and $S^2$, and M is nearly $=\sqrt{LL'}$.

From (3) and (4) at every instant,

$$-C=\frac{R'+L'\theta}{M\theta}C'=\left(\frac{R'}{M\theta}+\frac{L'}{M}\right)C'. \quad . \quad . \quad . \quad (15)$$

This result is of course derivable at once from equation (2). If R′ is small,

$$-C=\frac{L'}{M}C';$$

and even when R′ is not small, for frequencies and sizes of iron usual in transformers, the term $\frac{R'}{M\theta}$ is insignificant. In fact, the term $\frac{R'C'}{M\theta}$ even when $R'=\infty$ is merely the small value of C when the secondary is open, being then $\frac{-V}{R+L\theta}$. We may say then that it is true for all practical purposes that Effective $C=m+n\times$ Effective C′, where $n$ is nearly $\frac{L'}{M}$ or $\frac{S}{P}$, $m$ being a small constant which depends upon frequency, &c.

If R′ is small, or the loads are the usual loads on transformers,

$$C' = \frac{-V}{\frac{RL'+R'L}{M} + \frac{LL'-M^2}{M}\theta}, \quad . \quad . \quad (16)$$

$$C = \frac{V}{R+R'\frac{L}{L'} + \frac{LL'-M^2}{L'}\theta}. \quad . \quad . \quad . \quad (17)$$

Generally, then, unless when there is a very small load on the transformer :—

The secondary current is the same as if the primary voltage acted in one circuit of resistance

$$\frac{RL'+R'L}{M} \quad \text{or} \quad R\frac{S}{P} + R'\frac{P}{S},$$

and self-induction $\frac{LL'-M^2}{M}$, there being no other circuit; and the primary current is as if the primary voltage acted in one circuit of resistance,

$$R+R'\frac{L}{L'} \quad \text{or} \quad R+R\,\frac{P^2}{S^2},$$

and self-induction $\frac{LL'-M^2}{L'}$. When R′ is $\infty$, of course C is as before, the current in a circuit of resistance R and self-induction L, V being the voltage and $C'=0$.

Writing $M=\sqrt{LL'}(1-y)$, $y$ being small, we may write (16) as

$$-C' = \frac{V}{R\frac{S}{P} + R'\frac{P}{S} + 2Ly\theta}.$$

If, then, we can neglect $2Ly\theta$, we have the law when there is no leakage (9). But when $2Ly\theta$ cannot be neglected, of course the effect of it is to make the secondary voltage fall off more quickly as the load increases than would be indicated by formula (9). This I find shown by such actual experimental results on transformers as I have at command.

As I have already said, when we assume no magnetic leakage, it is of no importance whether we assume that permeability is constant or not, or whether there is hysteresis or not; the results given by (2) have been shown to be practically correct for such frequencies and amounts of iron &c. as are usual in transformers.

But if there is leakage $y$, and if $\mu$ and therefore L varies from instant to instant during a cycle, it is certain that $y$ will alter in an inverse way. Making, then, the very unnecessary assumption that there is hysteresis in a transformer, it is obvious that $Ly$ will not vary very much during a cycle, and the results of calculation will not be very different from what they are on the assumption that $\mu$ is constant. I have elsewhere given reasons for assuming that there is really no hysteresis in transformer working.

Taking the sizes of iron and other dimensions of any working transformer, and using them for calculating such tables as I have given, it will be found that on calculating the true power P given to the primary coil and comparing it with W, if W is

Effective Primary Volts × Effective Primary Current,

then $\frac{P}{W}$ is nearly 1, even when the load is rather small, and may be said to be exactly 1 for ordinary and all greater loads, if there is no magnetic leakage. But if there is magnetic leakage, $\frac{P}{W}$ is, as before, much less than 1 for very small loads, getting greater with the load until for heavy loads it reaches a maximum value, and for very heavy loads diminishes again. But it is always less than 1, and is less and less at its maximum value as the current departs further from a simple sine function of the time. In Mr. Eliott's two tables,

$$\frac{P}{W} = \cos\epsilon,$$

and without magnetic leakage $\epsilon$ is 0 for nearly the whole range of load—that is:—If there is no magnetic leakage, the power given to the transformer is obtained by multiplying effective primary volts by effective primary amperes. But when there is magnetic leakage, this rule is wrong. The ratio of $\frac{P}{W}$, for example, in the second table is never greater than cos 20°. Such experimental results as are at my command confirm my view that

Effective Primary Volts × Effective Primary Amperes

give a result always greater than the true power, even for very great loads; a result to be expected if there is considerable magnetic leakage.

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## XXVI. Mr. Blakesley's *Method of Measuring Power in Transformers.* *By* Prof. J. Perry, *F.R.S.**

Mr. BLAKESLEY'S method of measuring the power given to the primary coil of a transformer becomes more important the more it is studied. Mr. Blakesley proved it to be correct if currents followed the simplest periodic law; if there was no magnetic leakage; if magnetic permeability was constant. Any person who has used Fourier's theorem knows that if Mr. Blakesley's rule is right for a sine function, it must be right for any periodic function whatsoever; as any periodic function may be expressed in sine functions, and each of these enters into the equations as if it were alone†.

* Communicated by the Physical Society: read May 22, 1891.

† This assertion was challenged in the discussion. Perhaps I ought to have explained myself more fully. At the time I happened to be working with Fourier's Series very much, and I lost sight of the fact that what was very evident to me might not be evident to others.

If $$x=\Sigma_1^\infty (a_i \sin ikt+b_i \cos ikt),$$

and $$y=\Sigma_1^\infty (\alpha_i \sin ikt+\beta_i \cos ikt),$$

where $k=\frac{2\pi}{\tau}$ and $\tau$ is the periodic time, then the average value of $xy$ between the limits 0 and $\tau$ is

$$\tfrac{1}{2}\Sigma(a_i \alpha_i+b_i \beta_i),$$

and does not involve any term such as $a_i \alpha_r$ or $b_i \beta_r$; that is, into the expression for the average value each term of the Fourier's Series enters just as if there were no other terms. Nearly all practical Electrical Engineers are in the habit of ignoring calculations which assume that a current is a sine function of the time; they say that such calculations are useless because the current never is a true sine function of the time. I have here given one of many examples which might be given in which a proposition concerning any periodic function need only be proved for one of the Fourier terms of that function. And in all cases, the result of the study of a sine function is at once applicable to any periodic function whatsoever.