IDEALS IN THE TEACHING OF MATHEMATICS.

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A suspicion that there may be wide room for improvement in respect to the teaching of Mathematics is aroused by any serious attempt to explain the dislike for this study shown equally by high school pupils, college students, and people in business and professional life. That such dislike exists among a considerable percentage of school and college trained men and women cannot be denied, nor can it be satisfactorily explained solely on the basis of predilection and home environment. Certainly these play an important part in determining the likes and dislikes of students, but who can doubt that the influence of a strong and inspiring teacher is more potent in this respect than any other factor?

There is this distinguishing characteristic of the subject matter of mathematics—it means very little or else exceedingly much to the average student; there is no middle ground; his work is either that of the parrot or else that of the logical thinker. In memorizing a poem for the English class, or a chapter for the recitation in History he may gain some real benefit even without much comprehension of the laws of poetry or of the logic of events, but merely to memorize a demonstration in Geometry is worse than useless; the language is jargon and the successive steps are meaningless without a clear comprehension of the logic involved.

This well defined difference between mathematics and other branches of the curriculum would seem to explain further why so large a number of students speak with warmth, or at least with toleration, of their work in most other subjects, while they delight in inveighing against mathematics. Believing as we do that the study of mathematics under thoroughly and highly qualified teachers, and with wise selection and clear presentation of subject-matter, should lead not only to fair logical understanding but to a reasonable degree of appreciation and interest on the part of practically all students in high school and freshman courses; and yet admitting, as we must, that this status is far from realization, no apology is needed for suggesting some of the ideals which, it seems, should be kept in view with respect to the teaching of mathematics.

In teaching more than in any other profession, save the ministry, are high standards of life and conduct demanded. The teacher more than anyone, save the parent, and frequently even more than the parent, exerts a determining influence upon the young people under his care—a deteriorating influence if his moral standards are low, an elevating and refining influence, if they are high. Likewise the intellectual standards of a teacher are reflected in the pupils, both directly and indirectly. A teacher who is continually striving to grow in mental breadth and power and especially to gain greater mastery in his particular field of work, not only becomes a better teacher thereby, but also widens and deepens his influence among his pupils. In fact if we may rescue a good word from the technical vocabulary of philosophy, the teacher should be an *idealist;* not, indeed, a believer in the unreality of matter and things, not a visionary with head in the clouds, proposing and attempting impossible and impracticable schemes; but one whose ideals are high and at the same time clearly defined and capable of approximate realization, and whose standards of life, both moral and intellectual, are pure, inspiring, and uplifting.

The teacher should also be an *optimist*, not in the sense of ignoring the bad nor of condoning the mediocre, but in the sense of believing the best, looking for the best, and finding the best in his students. Many a man owes his career in life to some teacher who believed in him while a student. Sarcastic criticism, cynical taunts, and pessimistic predictions may serve to goad a certain few to harder work for the time being; but in general, latent intellectual powers do not develop in such an atmosphere. On the other hand timidity and self-distrust and even indifference and carelessness are banished in the presence of a teacher whose words and manner indicate confidence in honest effort and the expectation that everyone will do his best.

Again the teacher must be an *enthusiast*; not one who poses in sensational attitudes, or who astonishes with freakish statements, or who gushes with lightsome sentiment; but one who, believing in his work and filled with his subject, is capable of warming up with genuine enthusiasm in the presence of his pupils. It takes fire to kindle fire and enthusiasm is contagious. Pupils catch it from teachers and spread it among their mates. They are also quite as easily affected by the soporific influence of the lifeless

monotony and humdrum routine which pervades the class whose teacher has no enthusiasm in the work. Enthusiastic interest in ones work lies near the foundation of successful attainment and the teacher who is wanting in this characteristic falls far short of the power a teacher should wield.

Finally the teacher must be a *specialist*. It is no longer so common a practice for schools to "breed in" their immediate graduates as teachers. The grammar schools now demand special preparation for all phases of their work. The high schools demand college graduates. The colleges demand men of University training. A teacher of algebra or geometry whose mathematical experience does not extend far beyond these subjects has too limited a range of knowledge and too narrow a view of related truth to be entrusted with the guidance of students in these subjects. The successful teacher will not only possess thorough knowledge of the subjects relating to his work, but will himself be an independent thinker, at least, to the extent of formulating afresh in terms of his own perception the truths already known, and thus he will be constantly incited to search for truths which, new to him, may lie close at hand, and the discovery of which constitutes the strongest mental stimulus for the teacher.

Let us then consider a teacher of mathematics whose mental and moral standards are high, whose personality elicits from the students their best endeavor, whose enthusiasm dispels the monotony of the daily routine and whose special preparation implies a certain and wide command of the work in hand: What may we assume should be accomplished by a teacher who approximates these ideals?

It will be admitted that the successful teacher must have good order in the class room, must hold the attention of the pupils to the matter in hand, must induce them to learn the lessons assigned and must secure all these results not in a perfunctory manner nor by virtue of the commanding authority vested in him, but by reaching in some way the inner impulses and controlling motives of the student. Without this deeper influence the teacher is only a lesson-hearer, a task-master. Even good order, attention, and tasks performed, are not necessarily indications that the highest end is being attained in the teaching of mathematics; for, if the true spirit is lacking, even these may be only forms of compulsion

which lead to dislike or even disgust on the part of the student. They do, however, follow as natural and certain results when once the student is aroused to a genuine and enthusiastic interest in his work. Without this inner desire to investigate a subject, and without the personal interest in following its development, which genuine teaching inspires, the student's work is lifeless and fruitless.

The most important duty, the highest privilege and the crowning success of a teacher is to inspire the students with a desire to learn, to awaken and sustain in them a genuine interest in what they are learning, and thus to stimulate them to put forth their highest endeavor because of this desire and interest.

What resources may the teacher of mathematics draw upon in accomplishing this great end? What are some of the elements which contribute to the development of such interest on the part of the student?

1. Interest in a subject is aroused by discovering its relation to previously known subjects and to possible later investigation. A student finds no pleasure in working blindly; like a sailor he is always interested in taking his "bearings." Mathematics is a great field of interrelated truth and it cannot be assumed by the teacher that the average student, if put to work *anywhere* in this field will find his own bearings before he becomes discouraged and lost. To pore over rules and exercises and formulate laws, without any idea of what it is all about; or even to draw graphs and compute tables and perform concrete experiments with levers and weights and balances, without some conception of the relation of these things to the end in view, is too great a strain on the faith of the average pupil; and without faith in the outcome there is loss of interest in the process and finally failure or disgust with the whole matter. But a preliminary study which gives the student his bearings and leads him to appreciate the relation of the work in hand to his previous mathematical study puts the student at ease with the teacher in a field already familiar and at the same time affords a new and broader view of known subjects while leading up to the new. Such a scheme is effective with every new topic undertaken, not only at the beginning, but whenever the maze of details threatens to confuse the outcome and the student is in danger of losing his bearings and becoming discouraged.

2 A second essential in arousing interest is that the student shall become convinced of the utility of the work. Let him begin to experience the pleasure of solving some practical problems by means of the cold, dry formulae and at once his interest is stimulated. Digging bait is monotonous and uninteresting work except for the pleasurable anticipation of catching fish. So deducing rules and memorizing formulae have little power of attraction to the average student until it is seen that these are components of a powerful machine for removing obtsacles in the path of scientific discovery—and, for solving the problems which arise in connection with the practical affairs of life. The teacher should constantly aim to show the useful side of mathematical study. This is comparatively easy in arithmetic, geometry and trigonometry, and yet how often even these subjects are taught in the most formal and perfunctory manner, with no sign of life in the subject and none in the pupil. With algebra, and especially college algebra, it is more difficult to hold to the concrete and practical applications, but with analytics and calculus, the field widens out and no good excuse can be given for mere formula work in these subjects. Perhaps the most important service which can be rendered by the teachers of mathematics of the present generation is to rescue the mathematical curriculum from the stigma of "impracticability" which has long rested upon it.

3. A third element which may be made to appeal strongly to the interest of the student is the beauty of many mathematical forms and processes. Symmetry in geometrical figures and in algebraic expressions, remarkable properties and relations, as in Trigonometry, analytic geometry and calculus, may be made to play an important part incidentally in sustaining interest. Formulae may be stored away in the dark like dried fruit or they may be made to satisfy a certain aesthetic taste by properly exhibiting their "display" qualities, as the merchant adorns his store window with cans of fruit so treated as to preserve the symmetrical forms, attractive colors, and beautiful curves. A recognition of the attractive features of mathematics may have little weight in this practical age as over against the utilitarian aspect, but the skillful teacher will discover many suggestions of beauty in mathematical forms and processes with which to freshen and stimulate the interest of students who might otherwise rebel

against the hard work or grow weary of the monotony involved in the cold logical processes.

4. A fourth element, perhaps the most effective, in arousing and maintaining the interest of the student is clearness of presentation and explanation on the part of the teacher. This involves not only a clear understanding by the teacher of the subject in all its bearings, but also an appreciation by him of the difficulties experienced by the average or slow student. This means not lecturing but eliciting, drawing out, awakening. It means studying the case of each student and adapting the treatment to his need. It requires patience, sympathy and devotion to the work. Contributory to clearness are (1) a careful introduction leading up to each new topic before assignment; (2) exact and clear statements of principles involved; (3) translation of every formula into words; (4) many sided discussion of every topic by teacher and student, for example, several statements of the same problem from different points of view and discussion of the relative merits of these statements; and (5) frequent tabulation of results and summing up of principles.

It seems comparatively easy to tell how to arouse interest and to make a subject clear to a class of students, to tell how to be a successful and inspiring teacher, and yet how difficult it is to accomplish! Those who can do it most satisfactorily may be least able to tell how, and those who can tell how most smoothly may be least able to do it. It is sometimes said that a teacher is born not made. At any rate to be an inspiring teacher is surely an ideal worthy of the highest aspiration and whatever may be our gifts by birth we may certainly gain much in this direction by observation, comparison and study. To this end the great activity during the past few years in the organization of associations of teachers of mathematics is full of promise for the future. These associations indicate the awakening of an interest among the teachers which is certain to produce results.

The lines of renewed activity which now seem to be receiving the most attention are (1) graphic representation in connection with all analytic work; (2) correlation of related subjects such as physics and mathematics, and the closer union of the different branches of mathematics as algebra, geometry and trigonometry; (3) the application of laboratory methods to mathema-

tics which means, in brief, more actual work by the student in class and less talking by the teacher.

These constitute the triune watchword of the present mathematical renaissance,—*graphics, correlation, laboratory.* These words do not indicate new methods but they indicate renewed activity along lines which are sure to create fresh interest in mathematical study and to transform cold, dry, theoretical processes into living and concrete contact with the practical world.

With respect to graphic methods we are coming to understand that pictures which are so effective in fixing ideas in childhood are just as useful for this purpose as we grow older. Indeed the graph may not only serve to illustrate the idea already conceived but may also be used to lead up to the discovery of ideas. The graph has not only become indispensable for tabulating and showing to the eye at a glance long series of results or changes such as commercial statistics, weather indications, etc.; but it has come to be recognized as an instrument of investigation for determining results in architecture, mechanics, engineering, etc. The chief caution necessary is that the graph is not an end in itself but a means, and should be used, therefore, not to supplant analysis but to supplement it. They must go hand in hand. The student who can devine a result by a graph but does not know the analytic basis is as far from a real mastery of the subject as the one who can reach the conclusion by a rule or formula but cannot interpret it by a picture. Both are essential to a complete comprehension. The analysis and formal solution have been emphasized too much and the graphic interpretation too little. The present tendency is likely to throw the balance the other way while what is needed is an equilibrium between the two.

With respect to correlation of subjects we are coming to realize more fully the close inter-relation between pure and applied mathematics and between the various branches of pure mathematics; and that the real interest of a student is bound to center not in the abstract principles of the subject but rather in the concrete things which can be done by the use of those principles; and hence that theory and application should go together; and further that the various subjects, such as algebra, geometry and trigonometry are only different tools for working toward the same end and should be studied as well as applied in correlation with each other.