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are continuous functions of the real variable x in the interval $|x-a| \leq r$ and whole rational functions of the complex parameter μ . A solution y which is such that the initial values of $y, y' \dots y^{(n-1)}$ for $x=a$ are independent of μ is a whole transcendental function of μ for values of x within the given range (see p. 297), a theorem which corresponds to an important result in the theory of integral equations.

The printing is good in both cases and I have not noticed any misprints.

H. BATEMAN.

MATHEMATICAL NOTES.

205. [L. 2. b.]. [A solution, not by elliptic functions is wanted of the following :

Given five lines $abcde$ in a plane, it is known that the pairs of points ab, ce ; bc, da ; cd, eb ; de, ac ; ea, bd are in a collineation. Prove that the fixed triangle of this collineation is self polar as to both the conic on (ab, bc, cd, de, ea) and the conic on (ac, ce, eb, bd, da) . (F. MORLEY).]

Solution by H. BATEMAN.

Lemma. Let a collineation referred to its fixed triangle be given by the equations

$$x' = ax, \quad y' = by, \quad z' = cz,$$

then the conic

$$S' \equiv Ax^2 + By^2 + Cz^2 + 2Fyz + 2Gzx + 2Hxy = 0$$

will correspond to

$$S \equiv Aa^2x^2 + Bb^2y^2 + Cc^2z^2 + 2Fbcyz + 2Gcaxx + 2Habyx = 0,$$

and a point $P(\xi, \eta, \zeta)$ will correspond to the point $P'(\frac{\xi}{a}, \frac{\eta}{b}, \frac{\zeta}{c})$ on the one hand and to $P'(a\xi, b\eta, c\zeta)$ on the other.

The polar of P' with regard to S is

$$Aax\xi + Bby\eta + Ccz\zeta + F(c\eta z + b\xi y) + G(c\xi z + a\xi x) + H(a\eta x + b\xi y) = 0,$$

and the polar of P with regard to S' is

$$Aax\xi + Bby\eta + Ccz\zeta + F(b\eta z + c\xi y) + G(a\xi z + c\xi x) + H(b\eta x + a\xi y) = 0.$$

These lines will be the same if

$$\frac{Aa\xi + Ha\eta + Ga\xi}{Aa\xi + Hb\eta + Gc\xi} = \frac{Hb\xi + Bb\eta + Fb\xi}{Ha\xi + Bb\eta + Fc\xi} = \frac{Gc\xi + Fc\eta + Cc\xi}{Ga\xi + Fb\eta + Cc\xi} = \lambda, \text{ (say).}$$

Eliminating ξ, η, ζ we get a cubic for λ , hence there will in general be three positions of P for which the lines are the same; if there are more than three positions of P , we must have $F=G=H=0$, and then the conics S and S' will have the fixed triangle as a self-conjugate triangle, and the two lines will be the same for all positions of P .

To apply this lemma to the theorem in question, let $ABCDE$ be the pentagon formed by the five lines $abcde$, $A'B'C'D'E'$ the pentagon formed by the alternate sides. We can show at once that $A, A'; B, B'; C, C'; D, D'; E, E'$ are in a collineation, by comparing the cross-ratios of the pencils $A\{BCDE\}$, $A'\{B'C'D'E'\}$, and the corresponding pencils for the other corners.

Let S be the conic $ABCDE$, S' the conic $A'B'C'D'E'$. Now the two points which correspond to A in the collineation are A'' on the one hand (for EC corresponds to $E'C'$ and BD to $B'D'$) and A' on the other; also the polar of A'' with regard to S is easily seen to be the line $A'G$, and by considering

the Pascal hexagon $A'D'B'E'C'A'$ we see that this line is the tangent at A' to the conic S' ; accordingly the polar of A'' with regard to S is the same line as the polar of A' with regard to S' . Taking the five points $ABCDE$

in turn, we have five pairs of points for which this condition is satisfied; hence from the above lemma we conclude that the conics S and S' must have the fixed triangle of the collineation as a self-conjugate triangle.

206. [D. 2. a.]. From Prof. Elliott's remark, I gather that he *presupposes* Dedekind's "Schnitt" theory. Of course, if this is done, and thus real numbers are defined (as 'Schnitte'), the sufficiency of the criterion of convergence can be proved, and this was done by Dedekind himself, at the end of his *Stetigkeit und irrationale Zahlen*, more shortly and, I think, more simply than in Prof. Elliott's note.

But I thought that Prof. Elliott was attempting to prove the sufficiency of the criterion *without* any previous arithmetical theory of irrationals (since he did not mention any), and I pointed out the impossibility of this.

PHILIP E. B. JOURDAIN.

207. [X. 4.]. "*Graphs*."—If in many examinations you set a question on tracing a simple conic, say, $x^2 - y^2 + x + y - 1 = 0$, you are ruled out because "General Equation of the Second Degree is not in the syllabus." The remedy is to replace the simple conic by a difficult *cubic* curve, the question then becomes a "graph" and is accepted without opposition. G. H. BRYAN.

208. [X. 4.]. *Figure of a Bicycle*.—The annexed figure or something like it was drawn by a candidate in an examination in Applied Mathematics in a question on Rigid Dynamics. The writer *believes* that that candidate passed.

G. H. BRYAN.