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To cite this article: G. Johnstone Stoney M.A. Sc.D. F.R.S. (1905) XIV. On 'An optical paradox', Philosophical Magazine Series 6, 10:55, 126-129, DOI: [10.1080/14786440509463351](https://doi.org/10.1080/14786440509463351)

To link to this article: <http://dx.doi.org/10.1080/14786440509463351>

Published online: 16 Apr 2009.

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XIV. On 'An Optical Paradox.'

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IN the June number of the Phil. Mag. vol. ix. p. 779, Lord Rayleigh describes and discusses an experiment by which he was at first puzzled. The experiment is illustrated by a diagram, which is here reproduced.

A is a point-source of light of nearly wave-length λ , L is a lens forming an image of A upon the middle of C, which last is the objective of a telescope T. The telescope is to be focussed upon lens L; and the boundary of this lens is then the object which will be more or less satisfactorily seen by an observer who looks through the eyepiece of the telescope. The excellent definition of L as seen in the telescope was unexpected. It would seem paradoxical to a person who, remembering that most of the light from A is concentrated within the minute "spurious disk" of its image formed at C, was disposed at first to infer that this little disk may be regarded as determining the effective aperture of telescope T.

Lord Rayleigh interposed a material screen cutting down the aperture of the telescope to nearly the above small size. This greatly impaired the image of L as seen through the telescope; and this satisfied him that the effective aperture of the telescope before the interposition of the screen was not so small as had been supposed. Lord Rayleigh was led by this to make a closer theoretical discussion of the phenomenon, and obtained the definite integral upon which the question turns.

Lord Rayleigh also considers the effect of enlarging A, the source of light; and expresses the opinion that if the enlarged source is a self-luminous body, there will be "no possibility of bringing into play the interferences upon which the advantage of a large aperture depends," inasmuch as with a self-luminous source of light it was supposed that there can be "no phase-relation between the lights which act at different parts of the object-glass." This view, as it appears to the present writer, is based upon an oversight. The interferences upon which resolution depends are of quite another kind as will be explained in a subsequent paragraph,

* Communicated by the Author.

and they in no degree depend upon phase relationships between the puncta of A, the enlarged source of light.

Lord Rayleigh's experiment is one familiar to the present writer, whether made with a collimator, an intermediate lens and a telescope; or with a microscope: he prefers the latter form of the experiment, as it lends itself more conveniently to an investigation of the successive phenomena with the powerful aid of an analysis of the light into its component undulations of flat wavelets. The experiment is one of those which it has been his intention to describe and explain in the next of the series of papers on "Flat-wavelet Resolution" which is being published by the Phil. Mag. The full discussion will be postponed till that paper appears; but some of its results may in anticipation be stated here.

The analysis into undulations of flat wavelets makes it easy to understand the way in which the image of L is formed in telescope T. It is formed by the mutual interference of all the undulations of flat wavelets which advance in the directions indicated by the so-called "rays" that pass into the telescope from the several optical puncta of the image of A produced at C by lens L. In the case where the source of light is a single punctum or physical point, this image consists of a distribution of light upon C which is nearly* Airy's "spurious disk" along with its diffraction appendages.

Telescope T is to be focussed on L, and then, in order that the observer may be able to distinguish between the parts of an object so small as lens L (and, therefore, to exhibit the outline of that object), the analysis indicates that, if A the source of light be a single luminous punctum, the aperture of the telescope-objective C *must take in, in addition to the spurious disk, some part of the appendage rings*; or else portions of two or more of these rings if all light from the spurious disk is excluded.

To exhibit lens L in the most satisfactory way the aperture of objective C must be large enough to include *the spurious disk and all its diffraction appendages* that have any appreciable brightness. If the objective be anything smaller than

* Conceive the lens L to be divided into two lenses L' and L'', of which L' parallelises the light from the point source A, and L'' concentrates this approximately parallel beam to a focus at C. Then the image at C would be exactly Airy's image, if the parallelised beam were an exact beam of parallel light; but this it cannot be, since it can be shown that to form an exact parallel beam would require a small amount of other light than that from A to be incident upon lens L'. Accordingly, the light which is concentrated by L'' upon C is only an approximately parallel beam of light, and the image it forms differs slightly from Airy's image of a star.

this, the definition of the image of L as seen in the telescope will fall short of what the light that passes through lens L is capable of producing; if the aperture of the objective is reduced to so small a size that it admits the light of the spurious disk only and excludes the whole of the diffraction rings, then the definition will have become so bad that even the outline of L cannot be seen. Moreover, we can by this analysis ascertain the poor character of the image which results from employing in its formation the light of the spurious disk and the innermost ring only; as also the greatly improved definition, which is obtained if the aperture of the objective admits the light of the central disk and of two rings; and the slightly further improvement which results from still more enlarging the aperture. All these particulars are easily investigated by employing the analysis into undulations of flat wavelets. A full examination of phenomena of this kind will be given in the next of the Flat-wavelet Resolution papers.

The same method of proof shows that when the source of light is enlarged there is no necessity, such as Lord Rayleigh speaks of, for any phase relation between the portions of light emanating from the several luminous puncta of the source A when enlarged. In fact, the phase relation which needs to be present is of a totally different kind. It is the phase relation which necessarily subsists between the light forming at C the *spurious disk of each separate punctum* of the enlarged A , and the light forming at C the *annular appendages associated with that particular spurious disk*. This is a phase relationship which will prevail, however unrelated may be the portions of light emanating from different puncta of A^* . Accordingly, it may safely be affirmed (and is fully confirmed by experiment) that, without any loss of definition, the source of light may be a self-luminous body. It seems necessary to point this out, because an insufficient appreciation of the way in which images are produced is still prevalent.

In Lord Rayleigh's paper, and in the present paper, the simplest case, viz. where the image of a point-source A falls

* This makes it plain, why when Lord Rayleigh employed as the source of light, light coming through a narrow slit situated at A , he found that the definition of L as seen in the telescope did not suffer, "the vertical parts of the circular edge (parallel to the slit) being as well defined as the horizontal parts." For, in fact, the light issuing from the several puncta of the slit independently formed good images of L , and the simultaneous presence of these independent images of L , made a resultant brighter image and somewhat modified in other respects, but of which the definition was equally good.

centrally upon the telescope objective C, is alone considered. In order to deal adequately with what happens when the source of light though central is of sensible size, the investigation needs to be extended to the case where the image of a point-source falls excentrically upon C. When this is done some instructive results emerge into view.

Lord Rayleigh's experiment, and others related to it, can be most conveniently made with the microscope, which, besides, enables us to experiment with numerical apertures up to 28 times the numerical aperture of an ordinary refracting telescope*.

To see that the experiment can be made with the microscope—Remove lens L of the apparatus represented by the figure in the text, and replace it by two lenses L' and L'', of which lens L' collimates the light from A, while L'' concentrates the collimated beam to a focus at C. It is obviously legitimate to make this substitution. When the experiment is made with a microscope, the source A is to be light passing through a small hole (or slit) in a stop placed under the condenser. The condenser of the microscope then takes up the duties of lens L', and at the same time the objective of the microscope discharges the functions of the combination consisting of lens L'' together with lens C. The image of A produced at C then becomes that image of the hole (or slit) which may be seen in the "Concentration Image" of the microscope—i. e., in the image which comes into view on removing the eyepiece and looking down the microscope tube. Furthermore, when the experiment is made with the microscope, any desired object can be put upon the stage of the microscope and becomes the object to be resolved. So far as the writer knows, far the best object to employ is one of the bands of Grayson's Rulings, supplemented by observations upon a single pair of lines such as may here and there be seen to project from one or other end of a band. The hole in the stop may, if desired, be made to behave as a self-luminous source of light, by focussing the light of the lamp-flame or other luminary upon the stop. This will be found in no degree to impair definition, whether the hole in the stop be large or small.

* The numerical aperture (i. e. the sine of half the angular aperture) of the objective of an achromatic telescope is seldom so large as 0.05; while the numerical apertures of dry microscope objectives and condensers range up to 0.95, and the numerical apertures of immersion objectives and immersion condensers range up to 1.4.