XVI.—*On Cases of Instability in Open Structures.* By E. Sang, LL.D.

(Read February 7, 1887.)

In the course of some remarks on the scheme proposed for the Forth Bridge, which remarks are published in the eleventh volume of the *Transactions of the Royal Scottish Society of Arts,* I was led to enunciate, among other theorems, one of a somewhat unexpected character, to the effect that any symmetric structure built on a rectangular basis, having no redundant parts, and depending on longitudinal strain alone, is necessarily unstable. This theorem was established by arguments restricted to the single matter under consideration; it is one of an extensive class, and I now propose to discuss the subject from a general abstract point of view.

The whole subject is evolved in the working out of two inverse geometrical problems and their corresponding mechanical applications. The relative positions of a number of points being prescribed, we may have to secure these by linear connections; or, the lengths of these connections being given, we may seek to discover the relative positions of the points. And we may have to compute the strengths needed to enable these connections to resist strains applied at the various points.

The relative position of *two* points is determined by the length of the straight line joining them, and the material connection can only serve as the medium for the equipoise of equal and opposite strains applied at its two ends; it can offer no resistance to stresses directed obliquely to it. The opposing pressures may be directed inwardly so as to cause compression, or outwardly so as to cause distension; the former is an example of unstable, the latter an example of stable equilibrium.

The instability in the case of compression is familiarly exemplified by an attempt to balance a load on the top of a walking-stick, or by the buckling of a long, thin rod; stability can be obtained only by the use of something aside of the straight line. In the case of distension we have to observe that no member of a structure acts upon a contiguous member except by compression; we do not pull an object toward us, we always push it; each link of a chain pushes the other link; the pulling is internal to the links themselves. Every case of stretching necessarily implies at each end compression changed first into transverse strain and then into distension. This phenomenon, which, from habit, we regard as simple, is indeed a most complex one, whose intimate nature as yet surpasses our understanding. Hence it is that, beyond the abstract arrangement of the parts as represented by straight lines, there is the problem, far more difficult, requiring much more constructive skill, of contriving the manner of the junctions.

In general, the relative positions of *three* points, as A, B, C, are determined by the lengths of the three lines AB, BC, CA, joining them two and two. A pressure applied at the point A can be resisted by the linear members AB, AC only when its direction is in the same plane with them, and they must be enabled to offer resistance by pressures applied at B and at C, which again, if they be not in the direction BA, CA, must cause a stress on BC. Hence we have here a system of six pressures in equilibrium all having their directions in one plane.

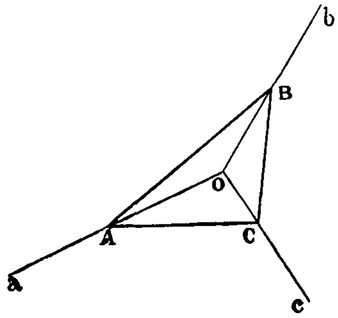

Fig. 1.

Let aA and bB be the directions of the pressures applied at A and at B, and let these be continued to meet in O; join also CO. According to the known law of equilibrium, the strains on AB and on AC are proportional to the sines of the angles OAC and BAO, which again are represented by the doubles of the expressions $\frac{\mathrm{AOC}}{\mathrm{CA.AO}}$ and $\frac{\mathrm{BOA}}{\mathrm{OA.AB}}$; wherefore, if we denote the strain on the member AB by the symbol $\overline{\mathrm{AB}}$, we have the equality $\overline{\mathrm{AB}}\cdot\frac{\mathrm{BOA}}{\mathrm{AB}}=\overline{\mathrm{CA}}\cdot\frac{\mathrm{AOC}}{\mathrm{CA}}\cdot$ And on examining the equilibrium at B, we find also $\overline{\mathrm{AB}}\cdot\frac{\mathrm{BOA}}{\mathrm{AB}}=\overline{\mathrm{CB}}\cdot\frac{\mathrm{COB}}{\mathrm{CB}}$, so that the direction of the pressure applied at C must also pass through the same point O.

Again, on comparing the pressure applied at A with the strain on AB, we find

$$\overline{a\mathrm{A}}:\overline{\mathrm{AB}}::\frac{\mathrm{ABC}}{\mathrm{CA.AB}}\ \frac{\mathrm{AOC}}{\mathrm{CA.AO}}$$

whence

$$\overline{a\mathrm{A}}\cdot\frac{\mathrm{AOC}}{\mathrm{AO}}=\overline{\mathrm{AB}}\cdot\frac{\mathrm{ABC}}{\mathrm{AB}},$$

and it follows that the six expressions

$$\overline{a\mathrm{A}}\cdot\frac{\mathrm{BOA.AOC}}{\mathrm{AO}},\quad \overline{b\mathrm{B}}\cdot\frac{\mathrm{COB.BOA}}{\mathrm{BO}},\quad \overline{c\mathrm{C}}\cdot\frac{\mathrm{AOC.COB}}{\mathrm{CO}},$$

$$\overline{\mathrm{AB}}\cdot\frac{\mathrm{ABC.BOA}}{\mathrm{AB}},\quad \overline{\mathrm{BC}}\cdot\frac{\mathrm{ABC.COB}}{\mathrm{BC}},\quad \overline{\mathrm{CA}}\cdot\frac{\mathrm{ABC.AOC}}{\mathrm{CA}},$$

are all of equal value.

On multiplying each of the first three by

$$\frac{\mathrm{AO.BO.CO}}{\mathrm{BOA.AOC.COB}}$$

we get the equalities

$$\overline{aA}\cdot\frac{BO.CO}{COB}=\overline{bB}\cdot\frac{CO.AO}{AOC}=\overline{cC}\cdot\frac{AO.BO}{BOA}$$

that is to say, the three external pressures applied at the points A, B, C balance each other just as if they had been applied directly to the point O.

When computing the internal strains caused by given external pressures, the area ABC occurs in every case as a division; if, then, the three points were in one straight line, that is, if the area ABC were zero, the internal strains would become infinitely great, unless the applied pressures were all in the same line with them. Here we have the first and very well known example of instability in construction.

If the point O be removed to a very great distance, the directions aA, bB, cC of the external pressures become parallel as in fig. 2. The intermediate pressure, in this figure bB, must be opposed to the direction of the others, its intensity being the sum of those at A and C.

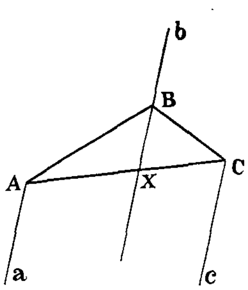

Fig. 2.

The relation of the strain on AC to the external pressure at B is then given by the formula

$$\overline{AC}=\overline{bB}\cdot\frac{AX.CX}{AC.BX};$$

so that if B were shifted along the line BX nearer to X, the strain on AC would be augmented in the inverse ratio of the new to the former BX; but the pressures $\overline{aA}$, $\overline{bB}$, $\overline{cC}$, would still remain proportional to the lines XC, CA, AX. Were B brought actually to X the strains would become infinite.

It is much to be regretted that, in lesson books on mechanics, the beginner is taught the properties of this impossible straight lever, without a hint of caution in regard to it. The strains on the arms, even that upon the fulcrum, are left out of view. In this way hazy notions are engendered; the load at A is said to balance that at C, although both be pressing in one direction.

The relative positions of four points, as A, B, C, D, fig. 3, are in general fixed by the lengths of the six lines AB, CD, AC, BD, AD, BC joining them two and two; these form the boundaries of a solid, called in Greek *tetrahedron*, which may get the English name *fournib*, shorter and quite as descriptive; the potters call it *crowfoot:* it is the simplest of flat-faced solids.

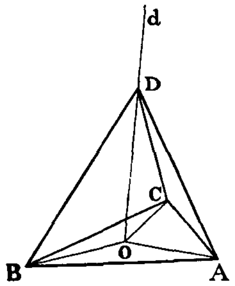

Fig. 3.

As in the triangle pressures applied at the corners can balance each other only when their directions meet in one point; so, reasoning by what is called *analogy*, we might infer that, of four pressures at the corners of a tetrahedron balancing each other, the directions must all tend to a single point. But this inference does not hold good; it may be that no two of these directions meet at all.

At each of the four points we have the equilibrium of four pressures, namely, the external pressure and the strains on the members meeting there. These strains can be computed when the direction and intensity of the applied pressure are known.

Thus let us continue the direction dD of a pressure applied at D until it meet the plane of ABC in some point O, and let AO, BO, CO be drawn. We have then the equalities

$$\frac{\overline{d\mathrm{D}}}{\mathrm{DO}\,.\,\mathrm{ABC}}=\frac{\overline{\mathrm{DA}}}{\mathrm{DA}\,.\,\mathrm{BOC}}=\frac{\overline{\mathrm{DB}}}{\mathrm{DB}\,.\,\mathrm{COA}}=\frac{\overline{\mathrm{DC}}}{\mathrm{DC}\,.\,\mathrm{AOB}}\,.$$

The points A, B, C and O remaining as they are, if D were brought nearer to O, the first of the above expressions would be augmented in inverse proportion to DO, and if D were brought actually to O, this term would become infinite, the strains $\overline{\mathrm{DA}}$, $\overline{\mathrm{DB}}$, $\overline{\mathrm{DC}}$ also infinite and the structure impossible.

When, in such an arrangement as fig. 3, the resistances at A, B, C are in a direction parallel to dD, their intensities are proportional to the opposite triangles, so that

$$\frac{\overline{d\mathrm{D}}}{\mathrm{ABC}}=\frac{\overline{a\mathrm{A}}}{\mathrm{BOC}}=\frac{\overline{b\mathrm{B}}}{\mathrm{COA}}=\frac{\overline{c\mathrm{C}}}{\mathrm{AOB}}\,,$$

and thus the distribution of the pressure among the ultimate resistances is independent of the distance DO.

In fig. 3 the point O is placed inside of the triangle ABC, and a pressure applied in the direction dDO causes compression in all the three members, DA, DB, DC. In fig. 4 O is placed outside of the line AC, and, with

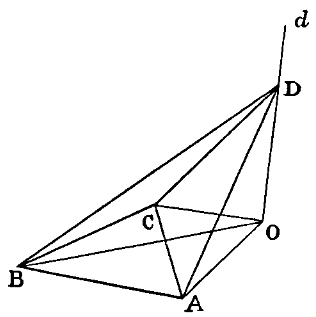

Fig. 4.

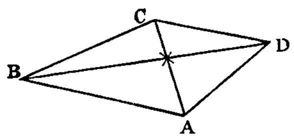

Fig. 5.

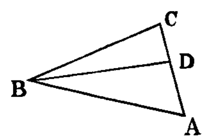

Fig. 6.

pressure in the direction dDO, the members DA, DC are compressed, while DB is distended.

If here the point D were brought down to O, the structure would take the form of a plane tetragon ABCD, with its two diagonals AC and BD, as shown in fig. 5. Such a structure can offer no resistance to pressures inclined to its plane.

If the point D were on the straight line AC, as in fig. 6, it might seem that the five distances DA, AB, BC, CD, DB would suffice to secure the straightness of ABC; but on consideration we perceive that the two triangles ABD, CBD are merely hinged upon the common line DB.

In the cases of two, three, and four points, we have seen that the length of every line joining them in pairs is needed for fixing the relative positions; this rule does not hold for higher numbers. Thus, if a fifth point E be connected with three of the four corners of the tetrahedron ABCD, its relative position is determined, provided always that E be not in the plane of the three points with which it is joined; so that nine lines suffice for five points. The line joining E with the fourth point of the tetrahedron would be redundant.

In all such structures, three of the points, as A, B, C in fig. 7, must each have four concurring lines, and the remaining two, D and E, only three; and if no four of the five points be in one straight line, the system is self-rigid.

This rigidity will subsist although the two triple points D, E be in the same plane with any one pair of the quadruple ones, as in fig. 8, which is intended to show D, B, E, C as in one plane. The scheme then takes the

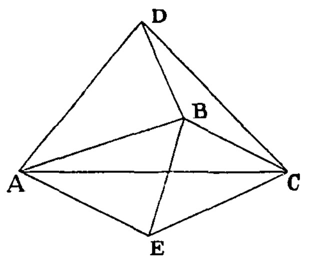

Fig. 7.

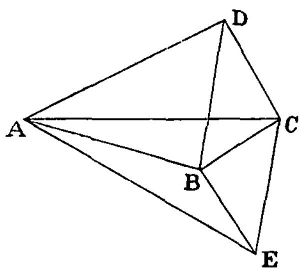

Fig. 8.

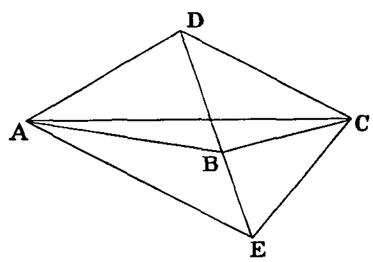

Fig. 9.

appearance of a pyramid, having A for its apex, and the quadrangle DBEC for its base. Thus the flatness of a tetragon may be secured by connecting each of its corners with a fifth point not in the same plane.

Moreover, the system still remains rigid although the points D and E be both in one plane with AB also. In this case DBE, the meeting of two planes, must be a straight line, as shown in fig. 9. Thus we see that, for the establishment of three points in a straight line, two auxiliary points must be introduced, with seven additional linear members.

We have now got a possible straight lever DBE. In order to examine the law of the balancing of pressures applied at B, D, E, we must trace out the strains on the various members, and their equilibriums at the five junctions, subject to the condition that there be no external pressure at A or at C. The result of this examination is, that the strains on the members are eliminated; that the directions of the applied pressures must all pass through one point;

and that their intensities must be proportional to the sines of the opposite angles,—this result being independent of the positions of the auxiliary points A and C.

Does it thence follow that we may omit those points altogether? Assuredly not; for our whole investigation proceeded on the ground that each member transmits from its one end to its other end the strain with which it is accredited.

Each additional point needs for its establishment three new linear members, so that in any self-rigid open structure, if n be the number of the points, there must be $3n-6$ linear connections; this formula failing only in the extreme case, $n=2$.

Hitherto we have been considering the self-rigidity of structures, and may now proceed to treat of the laws of stability in relation to the ground, taking first the case of a self-rigid structure to be kept firmly in position.

In every case the support must be derived from points in the ground, which points necessarily form by themselves a rigid structure, so that our problem assumes the general form of "how to connect one rigid structure with another." If f be the number of the points in the foundation, and n that of those in the supported structure, we have in all $f+n$ points in the compound, which must clearly be self-rigid. Hence the total number of linear members must essentially be $3f+3n-6$. But of these $3f-6$ are virtually included in the foundation, wherefore the number of the members above ground must in all possible cases be $3n$. Of these, however, $3n-6$ are already included in the supported structure, and thus we arrive at the important general law, "that the number of linear supports must be neither more nor less than six when the supported structure is self-rigid." This most elementary of the laws of support seems almost to be unknown, the enunciation of it takes even professional engineers by surprise.

All our portable direction-markers, our theodolites, alt-azimuths, levelling telescopes, have to be supported above the ground at a height convenient for the eye. It is essential that the stationary part of each be firmly held; yet in every case, with not one exception in the thousand, our geodetical instruments are set upon three slender legs, diverging almost from a point. In such an arrangement the steadiness in direction is derived exclusively from the stiffness of the legs, which, however, are very flexible. The well-known result is, that any strain in handling the instrument, even the pressure of a slight breeze, deranges the reading.

More than fifty years ago an instrument maker in London, Robinson by name, placed his beautifully made little alt-azimuths on a new kind of stand.

He connected three points in the stationary part of the instrument with three points in the ground by means of six straight rods inclined to each other. In this arrangement, the stability in every direction is derived from the resistance to longitudinal compression, the flexure of the rods having an infinitesimally small influence. It takes essentially the form of the octahedron or sixnib, as in fig. 10; a self-rigid structure having six connected points.

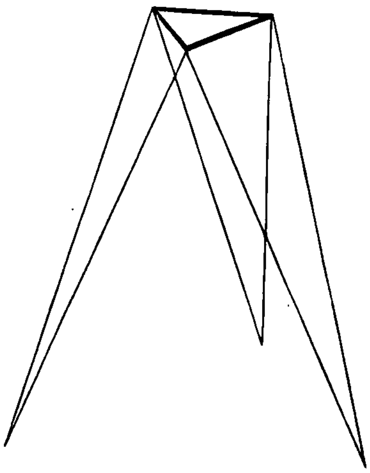

Fig. 10.

This beautiful Robinson stand keeps the alt-azimuth so firmly in position that, even during a heavy galė, the image of the moon may be seen to move without tremour across the cobwebs of the field-bar. Yet it has not been adopted by engineers and surveyors; the exigencies of the photographer, however, have determined his recourse to it.

It is not essential that the supports meet two and two as in this arrangement; they may be quite detached, connecting six points in the supported structure with six points in the ground; but in all cases they must be so disposed that any dislocation whatever would imply a change in the length of some of them.

It might have sufficed merely to remark that this condition excludes the parallelism of the supports; but it is expedient to insist, seeing that, in the deplorable case of the Tay Bridge, the fabric was set upon two rows of upright columns. The opinion is still held that the effective base is equal to the whole breadth of such a structure, whereas the most casual examination may show that, no matter how broad the structure may be, its effective base is only that of a single column.

When the superstructure is not rigid in itself, or indeed whether it be so or not, the entire number of the members above the ground must be thrice that of the supported points. If we attempt to do with fewer the fabric must fall; if we place more we cause unnecessary internal strains. I hope in a subsequent paper to treat of redundancy, meantime our attention may be confined to structures having the proper number of parts.

If the supported points belong to one system they must be mutually connected, and, at the least, there must be as many of these connections as there are points, less one, wherefore the number of supports can never exceed twice the number of the points by more than one.

Out of the endless variety of cases we may select one class for examination, that in which the supported points are connected so as to form a polygon, not necessarily all in one plane. The number of the connections being already n, that of the supporting members must be $2n$, which may rest on $2n$ separate points in the ground, but which may be brought together in pairs or otherwise. If they be placed in pairs there are as many supporting as supported points.

Robinson's octahedral stand shows this arrangement when there are three supported points; we shall now take the case when four points are supported from four points in the ground, as in fig. 11, where the connected points A, B, C, D are shown as supported by the eight members AE, EB, BF, FC, CG, GD, DH, HA.

In general, that is when there is no regularity, such a structure contains all the elements of stability. The positions of the foundation points being known, if the lengths of the twelve members be prescribed, we shall have twelve equations of condition whereby to compute the twelve co-ordinates of the four points A, B, C, D. Or, viewing the matter from the mechanical side, external pressures applied at A, B, C, D may be resolved into their elements in three assumed directions, x, y, z, and so may the stresses on the various parts; there must be equilibrium at each of the points, in each of the three directions, and so again we have twelve equations whereby to compute twelve unknown quantities.

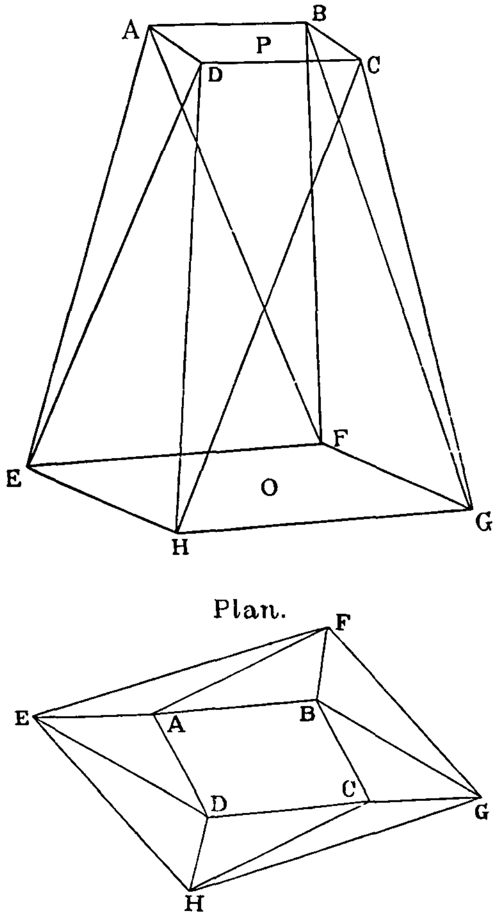

Fig. 11.

The algebraist at once perceives that the resulting divisor (or determinant as it is called) may happen to be zero, in which case the stress becomes infinite; that the dividend may be zero, showing that the particular member has no strain upon it; or even that both the dividend and the divisor may be zero at once, showing the structure to be indeterminate. But such investigations distract the attention from the objects under consideration to their mere representative symbols, and do not carry intellectual conviction along with them. Their true and highly important office is to determine accurately the various stresses, thereby enabling the constructor to apportion the strengths of the various parts.

The determinateness, that is the stability, of a structure typified by fig. 11 ceases when we introduce symmetry or even semi-regularity. Let, for example, the figures EFGH, ABCD be rhomboids, having their middle points O and P in the same vertical line, in which case the opposite members are of equal lengths, AE to CG; EB to GD, and so on. Such a structure is clearly instable.

Since the lengths HA, AE are fixed, the point A must be on the circumference of a circle, having HE for its axis of rotation; and similarly for the points B, C, D. If now we suppose the point A to be pushed inwards, the members AB, AD will push the points D and B outwards, and, consequently, C will move inwards by exactly as much as A; the structure will adapt itself perfectly to its new position. In truth, we have here not twelve, we have only eleven data; for, if one of the connections, say CD, were removed, and the structure thus made obviously mobile, the distance CD would yet remain always equal to AB; that distance cannot be reckoned among the data.

So much for the geometrical mobility; let us examine the strains. Any horizontal pressure at A is decomposable into two,—one in the direction AB, the other in the direction AD. The stress $\overline{AB}$ transferred to the point B may again be decomposed into two; the one of these in the plane EBF parallel to EF is completely resisted at E and F by the stresses $\overline{EB}$, $\overline{EF}$, but the other, perpendicular to EF, meets with no resisting obstacle; it and the corresponding pressure at D may be counteracted by extraneous pressure there, or by a single pressure applied at C, equal to the pressure at A, and in the same direction with it. Thus the distortion of the fabric by an eastward pressure at A is prevented by a like pressure applied at C, not westward, but eastward also; in respect, however, to the strains on EB, BF, HD, DG, the effects of these counteracting pressures are cumulative.

These considerations would seem to warrant the conclusion that all structures of this class are necessarily unstable; however, before venturing to accept of this conclusion, it may be prudent for us to inquire whether the arguments on which it is founded be strong enough to bear such a weighty superstructure. Now the chief argument was that the longitudinal stress on AB, acting at B, tends to turn the triangle EBF on EF as an axis; but this tendency exists only so long as AB is out of the plane EBF, and ceases whenever AB comes to be in that plane; in other words, whenever AB is parallel to EF. Hence it follows that structures, represented in plan by fig. 12, having the two rhomboids ABCD and EFGH placed conformably, are rigid. The conclusion was not absolutely general.

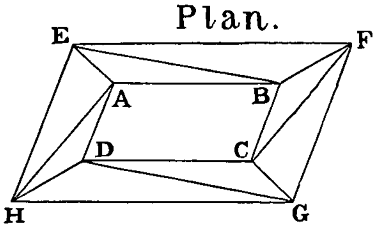

Fig. 12.

Though every rhomboid be not a rectangle, every rectangle is a rhomboid, and we might hastily thence conclude that these remarks concerning rhomboidal structures may be at once extended to rectangular ones. But we have just come from seeing an example in which peculiarity precludes generalisation, and thus it is expedient for us to examine specifically the case of rectangular structures.

The examination at once shows that the remarks made in regard to figs. 11 and 12 apply when the rhomboids pass into the form of rectangles; but the rectangle is symmetric, while the rhomboid is not so; the arrangement of the diagonals, as shown in these figures, is unsymmetric, and it thus remains for us to inquire into the laws of symmetry.

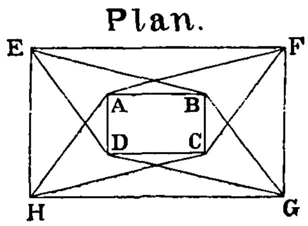

Fig. 13.

If, as shown in fig. 13, the rectangle ABCD be placed vertically over and conformably with EFGH, the arrangement is symmetric; it has already four out of the requisite twelve members, and the eight supports remain to be placed.

These may be inserted symmetrically as the eight diagonals, HA, AF, FC, CH; DE, EB, BG, GD, but then the structure becomes perfectly mobile; the points A and C move towards or from the central axis, while B and D move from or towards it.

In any other symmetric arrangement the four corner parts, EA, FB, GC, HD, must appear, leaving four members yet to be distributed. If one of these be placed as the diagonal AF, symmetry requires also DG, BE, and CH, as shown in fig. 14.

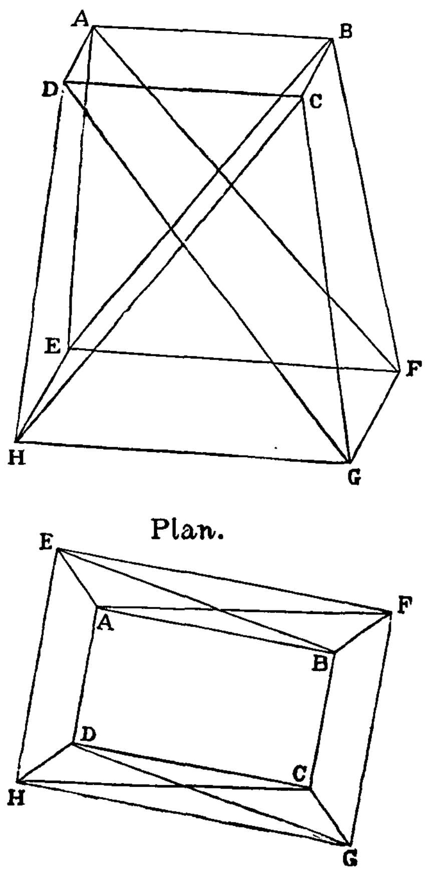

Fig. 14.

The insecurity of this arrangement is obvious at a glance; no more need to have been said about it, but for its adoption in the scheme for the central towers of the proposed Forth Bridge. It presents two instances of that most vicious arrangement, the flattened tetrahedron; vicious because, while incapable of resisting any pressure not directed in its own plane, such a structure as EABF converts any twisting pressure into indefinitely exaggerated stress. It also presents two attempts to determine the shape of a quadrangle by the lengths of the four sides. Were the three open figures, EFBA, ABCD, DCGH, replaced by flat rigid plates, we should have, turning on the four parallel hinges, EF, AB, DC, HG, a most familiar example of instability.

Among open structures built on a rectangular base, instability is not confined to those with rhomboidal tops; for if, as in fig. 15, the triangle HAE be set up equal to GCF and EBF to HDC, so that AC may be parallel to EF and BD parallel to FG, the structure is movable. The diagonals AC and BD may be on one level and so cross each other, or the one may pass above the other at a distance on the plumb line OP. Since the three lines, AO, OP, PB, are mutually perpendicular—

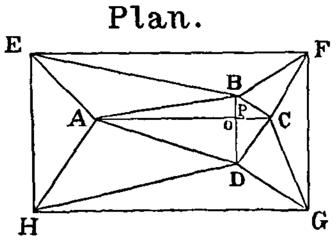

Fig. 15

$$AB^2 = AO^2 + OP^2 + PB^2$$

and $$CD^2 = CO^2 + OP^2 + PD^2;$$ wherefore

$$AB^2 + CD^2 = AO^2 + OC^2 + BP^2 + PD^2 + 2.OP^2.$$

But the sum of BC^2 and AD^2 is equal exactly to the same quantity, and consequently

$$AB^2 + CD^2 = BC^2 + DA^2,$$

so that one of these four is deducible from the remaining three; there are then only eleven data in this structure, instead of the twelve needed for rigidity. But it is to be observed that a dislocation must change the horizontality of AC and BD, so that the mutability may be only instantaneous, as in the case of maximum or minimum.

Passing now to the case of five supported points, we may remark that, by the introduction of a fifth point, a symmetric rigid structure may be built on a rectangular base.

Thus, if we place, as in fig. 16, the rhombus ABCD vertically over the rectangle EFGH, and complete the construction as in fig. 11, there results a symmetric structure, which, like all those of the same class, is changeable. On assuming, however, a point Z in the vertical axis of the system, and connecting it with each of the points A, B, C, D, we get a fabric both symmetric and rigid. The rigidity is confirmed thus:—If, supposing Z and its connections to be away, the points A and C be brought nearer, B and D would move apart; now, in virtue of the connections AZ, ZC, the shortening of AB would cause Z to rise, while, in virtue of the connections, BZ, ZD, the widening of BD would bring Z down; the opposition of these two tendencies keeps Z in its place.

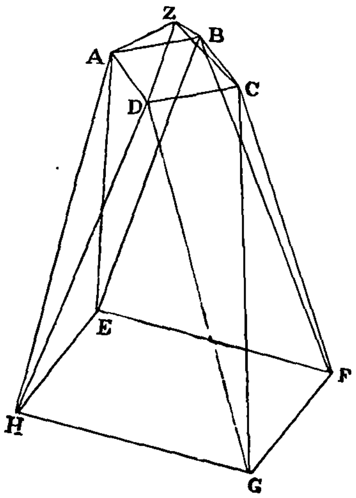

Fig. 16.

Here, in order to support five points, sixteen members are conjoined, and yet there does not seem to be any redundancy; moreover, the arrangement is quite symmetric. The equilibrium at each point gives rise to three equations of condition, and these fifteen equations cannot possibly serve to determine sixteen strains. But if we apply a pressure at any one of the five points, the fabric resists it, the various members are strained somehow, the law of equations notwithstanding. The explanation of this paradox may afford an instructive exercise to the student.

When the five points are arranged in the corners of a pentagon, each being carried by two supports, as shown in plan by fig. 17, the structure is rigid, provided the polygons be convex. Of this we easily convince ourselves by supposing one of the connections, say EA, to be removed, and by examining the motion of the link system thus left. The point A can move only in a circle, having KF for its axis; let A be moved inwards, the member AB will then cause the triangle FBG to turn outwards on FG as a hinge; BC will draw C inwards, CD will push D outwards, and lastly, DE will draw E inwards; wherefore the distance AE will be shortened, and the member AE can be replaced only when the structure is brought back to its former position.

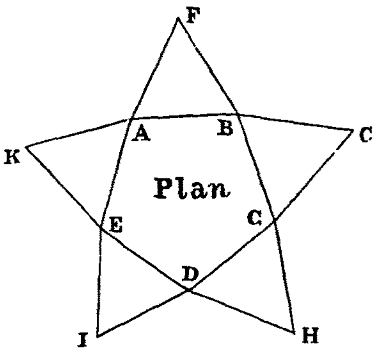

Fig. 17.

Following this line of argument one step further, we see that in the case of a hexagon the first and last points would move, the one outwards, the other inwards, and that so the distance might remain unchanged. When the hexagons are semi-regular or halvable, the distance remains absolutely unchanged, and the structure is indifferent as to position. This same remark applies to all polygons of an even number of sides.

If, however, the upper and lower polygons be placed conformably, as in fig. 18, the structure is rigid, whether the number of supported points be even or odd.

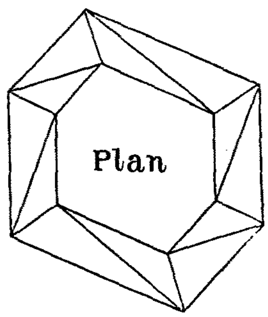

Fig. 18.

These truths may be illustrated experimentally by preparing a few isosceles triangles as AFB, having perforations at A and B, through which an elastic string may be passed. On connecting a number of these, say seven, by a continuous thread, and spreading them out on a table, we form a flexible equal-sided heptagon, and when this is arranged regularly the points F are in the corners of a larger regular heptagon. If we now trace on a flat board a regular seven-sided figure, intermediate in size between these two, and secure the points F of the triangles in holes made at the corners, we shall have erected a structure analogous to that shown in fig. 17, and shall find it to be rigid.

If one of the triangles be removed, and the same process of construction followed with the remaining six, the resulting regular hexagonal structure is found to be instable.

Another reduction of the number brings us to the pentagonal structure, which again is stable; and still another removal gives the tetragonal instable fabric; and, lastly, when only three triangles are left, we have Robinson's octahedral stand.

The important distinction between the two cases of conformable or of unconformable polygons may be illustrated by preparing two pairs of triangles, one pair as EAB, GCD, of fig. 12, the other pair as FBC, HDA, and by connecting the sides, AB, BC, CD, DA, so as to form a flexible tetragon.

When the feet, E, F, G, H, are secured in the corners of a rhomboid or of a rectangle, the structure is rigid, if AB, BC be parallel to EF, FG; in all other cases it is instable.

These cases of instability in open structures have been elicited by means of the simplest considerations in Geometry and Statics; they lie indeed on the very surface of mechanical inquiry. They do not occur as isolated examples—they are arranged in extensive groups; and, being found in those classes of structures which may be called shapely, they stand out as warning beacons to those engaged in engineering pursuits.