LXIV. Supplement to Mr. Herapath's paper in the Philosophical Magazine for August 1825, on functional equations

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three rectangular coordinates of a sphere, that is, of the quantities,

$$\mu, \sqrt{1 - \mu^2} \cos \varpi, \sqrt{1 - \mu^2} \sin \varpi;$$

then, r being the radius of a stratum, we shall have,

$$r = a \{ 1 + e \phi^{(2)} + e^2 \phi^{(4)} + e^3 \phi^{(6)} + \&c. \},$$

e being a quantity according to which the steps of approximation are to be arranged. This general expression supposes that the central nucleus is an ellipsoid; but if it be a spheroid of revolution, then we must make $\phi^{(2)}$ equal to μ^2 , or $1 - \mu^2$; and $\phi^{(4)}$, $\phi^{(6)}$ &c. must be trinomial, quadrinomial, &c. functions of μ^2 . The quantity e , the density of the stratum, and the coefficients of all the angular quantities are functions of a ; and they must be determined so that a stratum shall satisfy the conditions of equilibrium, and, when it comes to the centre, shall rest upon the solid nucleus. In this manner we shall have the most general solution of the equilibrium of a heterogeneous mass of fluid, pushed to any proposed degree of approximation.

I am, &c.

June 14, 1826.

J. IVORY.

LXIV. *Supplement to Mr. HERAPATH'S Paper in the Philosophical Magazine for August 1825, on Functional Equations.*
By JOHN HERAPATH, Esq.

To the Editor of the Philosophical Magazine and Journal.

Sir,

FROM the period of sending my last communication to the Phil. Mag., my mind has been so much estranged from the consideration of functions, that I might almost be said to have entirely abandoned them. This is the reason why so long a time has elapsed without my having noticed the omission of a restriction, which, on reperusing the paper alluded to a few days since, I was surprised to find I have no where distinctly mentioned. It should have been stated, that *the arguments of the paper relate to those periodical functional equations only, whose solutions contain arbitrary functions.* We may indeed understand this from "*arbitrary functions,*" "*complete solutions,*" &c. in the heading of the paper; but as there are an infinite number of periodical functional equations, having their solutions complete, without containing arbitrary functions, and therefore wanting the properties which render our reasoning applicable, I hasten to supply the omission.

As

As an example will best illustrate the propriety and necessity of the above restriction, let us take our old equation

$$\psi x = fx \cdot \psi ax + f_1 x, \quad (1)$$

supposing the only condition for the present to be $\alpha x = x$, and leaving fx and $f_1 x$ entirely unlimited. By changing in (1) x into αx and eliminating between the resulting equation and (1) $\psi \alpha x$, there is obtained

$$\psi x = \frac{f_1 x + fx \cdot f_1 \alpha x}{1 - fx \cdot f \alpha x}; \quad (2)$$

which is the complete solution of (1) without an arbitrary function. To prove this, let us add some function ϕx to the solution (2), which for brevity we denote by Px ; and it is evident that if Px be not the complete solution, ϕ may have such a form that

$$Px + \phi x$$

shall contain it, or at least some one of the other solutions. Substitute $Px + \phi x$ and $P\alpha x + \phi \alpha x$ for ψx and $\psi \alpha x$ in (1), and we get

$$Px + \phi x = fx \cdot P\alpha x + fx \cdot \phi \alpha x + f_1 x;$$

$$\text{or since by (2) } Px = fx \cdot P\alpha x + f_1 x$$

$$\phi x = fx \cdot \phi \alpha x.$$

Changing in this equation x into αx it becomes $\phi \alpha x = f \alpha x \cdot \phi x$, and of course gives

$$\phi x = fx \cdot f \alpha x \cdot \phi x;$$

an equation in which, if fx be as we suppose it unlimited, ϕx must be null, and consequently Px or (2) is the complete solution. But if ϕx be assumed to have a real value, then must $fx \cdot f \alpha x = 1$, and as a consequence by (2) $f_1 x = -fx \cdot f \alpha x$; which are what I have termed in my above-cited paper the *conditions of possibility*.

The trouble of working out and printing the solution of a periodic of a higher order than the second, is the only reason why I do not here avail myself of it in corroboration of the position advanced.

From what has been shown, there is manifestly a marked distinction between periodical equations considered indefinitely, and those of which I have treated in my paper. In the former, single solutions flow from the given functions being indefinite; in the latter, innumerable solutions are the consequences of certain restrictions in the functions proposed. For the sake of distinction I shall hereafter call the former simply *periodic functions*, and the latter *porismatic periodical functions*.

It would be hazardous in subjects so very general to pronounce

nounce positively, that the above distinction is universally true. Possibly there may exist certain equations, perhaps partially indefinite, which may contain in their solutions functions partially arbitrary, or arbitrary between certain limits; but I confess I do not know of any such.

Though Mr. Babbage and others have solved several equations, both simple and porismatic, I do not know that this essential line of distinction has occurred to any of them. As a proof, however, of the importance of well understanding the difference between these two species of equations, I may observe, that notwithstanding their solutions are obtained by means so very similar, yet the solution of neither species, as it appears to me, contains nor is contained in that of the other. For example (2), which is the solution of (1) when fx, f_1x are indefinite, neither contains nor is contained in the solution of the same (1), when this equation is limited to the conditions of possibility, namely $fx \cdot fax = 1$ and $f_1x = -fx \cdot fax$.

By the above considerations the solution of

$$f\{x, \psi x, \psi ax, \psi a^2x, \dots\} = 0, \quad (3)$$

taken generally, has but one form, not accounting of course for any thing those changes which the double signs of radical quantities may occasion. This single solution is in general to be deduced from the elimination of $\psi ax, \psi a^2x, \dots$ between the equations which result from the successive substitution of $ax, a^2x, \dots$ for x in the primitive equation.

The same observations extend to

$$f\{x, \psi x, \psi^2x, \dots\} = 0. \quad (4)$$

For since $\psi^r x$ can always be equated with $\phi \alpha^r \phi^{-1} x$, in which α may have any given form at pleasure, provided only that α be confined to the same order of periodicity as ψ is, we may transform, by putting $\phi \alpha \phi^{-1} x, \phi \alpha^2 \phi^{-1} x, \dots$ for $\psi x, \psi^2 x, \dots$ and then changing x into ϕx , our (4) into

$$f\{\phi x, \phi \alpha x, \phi \alpha^2 x, \dots\} = 0,$$

in which ϕ is the form to be determined. This equation is similar to (3), and consequently in a general point of view limits ϕ , and hence likewise ψ , to one form. It is obviously essential in this solution that we have the order of periodicity of ψ given, otherwise the problem cannot generally be solved; and that α must be of precisely the same order. The gentleman who first treated of the solution of (4) seems not to have included the order of periodicity of ψ amongst the necessary data; and seems to have imagined that if αx be not taken $= x$ its order is not of great consequence, as it would merely
limit

limit the generality of the solution. It was, however, not then known that an indefinite form to f gives but one solution, which therefore cannot be further limited.

Cranford.

J. HERAPATH.

P. S. Since writing the above, I have turned my attention to the solutions of non periodical functional equations and other subjects connected with them; in which, with the help of an improved notation, I have met with a degree of success beyond what my most sanguine expectations could have anticipated. Indeed, if the views which have opened to me, should turn out as prolific as they seem to promise to be, they will entirely new face most of the instruments of analysis, and will leave but little to desire except the perfection of common algebra.

LXV. *On the Ignition of Gunpowder by the Electric Discharge; and on the Transmission of Electricity through Water, &c.* By Mr. W. STURGEON.

To the Editor of the Philosophical Magazine and Journal.

Sir,

IT is generally admitted, that the present state of knowledge relative to the phenomena of electricity is enveloped in much obscurity; and perhaps no instance of electrical action manifests our ignorance of this branch of science more, than that of igniting gunpowder. Yet, so little notice is taken of this isolated fact, that no satisfactory attempt at explanation, that I am aware of, has ever appeared in the pages of any writer on this subject.

That gunpowder has frequently, by various individuals, been ignited by the electric fluid, is a truth that cannot be denied. But *why* those experimenters happened to succeed, and *why* others so frequently and still more constantly *fail*, are circumstances the cause of which has hitherto been left unexplained,—perhaps not understood. I am well convinced that no individual experimenter has been more embarrassed than myself, by fruitless attempts to ignite gunpowder by the electric fluid: and although I have varied the experiment according to all the directions I could either read or hear of, yet I candidly confess that I never succeeded by any of them.

Fruitless as these experiments were with respect to the object in view, it was observed, by passing the discharge of a jar through water (which is the method given by some authors), that the force of the shock is considerably abated; and that the report
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