A GRAPHICAL METHOD OF DETERMINING THE P CORRECTION IN THE DEFLECTION EXPERIMENT.

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Some time ago, in connection with some experiments on the question of suitable relation between the deflecting and the deflected magnets for use in the measurement of H , it was necessary to calculate the value of the P corrections for a number of observations. This calculation, if we adopt the ordinary expression, or even the approximate one given by Rücker and Thorpe,¹ takes some little time, and the following graphical method was devised. Although it is so simple that it is very unlikely that it has not already been used, yet, since I never remember having seen it described, it may be of some interest to readers of "Terrestrial Magnetism."

The expression connecting the magnetic moment M of the deflecting magnet when placed at a distance r , the value of H and the angle θ through which the instrument is turned may be put in the following form:

$$\frac{H}{M} = \frac{2}{r^3} \left(\operatorname{cosec} \theta (1 - qt - q't^2 - \frac{2\mu}{r^3}) \left(1 + \frac{P}{r^2} \right) \right)$$

in which t is the temperature reckoned from some standard temperature, q and q' are, respectively, the first and second temperature coefficients of the deflecting magnet, μ , the induction coefficient, and P the distribution coefficient, the relative dimensions of the deflecting and deflected magnet being such as to make the succeeding distribution coefficients, Q , R , etc., negligible.

However, for simplicity of calculation, it is usual to make use of the approximation

$$\left(1 + \frac{P}{r^2} \right)^{-1} = 1 - \frac{P}{r^2},$$

so that

$$\frac{H}{M} \left(1 - \frac{P}{r^2} \right) = \frac{2}{r^3} \operatorname{cosec} \theta \left(1 - qt - q't^2 - \frac{2\mu}{r^3} \right) \quad (1)$$

In obtaining this expression, powers of $\frac{P}{r^2}$ above the first have been neglected, and hence it is useless to employ a formula for deducing P which takes account of higher powers of $\frac{P}{r^2}$ than the first.

Now if A and A_1 are the reciprocal values of the right hand side of equation (1) above for two values of r and r_1 , r being the shorter distance, the ordinary expression for calculating P is:

$$\frac{A}{A_1} = \frac{1 - \frac{P}{r_1^2}}{1 - \frac{P}{r^2}}$$

¹ Phil. Trans. 1889 A, p. 71.

Hence,

$$\begin{aligned}\log_{10} A - \log_{10} A_1 &= \log_{10} e \left[\log_e \left(1 - \frac{P}{r_1^2} \right) - \log_e \left(1 - \frac{P}{r^2} \right) \right] \\ &= \log_{10} e \cdot P \left(\frac{1}{r^2} - \frac{1}{r_1^2} \right),\end{aligned}$$

in which we have neglected powers of $\frac{P}{r^2}$ above the first.

Or writing the result thus:

$$\frac{\log_{10} A - \log_{10} A_1}{P} = \log_{10} e \left(\frac{1}{r^2} - \frac{1}{r_1^2} \right) \quad (2)$$

That is, if the two distances at which the observations are made are kept constant, so that $\frac{1}{r^2} - \frac{1}{r_1^2}$ is constant, we have that there is a linear relation between $(\log A - \log A_1)$ and P , so that if by any method we calculate a value of P , which gives the correct value for the expression $(\log A - \log A_1)$, then the value so obtained must be the correct one.

Suppose now we plot a curve in which the ordinates are the values of P , and the abscissae the corresponding values of $(\log A - \log A_1)$. Since (2) shows that this curve will be a straight line, it will only be necessary to calculate two points, or, if we make use of the fact that when P is zero $(\log A - \log A_1)$ is also zero, only one point. This curve which is represented by the straight line OA in the figure will enable us to immediately find P when we know the value of the difference between the values of $\log A$, which is obtained in the process of reducing the observations.

What is really required in reducing the observations for the value of H is not, however, P , but the values of $\log (1 - P/r^2)$ for the two distances employed. Now to the approximation we are using, we have

$$\log_{10} \left(1 - \frac{P}{r^2} \right) = - \log_{10} e \cdot \frac{P}{r^2} = c P, \quad (3)$$

where c is a constant for any particular value of r —thus there is a linear relation between $\log (1 - P/r^2)$ and P . Hence we may draw a straight line to represent the connection between these two quantities. This line is represented in the figure by BD for $r = 30$ cm., and by CD for $r_1 = 40$ cm. Thus by means of these two lines we can immediately find the values of $\log (1 - P/30^2)$ and $\log (1 - P/40^2)$ if we know the value of P , this value being obtained from the line OA .

By drawing these lines on a sheet of squared paper, the distance between the parallel lines being a tenth of an inch, and taking one inch to represent one unit in P and 50 units in the fifth place of decimals in the logarithms, it is quite easy to read off the values of the quantities $\log (1 - P/30^2)$ and $\log (1 - P/40^2)$ correct to a unit in the fifth place.